Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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The individual ergodic theorem on IF events

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Abstract

In this paper the individual ergodic theorem for MV algebras is applied on the set of all IF events $\mathcal{F}$. We use the embedding $\mathcal{F}$ into the corresponding MV algebra and we show that this MV algebra has necessary properties.

Keywords: IF event, MV algebra, ergodic theorem.

1 Introduction

The concept of Intuitionistic Fuzzy Sets (IFS) was proposed by ATANASSOV [2] in 1983. It is an extension of the well-known notion of fuzzy set defined by ZADEH.

Every element a of an IFS set has:
- a degree of membership: $\mu : a \to \langle 0, 1\rangle$
- a degree of non-membership: $\nu : a \to \langle 0, 1\rangle$

The sum of the two degrees have to be less than 1: $\mu(a) + \nu(a) \leq 1$.

In this text we will work with IF event, which is a special kind of the IF set. It is introduced in the following definition.

Definition 1 *Let (Ω, S) be a measurable space. By an IF-event we mean any pair $A = (\mu_A, \nu_A)$ of S-measurable functions, such that $\mu_A \geq 0, \nu_A \geq 0$ and $\mu_A + \nu_A \leq 1$.*

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The function μ_A is the membership function and the function ν_A is the non-membership function. The family $\mathcal{F}$ of all IF events is ordered by the following way:

$$A \le B \Leftrightarrow \mu_A \le \mu_B, \nu_A \ge \nu_B.$$

Evidently the notion of an IF-event is a natural generalization of the notion of the fuzzy events. Given a fuzzy event μ_A, the pair $(\mu_A, 1 - \mu_A)$ is an IF event, so IF events can be seen as generalizations of fuzzy events.

The mapping 1_Ω is denoted the function, for which the image of each element from Ω is 1. Similarly 0_Ω maps each element from Ω to 0. The binary operations on $\mathcal{F}$ can be defined by this way:

$$\begin{aligned} A \oplus B &= ((\mu_A + \mu_B) \wedge 1_\Omega, (\nu_A + \nu_B - 1_\Omega) \vee 0_\Omega), \\ A \odot B &= ((\mu_A + \mu_B - 1_\Omega) \vee 0_\Omega, (\nu_A + \nu_B) \wedge 1_\Omega). \end{aligned}$$

These operations are called Lukasiewicz connectives.

The smallest element in this structure is $(0, 1)$ and the largest is the element $(1, 0)$.

The aim of this paper is apply the results on MV algebras for the set of all IF events. As first we show, how we can the set of all IF events embedded to the MV algebra with product.

Let $(\Omega, \mathcal{S})$ be a measurable space, let $\mathcal{F}$ be the set of all IF event over Ω.
Let M is the set

$$M = \{A = (\mu_A, \nu_A);\ \mu_A, \nu_A : \Omega \to [0, 1];\ \mu_A, \nu_A \quad \texttt{are} \quad \mathcal{S}\texttt{-measurable}\}.$$

Then $\mathcal{M} = (M, 0_M, 1_M, ^*, \oplus, \odot, \cdot)$ is MV algebra with product, where $1_M = (1_\Omega, 0_\Omega)$, $0_M = (0_\Omega, 1_\Omega)$, $A^* = (1_\Omega - \mu_A, 1_\Omega - \nu_A)$ and binary operation product is defined by this equality

$$A \cdot B = (\mu_A \mu_B, 1_\Omega - (1_\Omega - \nu_A)(1_\Omega - \nu_B)).$$

The set of all IF event $\mathcal{F}$ is embedding to the corresponding MV algebra

$$M = \{A = (\mu_A, \nu_A);\ \mu_A, \nu_A : \Omega \to [0, 1];\ \mu_A, \nu_A \quad \texttt{are} \quad \mathcal{S}\texttt{-measurable}\}.$$

2 Probability on IF-events

We want to work with the probability on IF event. So we need define two important mappings the state and the observable.

Definition 2 *The state on the set of all IF events $\mathcal{F}$ is the mapping $m : \mathcal{F} \to [0,1]$ if satisfies the following conditions:*

1. $m(1_\Omega, 0_\Omega) = 1$, $m(0_\Omega, 1_\Omega) = 0$;
2. *if $A \odot B = (0_\Omega, 1_\Omega)$, then the following equality holds:*
$$m(A \oplus B) = m(A) + m(B);$$
3. *let $\{A_n\}_{n=1}^{\infty}$ be the sequence from the set $\mathcal{F}$ and hold $A_n \nearrow A$, then $m(A_n) \nearrow m(A)$.*

This mapping we illustrate on the following example.

Example 1 *The mapping m is defined:*

$$\forall A \in \mathcal{F} : m((\mu_A, \nu_A)) = \frac{1}{2}\left(\int \mu_A dP + 1 - \int \nu_A dP\right).$$

We proved that this mapping has a properties of a state.

1. $m((1_\Omega, 0_\Omega)) = \frac{2}{2} = 1, \quad m((0_\Omega, 1_\Omega)) = \frac{0}{2} = 0.$
2. *Let for $A, B \in \mathcal{F}$ hold $A \odot B = (0_\Omega, 1_\Omega)$ then: $\mu_A + \mu_B - 1 \leq 0$ and $\nu_A + \nu_B \geq 1$. We get the following equality with using previous inequalities:*
$$m(A \oplus B) = m((\mu_A + \mu_B) \wedge 1_\Omega, (\nu_A + \nu_B - 1_\Omega) \vee 0_\Omega) = m((\mu_A + \mu_B), (\nu_A + \nu_B - 1_\Omega)) = \frac{1}{2}\left(\int \mu_A + \mu_B dP + 1 - \int \nu_A + \nu_B - 1_\Omega dP\right) = \frac{1}{2}\left(\int \mu_A + \mu_B dP + 2 - \int \nu_A + \nu_B dP\right) = \frac{1}{2}\left[\left(\int \mu_A dP - \int \nu_A dP + 1\right) + \left(\int \mu_B dP - \int \nu_B dP + 1\right)\right] = m(A) + m(B).$$
3. *We have $(\mu_{A_n}, \nu_{A_n}) \nearrow (\mu_A, \nu_A)$ then $\mu_{A_n} \nearrow \mu_A, \quad \mu_{B_n} \searrow \mu_B$ and holds:*
$$\int \mu_{A_n} dP \nearrow \int \mu_A dP, \quad 1 - \int \mu_{B_n} dP \nearrow 1 - \int \mu_B dP.$$
With using these properties we have:
$$m(\mu_{A_n}, \nu_{A_n}) = \frac{1}{2}\left(\int \mu_{A_n} dP - \int \nu_{A_n} dP + 1\right) \nearrow \frac{1}{2}\left(\int \mu_A dP - \int \nu_A dP + 1\right) = m(\mu_A, \nu_A).$$

Now we use the embedding $\mathcal{F}$ to the corresponding MV algebra $\mathcal{M} = (M, 0_M, 1_M, ^*, \oplus, \odot)$, where M is a set

$$M = \{A = (\mu_A, \nu_A);\ \mu_A, \nu_A : \Omega \to [0,1];\ \mu_A, \nu_A \ \ \texttt{are} \ \ \mathcal{S}\texttt{-measurable}\}.$$

The following theorem enable us to apply the results on MV algebra to the set of all IF event.

Theorem 1 *To each state $m' : \mathcal{F} \to [0,1]$ exists exactly one state $m : \mathcal{M} \to [0,1]$ with the property $m|\mathcal{F} = m'$.*

Proof: [Theorem 2, [14]].

The observable in the probability theory of IF events play the same role as the random variable in the classical probability space. The observable satisfies the condition $x(C) \odot x(D) = 0_{\mathcal{M}}$, whenever the borel sets C and D are disjunct. This condition guarantee us, that if C and D are disjunct and $x(C), x(D) \in \mathcal{F}$, then $x(C) \oplus x(D) \in \mathcal{F}$.

Definition 3 *The observable on the set of all IF events $\mathcal{F}$ is called a mapping $x : \mathcal{B}(R) \to \mathcal{F}$ if satisfies the following conditions:*

1. $x(R) = (1_\Omega, 0_\Omega)$,
2. *if $C, D \in \mathcal{B}(R),\ C \cap D = \emptyset$, then holds $x(C) \odot x(D) = (0_\Omega, 1_\Omega)$ and $x(C \cup D) = x(C) \oplus x(D)$,*
3. *if $C, C_n \in \mathcal{B}(R),\ C_n \nearrow C$, then holds $x(C_n) \nearrow x(C)$.*

It holds $\mathcal{F} \subset M$, so each observable $x : \mathcal{B}(R) \to \mathcal{F}$ is the observable in the sense of probability theory on MV algebras, so it is an observable on the corresponding MV algebra $\mathcal{M}$ (Definition). We recall, that for the invariance of Cesaro mean is the other property of the observable required. We will work with P-observable, that is the observable, which satisfies the condition

$$\forall C, D \in \mathcal{B}(R) : x(C \cap D) \leq x(C) \cdot x(D).$$

We define two important mappings m, x, which we use on making probability measure $m_x : M \to M$:

$$\forall A \in \mathcal{F} : m_x(A) = m(x(A)).$$

We will work with an integrable observable. Now we define the expected value of an observable on $\mathcal{F}$.

Definition 4 *The expected value $E(x)$ of the observable x on $\mathcal{F}$ is defined by the equality:*

$$E(x) = \int_R t dm_x(t),$$

if the integral exists.
If the observable has the expected value, then it is called integrable observable.

Definition 5 *Let x, y be observables on $\mathcal{F}$. The joint observable is the mapping $h : \mathcal{B}(R^2) \to M$ with properties:*

1. $h(R^2) = 1$;
2. $\forall A, B \in \mathcal{B}(R^2) : \quad A \cap B = \emptyset$, *then* $h(A \cup B) = h(A) \oplus h(B)$;
3. $A_n, A \in \mathcal{B}(R^2) : \quad A_n \nearrow A$, *then* $h(A_n) \nearrow h(A)$;
4. $h(C \times D) = x(C) \cdot y(D)$ *for each* $C, D \in \mathcal{B}(R)$.

It is clear, that this definition can be generalize for more observables. Let $x_1, ..., x_n$ be the observables, then their joint observable will be the mapping $h : \mathcal{B}(R^n) \to \mathcal{F}$ with similar properties like in previous Definition 5.

Recall that in $\mathcal{F}$ for each pair of observables x, y their joint observable exists (see Riečan and Neubrunn [[16]], Theorem 8.3.2).

The convergence m-almost everywhere and equality m-almost everywhere on the set of all IF events is defined analogous like on other algebraic structures.

We define convergence almost everywhere with using limes superior a limes inferior. In classical case can be define limes superior of the sequence of random variables ξ_n with using the following equivalence:

$$\limsup_{n\to\infty} \xi_n(\omega) < t \Leftrightarrow \omega \in \bigcup_{p=1}^{\infty} \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} \xi_n^{-1}\left(\left(-\infty, t - \frac{1}{p}\right)\right).$$

Similarly we will proceed in our case.

Definition 6 *The sequence of observables $(x_n)_{n=1}^{\infty}$ on the set of all IF events $\mathcal{F}$ has limes superior, if there exists the observable $\overline{x}$, for which holds:*

$$\overline{x}((-\infty, t)) = \bigvee_{p=1}^{\infty} \bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} x_n\left(\left(-\infty, t - \frac{1}{p}\right)\right)$$

for every $t \in R$. If this observable exists, we will write: $\overline{x} = \limsup_{n\to\infty} x_n$.

Definition 7 *The sequence of observables* $(x_n)_{n=1}^{\infty}$ *on the set of all IF events* $\mathcal{F}$ *has limes inferior, if there exists the observable* $\underline{x}$*, for which holds:*

$$\underline{x}\left((-\infty, t)\right) = \bigvee_{p=1}^{\infty} \bigwedge_{k=1}^{\infty} \bigvee_{n=k}^{\infty} x_n \left(\left(-\infty, t - \frac{1}{p}\right)\right)$$

for every $t \in R$*. If this observable exists, we will write:* $\underline{x} = \liminf_{n \to \infty} x_n$*.*

In the following proposition we show the relationship between limes superior and limes inferior.

Proposition 1 *Let* $(x_n)_{n=1}^{\infty}$ *be the sequence of the observables on* $\mathcal{F}$*. Let there exist limes superior and limes inferior of this sequence, then for all real numbers* t *holds the following inequality:*

$$\limsup_{n \to \infty} x_n \left((-\infty, t)\right) \leq \liminf_{n \to \infty} x_n \left((-\infty, t)\right).$$

Proof:
Let t be a real number and p be a natural number. Then for each $k \in N$ are satisfying the inequalities:

$$\bigwedge_{n=k}^{\infty} x_n \left(\left(-\infty, t - \frac{1}{p}\right)\right) \leq \bigwedge_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \frac{1}{p}\right)\right);$$

$$\bigvee_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \frac{1}{p}\right)\right) \leq \bigvee_{n=k}^{\infty} x_n \left(\left(-\infty, t - \frac{1}{p}\right)\right).$$

Evidently the following inequalities hold:

$$\bigwedge_{n=k}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right) \leq \bigwedge_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right)$$
$$\bigvee_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right) \leq \bigvee_{n=k}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right).$$

It is clear then for every $k \in N$:

$$\bigwedge_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right) \leq \bigvee_{n=k+1}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right)$$

So we have for all $p \in N$

$$\bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right) \leq \bigwedge_{k=1}^{\infty} \bigvee_{n=k}^{\infty} x_n \left(\left(-\infty, t - \tfrac{1}{p}\right)\right).$$

Hence:

$$\bigvee_{p=1}^{\infty}\bigvee_{k=1}^{\infty}\bigwedge_{n=k}^{\infty} x_n\left(\left(-\infty, t-\tfrac{1}{p}\right)\right) \leq \bigvee_{p=1}^{\infty}\bigwedge_{k=1}^{\infty}\bigvee_{n=k}^{\infty} x_n\left(\left(-\infty, t-\tfrac{1}{p}\right)\right)(\omega)$$

and that is what we need.

Now we extend the convergence m-almost everywhere by the following way.

Definition 8 *A sequence of the observables $(x_n)_{n=1}^{\infty}$ on $\mathcal{F}$ converges m-almost everywhere to an observable x, if for any $t \in R$:*

$$m\left(\bigvee_{p=1}^{\infty}\bigvee_{k=1}^{\infty}\bigwedge_{n=k}^{\infty} x_n\left(\left(-\infty, t-\tfrac{1}{p}\right)\right)\right) =$$
$$m\left(\bigvee_{p=1}^{\infty}\bigwedge_{k=1}^{\infty}\bigvee_{n=k}^{\infty} x_n\left(\left(-\infty, t-\tfrac{1}{p}\right)\right)\right) =$$
$$= m\left(x\left((-\infty, t)\right)\right).$$

In the paper we will need besides the convergence m-almost everywhere also the equality of the observables m-almost everywhere.

Definition 9 *Let y, z be the observables on $\mathcal{F}$, $h : \mathcal{B}\left(R^2\right) \to \mathcal{F}$ is their joint observable and $\Delta = \left\{(u,v) \in R^2; u = v\right\}$. We say, that the observables y, z are equal m-almost everywhere, if holds $m\left(h\left(\Delta\right)\right) = 1$.*

Now we recall the result, which is known for MV algebras. We can find it for example in [8].

Definition 10 *Let $\mathcal{M} = (M, 0_M, 1_M, ^*, \oplus, \odot, \cdot)$ be MV algebra with product. The mapping $\tau : M \to M$ is called a state m-preserving transformation, if it satisfies the following conditions:*

1. *$\tau(1_M) = 1_M$;*
2. *if $a, b, c \in M$ and $a = b + c$, then $\tau(a) = \tau(b) + \tau(c)$;*
3. *if $a_n \in M, n \in N : \quad a_n \nearrow a$, then $\tau(a_n) \nearrow \tau(a)$;*
4. *$m(\tau(a) \cdot \tau(b)) = m(a \cdot b)$ for each $a, b \in M$.*

By the same way we define m-preserving transformation on $\mathcal{F}$. Now we formulate the individual ergodic theorem for MV algebras.

Theorem 2 *Let we have σ-complete weakly σ-distributive product MV algebra $\mathcal{M} = (M, 0_M, 1_M, ^*, \oplus, \odot, \cdot)$ with state m. Let x be an integrable P-observable and τ is state m-preserving transformation. The there exists integrable P-observable x^*, which satisfies following:*

1. $E(x^*) = E(x)$;
2. *a sequence* $\frac{1}{n}\sum_{i=0}^{n-1} \tau^i \circ x$ *converges m-almost everywhere to the observable* x^*;
3. $\tau \circ x^* = x^*$ *m-almost everywhere.*

Proof: We can find it in [8].

Now we show, that MV algebra $\mathcal{M}$, which corresponds to the set of all IF event is σ-complete weakly σ-distributive product MV algebra ($\forall (a_{ij})_{i,j} \subset M$: $a_{ij} \searrow 0_M \quad (j \to \infty, i = 1, 2, ..)$ holds $\bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i\varphi(i)} = 0_M$) and we can use the previous theorem in our case. The property of σ-completeness is clear. We have to prove, that this MV algebra is weakly σ-distributive.
Let $(A_{ij})_{i,j}$ be a sequence of elements of the set $\mathcal{F}$ for which holds

$$\mu_{A_{ij}} \searrow 0_\Omega, \quad \nu_{A_{ij}} \nearrow 1_\Omega \quad (j \to \infty, i = 1, 2, ...).$$

With using assumptions we get for $\forall \omega \in \Omega$ and each natural number i:

$$\forall \epsilon > 0, \quad \exists \varphi(i) : \quad \mu_{A_{i\varphi(i)}}(\omega) < \epsilon, \quad \nu_{A_{i\varphi(i)}}(\omega) > 1 - \epsilon.$$

We can see, that for $\forall \omega \in \Omega, \forall \epsilon > 0$ holds

$$\bigvee_{i=1}^{\infty} \mu_{A_{i\varphi(i)}}(\omega) < \epsilon, \quad \bigwedge_{i=1}^{\infty} \nu_{A_{i\varphi(i)}}(\omega) > 1 - \epsilon,$$

so

$$\bigwedge_{\varphi(i)}^{\infty} \bigvee_{i=1}^{\infty} \mu_{A_{i\varphi(i)}}(\omega) < \epsilon, \quad \bigvee_{\varphi(i)}^{\infty} \bigwedge_{i=1}^{\infty} \nu_{A_{i\varphi(i)}}(\omega) > 1 - \epsilon.$$

From these inequalities we get for $\forall \omega \in \Omega$

$$\bigwedge_{\varphi(i)}^{\infty} \bigvee_{i=1}^{\infty} \mu_{A_{i\varphi(i)}}(\omega) = 0, \quad \bigvee_{\varphi(i)}^{\infty} \bigwedge_{i=1}^{\infty} \nu_{A_{i\varphi(i)}}(\omega) = 1.$$

Now we see

$$\bigwedge_{\varphi(i)}^{\infty} \bigvee_{i=1}^{\infty} \mu_{A_{i\varphi(i)}} = 0, \quad \bigvee_{\varphi(i)}^{\infty} \bigwedge_{i=1}^{\infty} \nu_{A_{i\varphi(i)}} = 1,$$

so

$$\bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} A_{i\varphi(i)} = (0_\Omega, 1_\Omega) .$$

We finished the proof, that MV algebra $\mathcal{M}$ corresponding to the set of all IF events is weakly σ-distributive MV algebra.

Now we can formulated the expected individual ergodic theorem on the set of all IF events.

Theorem 3 *(Individual ergodic theorem on IF events) Let $\mathcal{F}$ be a set of all IF events with a state m. Let x be an integrable observable on $\mathcal{F}$ and τ is state m-preserving transformation. Then there exists integrable P-observable x^*, which satisfies the following:*

1. $E(x^*) = E(x)$;
2. *sequence* $\frac{1}{n} \sum_{i=0}^{n-1} \tau^i \circ x$ *converges m-almost everywhere to P-observable x^*;*
3. $\tau \circ x^* = x^*$ *m-almost everywhere.*

Proof: The proof of this theorem follows from Theorem 2 and properties of the MV algebra, which corresponds with the set of all IF events.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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