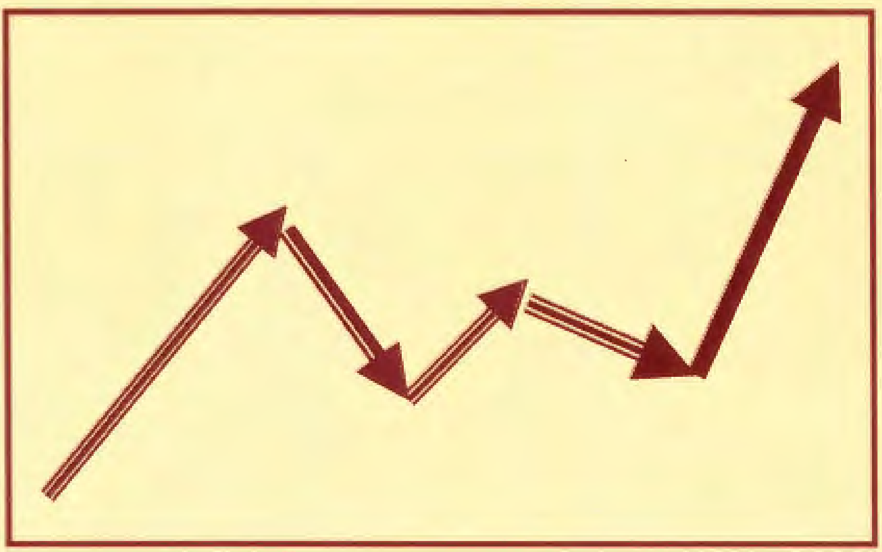
STANISŁAW PIASECKI

AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume II



Warsaw 2011

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Volume II

WARSAW 2011

ISBN 9788389475343

Warsaw, October 2011

INTRODUCTION

The purpose of the present book is to show the possibility of developing a quantitative description of the action of "invisible hand" on the market. This is why the text is full of mathematical expressions, even though they are kept purposefully at the possibly simple level.

At the same time, the book is a subsequent publication of results from the work on establishment of foundations for the theory of economic competition, limited, however, essentially to what is called price competition. Let us note at this point that the "competitors" here considered are the companies selling their produce on the common and limited market, and that price competition analysed takes place among the products of different companies, serving to satisfy the very same kind of demand from the side of the customers.

Price competition ought to be regarded as a dynamic market game, which takes place within the space of retail prices, i.e. in the "open", before the eyes of the consumers, or in the space of wholesale prices – behind the scenes. The strategies of the players consist in selection of prices, at which their products (services) are sold.

Under a close examination of the problem it turned out that most important for defining the market price game is determination of the "payoff function". The present book is devoted, therefore, mainly to this problem.

This volume constitutes a continuation of the considerations from the first volume of "*Introduction to a Theory of Market Competition*", in which territorial expansion strategy of companies has been analysed, this strategy allowing for expansion of sales and lowering of prices. Yet, sooner or later, the instant has to come when a company must enter an "alien" market, and, after a successful entry, face the problem of expanding its market share.

Hence, it is the last two issues that this book takes up and analyses.

The considerations forwarded therein are based on three fundamental assumptions:

- Increase of price of purchase entails a decrease of the number of products sold.
- Increase of the number of products turned out makes it possible to lower the cost of producing these products.
- The market secures the preservation of equilibrium between demand and supply. Customers are directed by reason when making purchasing decisions.

The first assumption results from the fact that each customer has limited financial capacities (of purchasing products and services).

The second assumption is justified by the commonly observed "production scale effect", which results from the continuous technological progress, taking place especially in the domain of production technologies. This fact finds its confirmation in the history of economic development – from handicraft through workshop production to the present-day mass (even if customised) production.

The third, double assumption is associated with the adoption of principles of free market.

In order to represent the "scale effect", the hyperbolic relation was used, resulting from the analysis of the constant and variable production costs.

To describe the dependence of demand upon the product price, stemming from the income structure of potential customers, the linear dependence was used, which is characteristic for the constant income density of customers.

Other adopted assumptions and simplifications are of technical character.

Many of the Readers shall certainly be disappointed, as they will not find in the book the statistical inquiries, based on what is called "real-life data", that would confirm the assumptions adopted and the results obtained. In order, though, for a theory to be subject to verification, it must first be formulated. It should be indicated that the precepts of this theory have been successfully implemented in economic reality, in the practice of quite a significant company in Poland.

Thus, the contents of this book ought to be regarded as an attempt of formulating a definite theory, by no means pretending to having exhausted the entire problem area. It should be added that the results contained in both volumes published so far result from the research done by the respective authors within the Systems Research Institute of the Polish Academy of Sciences. Separate thanks go to the NTT System S.A. company that supported financially the publication of both volumes.

The authors of both volumes hope that this modest contribution shall serve its purpose of providing to the Readers the very first insight into the possibility of representing and analysing in quantitative terms the processes we observe daily on the globalising markets. The authors would also like to announce the preparation of the subsequent volume, presenting the extension to the theory here expounded.

Warsaw, June 2011

Chapter III

STRATEGY OF COMPETITION ON A SHARED MARKET

1. Description of the situation

Consider a market, on which two companies function, A and B. These companies manufacture products having the same purpose. Assume, further, that at the initial time moment both companies, A and B, sell their products for the same prices, i.e.

$$C = C_A = C_B,$$

Where C denotes the price, for which both companies offer their products, while C_A and C_B denote, respectively, the prices of the products offered by the companies A and B.

Assume that as the two companies sell their SIMILAR products for the same price, then demand for these products, ON THE GIVEN MARKET is equal and hence total demand is split evenly, 50% each.

We introduce the notions of maximum potential demand and of the initial demand, i.e.

λ_{mx} – the maximum demand, attained when $C \to 0$, and

λ_0 – the initial demand for the products offered by companies A and B.

It can be reasonably assumed that dependence between demand and price is given by the expression

$$\lambda_0 = \lambda_{max}(1 - \frac{C}{C_{max}})$$

where C_{max} is the maximum price, when $\lambda \to 0$, i.e. the one, for which the companies A and B would just stop finding any buyer of their products here analysed.

Consider now the situation, when Company A decides to lower the price, for which it has been selling its products, by the value $\Delta C > 0$.

In this new situation we have

$$C_A = C - \Delta C,$$

$$C_A < C_B.$$

The key question is whether by lowering the price one can simultaneously achieve:

1. increase of the own share of the market, so that $\lambda_A > \lambda_B$,
2. securing of the own profit at the level higher than that of the competitor, so that we obtain $\Delta Z = Z_A - Z_B > 0$,
3. increase of the own profit of Company A in comparison with the profit Z_0 attained before the price decrease: $Z_A > Z_0$.

2. On the increase of demand and sales

After Company A decreases the price, the entire demand for the products manufactured by both companies, A and B, shall increase. There will be a definite group of customers, having lower incomes that will now be able to afford buying the cheaper products of Company A.

Under the new circumstances total demand λ for products manufactured by A and B shall be defined by the formula:

$$\lambda = \lambda_{\max} \, (1- \frac{C_A}{C_{\max}}) = \lambda_{\max}\left(1 - \frac{C - \Delta C}{C_{\max}}\right),$$

where, now, λ is the total demand for the products manufactured by companies A and B, after Company A has lowered its price for these products by ΔC.

Since demand before the price decrease can be represented by the formula

$$\lambda_0 = \lambda_{\max} \left(1 - \frac{C_B}{C_{\max}}\right),$$

Then the difference of demand after and before the price decrease is equal

$$\Delta\lambda = \lambda - \lambda_0 = \lambda_{\max}\left(1 - \frac{C_A}{C_{\max}}\right) - \lambda_{\max}\left(1 - \frac{C_B}{C_{\max}}\right),$$

$$\Delta\lambda = \lambda_{\max} \frac{C_B - C_A}{C_{\max}} = \lambda_{\max} \frac{\Delta C}{C_{\max}},$$

where $\Delta\lambda$ is the difference between the demand for products, manufactured by companies A and B, after Company A lowered its prices by ΔC and the initial demand for these products, λ_0.

In order to determine the division of demand – both before and after the price decrease, performed by Company A, we shall assume that the division (the market shares) is proportional to the prices of the competing companies.

This can be formally written down in the form

$$\lambda_A^{st} = \lambda_0 \frac{C_B}{C_A + C_B},$$

$$\lambda_B^{st} = \lambda_0 \frac{C_A}{C_A + C_B},$$

where λ_A^{st} is the demand for products manufactured by Company A, while λ_B^{st} is the demand for products manufactured by Company B.

Since after Company A has lowered its prices by ΔC we have $C_A = C_B - \Delta C$, we can reformulate the above expressions to the following ones:

$$\lambda_A^{st} = \lambda_0 \frac{C}{2C - \Delta C}, \text{ and}$$

$$\lambda_B^{st} = \lambda_0 \frac{C - \Delta C}{2C - \Delta C}.$$

Now, as we substitute the value for λ_0, we get

$$\lambda_A^{st} = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right)\frac{C}{2C - \Delta C},$$

$$\lambda_B^{st} = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right)\frac{C - \Delta C}{2C - \Delta C}.$$

After Company A lowers its price, the entire additional demand, $\Delta\lambda$, generated by inclusion of new customers, shall constitute the addition to the demand for products, manufactured by this company. Hence, demand volumes for the products, turned out by the two companies, after Company A has lowered its price, can be expressed as

$$\lambda_A = \lambda_A^{st} + \Delta\lambda,$$

$$\lambda_B = \lambda_B^{st},$$

where λ_A is the demand for products of Company A after it has lowered the respective price, and λ_B is the demand for products of Company B after the other company lowered its prices.

After the appropriate substitution we obtain

$$\lambda_A = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right)\frac{C}{2C - \Delta C} + \lambda_{\max}\left(\frac{\Delta C}{C_{\max}}\right),$$

$$\lambda_B = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right)\frac{C - \Delta C}{2C - \Delta C},$$

and, after further transformations:

$$\lambda_A = \lambda_{max}\left(\frac{C_{max} - C}{C_{max}}\right)\frac{C}{2C - \Delta C} + \lambda_{max}\left(\frac{\Delta C}{C_{max}}\right) = \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max} - C)C}{2C - \Delta C} + \Delta C\right] =$$

$$= \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max} - C)(C - \Delta C)}{2C - \Delta C}\right].$$

Now, using the above formulae for the values of λ_A and λ_B, we get

$$\Delta\lambda = \lambda_A - \lambda_B = \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max} - C)C}{2C - \Delta C} + \Delta C - \frac{(C_{max} - C)(C - \Delta C)}{2C - \Delta C}\right] =$$

$$= \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max} - C)C - (C_{max} - C)C + (C_{max} - C)\Delta C}{2C - \Delta C} + \Delta C\right] =$$

$$= \frac{\lambda_{max}}{C_{max}}\Delta C\left[\frac{(C_{max} - C)}{2C - \Delta C} + 1\right].$$

And, ultimately

$$\Delta\lambda = \frac{\lambda_{max}\Delta C}{C_{max}}\left[\frac{C_{max} + C - \Delta C}{2C - \Delta C}\right]$$

Since $\Delta C < C$, then the condition of increase of sales of products, manufactured by Company A is always fulfilled.

3. On the possibility of achieving advantage of profits

Let us now turn to the analysis of possibility of achieving higher profits by Company A in comparison with Company B, that is – the fulfilment of the condition:

$\Delta Z = Z_A - Z_B > 0.$

<u>The case of an arbitrary *C*</u>

Assume at the beginning that the initial price $C_A = C_B = C$ is arbitrary, but, of course, contained within the interval $b < C < C_{mx}$.

In the situation, when Company A lowered its price by ΔC, the financial outcomes for the two companies shall be as follows:

$$Z_A = \lambda_A(C - \Delta C) - \left(\frac{Q_A}{\lambda_A} + b_A\right)\lambda_A = \lambda_A(C - \Delta C) - b_A\lambda_A - Q_A = \lambda_A(C - \Delta C - b_A) - Q_A$$

$$Z_B = \lambda_B C - \left(\frac{Q_B}{\lambda_B} + b_B\right)\lambda_B = \lambda_B C - \lambda_B b_B - Q_B = \lambda_B(C - b_B) - Q_B .$$

It is assumed in the above formulae that production costs are composed of two components: constant cost (respectively, Q_A and Q_B for companies A and B) and variable cost (respectively, $b_A\lambda_A$ and $b_B\lambda_B$). Here, b_A and b_B denote the direct unit costs of production in the respective companies. For a broader treatment of the issue of costs, see Volume I.

Like in other considerations here forwarded, we shall assume that both companies, A and B, dispose of similar technologies. Since they operate on the same market, we can assume that they have access to the same kind of technologies and bear the same financial burden, related to the maintenance of their production lines.

Hence, we can assume that $Q_A \approx Q_B$.

Further, we can also accept the assumption that the two companies, disposing of production potentials, purchase components for their products at the same or very similar prices.

Hence, we can take $b_A \approx b_B$.

Then, by making appropriate substitutions, we get

$$Z_A = \lambda_A(C - \Delta C) - \left(\frac{Q}{\lambda_A} + b\right)\lambda_A = \lambda_A(C - \Delta C) - b\lambda_A - Q = \lambda_A(C - \Delta C - b) - Q$$

$$Z_B = \lambda_B C - \left(\frac{Q}{\lambda_B} + b\right)\lambda_B = \lambda_B(C - b) - Q .$$

Further, by introducing the expressions for λ_A and λ_B we can write down the formulae for profit of the two companies in the following manner:

$$Z_A = \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max}-C)C}{2C-\Delta C}+\Delta C\right](C-\Delta C-b)-Q,$$

$$Z_B = \lambda_B C-\left(\frac{Q}{\lambda_B}+b\right)\lambda_B = \frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max}-C)(C-\Delta C)}{2C-\Delta C}\right](C-b)-Q.$$

By subtracting the profit of Company B from that of Company A we obtain the difference of profits between them:

$$\Delta Z = Z_A - Z_B = \lambda_A(C-\Delta C-b)-Q-\lambda_B(C-b)+Q =$$

$$= \lambda_A C-\lambda_A\Delta C-\lambda_A b-\lambda_B C+\lambda_B b = C(\lambda_A-\lambda_B)-b(\lambda_A-\lambda_B)-\lambda_A\Delta C$$

$$= (C-b)(\lambda_A-\lambda_B)-\lambda_A\Delta C.$$

If Company A wishes to conquer the market, it is extremely important that the condition $\Delta Z > 0$ be satisfied.

Hence, we should check whether this is possible and if so, under what conditions.

As we introduce the values of $\Delta\lambda$ and λ_A into the last formula above for ΔZ we get:

$$\Delta Z = Z_A - Z_B = (C-b)\Delta\lambda-\lambda_A\Delta C =$$

$$= (C-b)\frac{\lambda_{max}\Delta C}{C_{max}}\left[\frac{C_{max}+C-\Delta C}{2C-\Delta C}\right]-\lambda_A\Delta C =$$

$$= (C-b)\frac{\lambda_{max}\Delta C}{C_{max}}\left[\frac{C_{max}-C}{2C-\Delta C}+1\right]-\frac{\lambda_{max}}{C_{max}}\left[\frac{(C_{max}-C)C}{2C-\Delta C}+\Delta C\right]\Delta C =$$

$$= \frac{\lambda_{max}}{C_{max}}\left\{\left[\frac{C_{max}-C}{2C-\Delta C}+1\right](C-b)-\left[\frac{C_{max}-C}{2C-\Delta C}C+\Delta C\right]\right\}\Delta C.$$

With these expressions let us analyse the conditions that have to hold in order for $\Delta Z > 0$ to be true.

Since $\Delta C > 0$, $\lambda_{max} > 0$, and $C_{mx} > 0$, then it suffices to have the inequality

$$\left[\frac{C_{max} - C}{2C - \Delta C} + 1\right](C - b) - \frac{C_{max} - C}{2C - \Delta C} C - \Delta C > 0$$

satisfied. By transforming now the left-hand-side expression, we obtain

$$\frac{(C_{max} - C)}{2C - \Delta C} C - \frac{(C_{max} - C)}{2C - \Delta C} b + C - b - \frac{C_{max} - C}{2C - \Delta C} C - \Delta C > 0$$

and then

$$-\frac{(C_{max} - C)b}{2C - \Delta C} + \frac{(C - b)(2C - \Delta C)}{2C - \Delta C} - \frac{\Delta C(2C - \Delta C)}{2C - \Delta C} > 0$$

and, ultimately,

$$\frac{-C_{max} b + Cb + 2C^2 - 2Cb - C\Delta C + b\Delta C - 2C\Delta C + \Delta C^2}{2C - \Delta C} > 0.$$

As $2C$ - ΔC is always bigger than zero, so, in order for ΔZ to be bigger than zero, it suffices to have

$$-C_{max} b + 2C_B{}^2 - C_B b + b\Delta C - 3C_B \Delta C + \Delta C^2 > 0.$$

This inequality involves a second order polynomial of the form

$$\Delta Z = \Delta C^2 - \Delta C(3C - b) + 2C^2 - b(C_{max} + C) > 0.$$

Let us now determine the roots of this polynomial, namely:

$$\Delta = b^2 - 4ac = [-(3C - b)]^2 - 4[2C^2 - b(C_{max} + C)] =$$
$$= 9C^2 - 6Cb + b^2 - 8C^2 + 4b(C_{max} + C) =$$
$$= C^2 - 2Cb + b^2 + 4bC_{max} = (C - b)^2 + 4bC_{max}.$$

It can be easily noticed that there is always $\Delta > 0$, and so the inequality has two roots:

$$\Delta C_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{(3C-b)-\sqrt{(C-b)^2+4bC_{max}}}{2}$$

and

$$\Delta C_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{(3C-b)+\sqrt{(C-b)^2+4bC_{max}}}{2}.$$

Let us first look at the value of ΔC_2. It can easily be noticed that $\Delta C_2 > 0$

$$\Delta C_2 = \frac{1}{2}(3C-b)+\frac{1}{2}\sqrt{(C-b)^2+4bC_{max}}.$$

Since $C \geq b$ and $\Delta C_{mx} > C_B + b$, then

$$\Delta C_2 = \frac{1}{2}(3C-b)+\frac{1}{2}\sqrt{(C-b)^2+4bC_{max}} > C+\sqrt{bC_{max}} > C_B + b$$

$$\Delta C_2 > C_B + b.$$

Note that the value of ΔC_1 has no economic sense, as it would push down the sales price, C - ΔC, to the domain of negative prices.

Let us now check the value of ΔC. Since ΔC must be bigger than zero, hence the admissible interval of the values of ΔC is contained in the interval

$$0 < \Delta C < \frac{(3C-b)-\sqrt{(C-b)^2+4bC_{max}}}{2},$$

under the condition, naturally, that

$$\frac{(3C-b)-\sqrt{(C-b)^2+4bC_m}}{2} \geq 0.$$

The latter condition is fulfilled, when the following inequality holds:

$$(3C-b)-\sqrt{(C-b)^2+4bC_{max}}\geq 0\,.$$

This is equivalent to the fulfilment of the condition

$$(3C-b)^2\geq (C-b)^2+4bC_{max}\,.$$

After transformations we obtain

$$9C^2-6Cb+b^2\geq C^2-2Cb+b^2+4bC_{max}\,,$$

and then

$$8C^2-4Cb\geq 4bC_{max}\,,$$

and, ultimately,

$$2C^2-Cb-bC_{max}\geq 0\,.$$

Thus, in order to have the inequality $Z_A - Z_B > 0$ fulfilled, the above inequality must hold.

We shall analyse this inequality in deeper detail, so as to see for what relations between C, C_{mx} and b it holds. As we equate the left-hand side of the inequality to zero, we shall find the roots of the inequality. The respective calculations are as follows:

$$\Delta = b^2-4ac=$$
$$=b^2-4*2(-bC_{max})=b^2+8bC_{max}\,,$$

and

$$C_1=\frac{-b-\sqrt{\Delta}}{2a}=\frac{b-\quad b^2+8bC_{max}}{4}$$

$$C_2=\frac{-b+\sqrt{\Delta}}{2a}=\frac{b+\quad b^2+8bC_{max}}{4}\,.$$

The value of C_1 has no economic sense, since it is negative. On the other hand, the calculated value of C_2 has an economic sense. As the inequality is represented by the parabola with the arms turned upwards, so the inequality

$$2C^2 - Cb - bC_{mx} > 0$$

is satisfied for all the values

$$C > \frac{b + \sqrt{b^2 + 8bC_{max}}}{4}.$$

Hence, we see that Company A can secure for itself the advantage of profits, $Z_A > Z_B$, if only the initial price C satisfies the inequality $2C^2 - Cb - bC_{mx} > 0$.

If we assume that in the initial conditions the companies have been selling their products at the prices guaranteeing for them the maximum profit, i.e. the price has been the optimum one, corresponding to maximum profits:

$$C = C^* = \frac{C_{max} + b}{2}.$$

<u>The case when $C = C^*$</u>

After we substitute C^* in the condition $2C^2 - Cb - bC_{mx} > 0$, we obtain

$$2\left(\frac{C_{mx} + b}{2}\right)^2 - \left(\frac{C_{mx} + b}{2}\right)b - bC_{mx} > 0,$$

and, after transformations

$$\frac{C_{mx}^2 + 2C_{mx}b + b^2}{2} - \frac{C_{mx}b + b^2}{2} - \frac{2bC_{mx}}{2} > 0,$$

$$\frac{C_{mx}^2 + 2C_{mx}b + b^2 - C_{mx}b - b^2 - 2C_{mx}b}{2} > 0,$$

leading to

$$\frac{C_{mx}(C_{mx}-b)}{2} > 0.$$

Since C_{mx} is always bigger than zero, the above condition is fulfilled, when $C_{mx} > b$. Yet, in view of the fact that the maximum sales price (the one implying the decrease of demand to zero) is virtually always higher than production costs, we can simply assume that this condition is also satisfied always.

As every enterprise tends to maximise its profits, we can assume, as we anyway did before, that in the initial conditions, before the decrease of price by ΔC, the companies have been selling their products form the optimum price, maximising their profits, that is,

$$C = \frac{C_{max}+b}{2}.$$

Taking into consideration the fact that $\Delta Z > 0$ when

$$0 < \Delta C < \frac{(3C-b)-\sqrt{(C-b)^2+4bC_{max}}}{2},$$

then, after we substitute $C = \frac{C_{max}+b}{2}$, we get:

$$0 < \Delta C < \frac{\left[3\frac{C_{max}+b}{2}-b\right]-\sqrt{\left(\frac{C_{max}+b}{2}-b\right)^2+4bC_{max}}}{2}.$$

As we transform the expressions involved, we obtain:

$$0 < \Delta C < \frac{\left[\frac{3C_{max}+3b-2b}{2}\right]-\sqrt{\left(\frac{C_{max}+b-2b}{2}\right)^2+4bC_{max}}}{2}$$

$$0 < \Delta C < \frac{\left(\frac{3C_{max} + b}{2}\right) - \sqrt{\left(\frac{C_{max} - b}{2}\right)^2 + 4bC_{max}}}{2}$$

$$0 < \Delta C < \frac{\left(\frac{3C_{max} + b}{2}\right) - \sqrt{\frac{C_{max}^2 - 2C_{max}b + b^2 + 16bC_{max}}{4}}}{2},$$

and then

$$0 < \Delta C < \frac{\left(\frac{3C_{max} + b}{2}\right) - \frac{1}{2}\sqrt{C_{max}^2 + 14bC_{max} + b^2}}{2}$$

$$0 < \Delta C < \frac{3C_{max} + b - \sqrt{(C_{max} + b)^2 + 12bC_{max}}}{4}.$$

As we can represent C_{mx} in the form of product of b and a certain coefficient, say $C_{max} = \varphi * b$, where $\varphi > 1$, then, after we introduce this expression to the above inequalities, we get

$$0 < \Delta C < \frac{3b * \varphi + b - \sqrt{b^2(\varphi + 1)^2 + 12b^2\varphi}}{4},$$

$$0 < \Delta C < b\frac{3\varphi + 1 - \sqrt{(\varphi + 1)^2 + 12\varphi}}{4}.$$

Fulfilment of the above inequality means, in practice, that ΔC_1 is positive, so that there exists such an area, within which Company A, by decreasing the price of its products, expands its market share and at the same time achieves higher profits than the competitor.

4. Remarks on the choice of the optimum price *C*

Let us analyse once more the expression for ΔZ under the assumption that before the price was decreased by ΔC by the Company A both companies have been selling their products for the optimum price, ensuring the maximum possible profit, and so the expression we consider

$$\Delta Z = \frac{\lambda_{\max}\Delta C}{C_{\max}}\left[\frac{\Delta C^2 - \Delta C(3C_B - b) + 2C_B^2 - b(C_{\max} + C_B)}{2C_B - \Delta C}\right],$$

should be treated in the light of the assumption made above, concerning the value of C, i.e.

$$C = \frac{C_{\max} + b}{2}.$$

As we now introduce this value to the nominator of the expression in the brackets in the formula for ΔZ, we get

$$\Delta C^2 - \Delta C\left(3\frac{C_{\max} + b}{2} - b\right) + 2\left(\frac{C_{\max} + b}{2}\right)^2 - b\left(C_{\max} + \frac{C_{\max} + b}{2}\right) =$$

$$= \Delta C^2 - \Delta C\left(\frac{3C_{\max} + 3b - 2b}{2}\right) + \frac{C_{\max}^2 + 2bC_{\max} + b^2}{2} - \frac{2bC_{\max}}{2} - \frac{bC_{\max} + b^2}{2} =$$

$$= \Delta C^2 - \Delta C\left(\frac{3C_{\max} + b}{2}\right) + \frac{C_{\max}^2 + 2bC_{\max} + b^2 - 2bC_{\max} - bC_{\max} - b^2}{2} =$$

$$= \Delta C^2 - \Delta C\left(\frac{3C_{\max} + b}{2}\right) + \frac{C_{\max}^2 - bC_{\max}}{2} =$$

and, finally,

$$= \Delta C^2 - \Delta C\left(\frac{3C_{\max} + b}{2}\right) + C_{\max}\frac{C_{\max} - b}{2}.$$

The formula for ΔZ shall now take the form of

$$\Delta Z = \frac{\lambda_{\max}}{C_{\max}}\left[\frac{\Delta C^3 - \Delta C^2\left(\frac{3C_{\max} + b}{2}\right) + \Delta C C_{\max}\frac{C_{\max} - b}{2}}{C_{\max} + b - \Delta C}\right].$$

After transformations we obtain

$$\Delta Z = \frac{\lambda_{\max}}{C_{\max}}\left[\frac{2\Delta C^3 - \Delta C^2(3C_{\max} + b) + \Delta C C_{\max}(C_{\max} - b)}{2(C_{\max} + b - \Delta C)}\right].$$

In the nominator of the formula for ΔZ there are three components. Since ΔC is a small quantity, the biggest significance for ensuring the positive value of ΔZ will be associated with the components, in which ΔC appears in the lowest powers. The component with the lowest power of ΔC has a positive sign. In this component, the ΔC is multiplied by $C_{mx}(C_{mx}-b)$.

Of the three components, the negative sign appears at the one, where ΔC is squared. This factor is multiplied by $3C_{mx}+b$.

Hence, for constant ΔC the value (and the sign) of ΔZ shall depend upon the relation between the values of C_{mx} and b.

As we introduce again the expression for C_{mx} in the form of φb, we get

$$\Delta Z = \frac{\lambda_{\max}}{2C_{\max}}\left[\frac{2\Delta C^3 - \Delta C^2 b(3\varphi + 1) + \Delta C \varphi b^2(\varphi - 1)}{\varphi b + b - \Delta C}\right].$$

In order to establish the value of ΔC, for which Company A achieves the highest (positive) difference of profits, ΔZ, in comparison with the competitor, we should determine the derivative of ΔZ with respect to ΔC and determine, for what value of ΔC this derivative equals zero (as the necessary condition). Let us, therefore, recall:

$$\Delta Z = \frac{\lambda_{\max}}{2C_{\max}}\left[\frac{2\Delta C^3 - \Delta C^2(3C_{\max} + b) + \Delta C C_{\max}(C_{\max} - b)}{C_{\max} + b - \Delta C}\right].$$

Having in mind the fact that the above expression is a ratio, its derivative is calculated as

$$\frac{\partial \Delta Z}{\partial \Delta C} = \frac{f'g - g'f}{g^2},$$

where f represents the nominator, and g represents the denominator.

The derivatives of the nominator and of the denominator are equal, respectively

$$f' = 6\Delta C^2 - 2\Delta C(3C_m + b) + C_m(C_m - b) \text{ and } \quad g' = -1.$$

And so:

$$\frac{2C_{\max}}{\lambda_{\max}} * \frac{\partial \Delta Z}{\partial \Delta C} = \frac{[6\Delta C^2 - 2\Delta C(3C_m + b) + C_m(C_m - b)](C_m + b - \Delta C)}{(C_m + b - \Delta C)^2} +$$

$$\frac{1[2\Delta C^3 - \Delta C^2(3C_m + b) + \Delta C C_m(C_m - b)]}{(C_m + b - \Delta C)^2} =$$

$$= \frac{[6\Delta C^2 - 2\Delta C(3C_m + b) + C_m(C_m - b)](C_m + b) - 4\Delta C^3 + \Delta C^2(3C_m + b)}{(C_m + b - \Delta C)^2}$$

Since, as said, ΔZ attains an extreme value (the maximum or the minimum) when the derivative equals zero, let us check this condition:

$$-4\Delta C^3 + \Delta C^2(9C_{\max} + 7b) - 2\Delta C(3C_{\max} + b)(C_{\max} + b) + C_{\max}(C_{\max} - b)^2 = 0,$$

for $\Delta C > 0$.

The equation obtained has a solution. Yet, its determination in an analytic manner is of little use, since it requires complicated calculations. We have also already noted that the value of ΔC, for which the expression analysed attains zero, depends upon two parameters, namely C_{mx} and b. By expressing, again, C_{mx} as φb we obtain dependence upon just one parameter and we can produce appropriate diagrams that may be used for determination of the respective zeroes. Yet, actually, it may be simpler to use the numerically produced with the help of the computer diagrams corresponding to the function $\Delta Z(\Delta C)$ for this particular purpose.

5. Remarks on the choice of *C* maximising *Z*

Let us now pass over to the direct analysis of the value of profit of Company A. We shall, namely, try to choose the value of

ΔC in such a manner as to ensure the increase of the value of profit, Z, in comparison with its initial value, i.e. for $\Delta C = 0$.

Hence, we shall be looking at the profits of Company A after it has lowered by ΔC the price of its products. As we assume that the other company, Company B, does not change its prices, we can write down $C_B = C$. Then:

$$Z_A = \frac{\lambda_{\max}}{C_{\max}}\left[\frac{(C_{\max} - C)C}{2C - \Delta C} + \Delta C\right](C - \Delta C - b) - Q$$

and

$$Z_A = \frac{\lambda_{\max}}{C_{\max}}\left[\frac{(C_{\max} - C)C + \Delta C(2C - \Delta C)}{2C - \Delta C}\right](C - \Delta C - b) - Q =$$

$$= \frac{\lambda_{\max}}{C_{\max}} * \frac{(C_{\max}C - C^2 + 2C\Delta C - \Delta C^2)(C - \Delta C - b)}{2C - \Delta C} - Q =$$

$$= \frac{\lambda_{\max}}{C_{\max}} * \frac{\left[C_{\max}C - (C - \Delta C)^2\right](C - \Delta C - b)}{2C - \Delta C} - Q$$

In the initial state, when both companies sell their products for the same prices ($C_A = C_B = C$), we have:

$$Z_A = \lambda_A(C_A - b) - Q,$$

where Z_A is the profit of Company A under the initial conditions, when both companies sell their products for the same prices.

The initial demand for the products turned out by Company A is expressed, therefore, as

$$\lambda_A = \lambda_{\max}\left(1 - \frac{C_A}{C_{\max}}\right) * \frac{C_B}{C_A + C_B} = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right) * \frac{1}{2},$$

that is

$$\lambda_A = \lambda_{\max}\left(1 - \frac{C}{C_{\max}}\right) * \frac{C}{2C} = \frac{\lambda_{\max}}{2C_{\max}}(C_{\max} - C),$$

and so

$$Z_A = \frac{\lambda_{\max}}{2C_{\max}} (C_{\max} - C)(C - b) - Q.$$

Next, Company A lowers the price, in order

- not only to expand its market share, but also
- to increase its profits.

Let us check, then, whether there exists such a value of ΔC, for which the company that increases its market share by lowering the price, achieves higher profits than it had had when $C_A = C$.

We analyse

$$\Delta Z_A = Z_A(\Delta C > 0) - Z_A(\Delta Z = 0),$$

$$\Delta Z_A = \frac{\lambda_{\max}}{C_{\max}} \left[\frac{(C_{\max} - C)C + \Delta C(2C - \Delta C)}{2C - \Delta C} \right] (C - \Delta C - b) - \frac{\lambda_{\max}}{2C_{\max}} (C_{\max} - C)(C - b),$$

$$\Delta Z_A = \frac{\lambda_{\max}}{C_{\max}} \left\{ \frac{\left[C_{\max} C - C_A^2\right](C_A - b)}{C - C_A} - \frac{(C - \Delta C)(C - b)}{2} \right\}$$

under the assumption that $b \le C_A \le C = \dfrac{C_{\max} + b}{2}$, and we get

$$\Delta Z_A = \frac{\lambda_{\max}}{C_{\max}} * \frac{\left[C_{\max} C - (C - \Delta C)^2\right](C - \Delta C - b)}{2C - \Delta C} - \frac{\lambda_{\max}}{C_{\max}} * \frac{(C_{\max} - C)(C - b)}{2} =$$

$$= \frac{\lambda_{\max}}{2C_{\max}} * \frac{2\left[C_{\max} C - (C - \Delta C)^2\right](C - \Delta C - b) - (C_{\max} - C)(C - b)(2C - \Delta C)}{2C - \Delta C}$$

Let us simplify the expression in the nominator:

$$2C_{\max} C(C - \Delta C - b) - 2(C - \Delta C)^2 (C - \Delta C - b) - 2C(C_{\max} - C)(C - b) + \Delta C(C_{\max} - C)(C - b) =$$
$$= 2C_{\max} C(C - b) - 2C_{\max} C\Delta C - 2(C^2 - 2C\Delta C + \Delta C^2)(C - \Delta C - b) - 2CC_{\max}(C - b) + 2C^2(C - b) +$$
$$+ \Delta C(C_{\max} - C)(C - b) =$$
$$= -2C_{\max} C\Delta C - 2C^2(C - \Delta C - b) - 2(-2C\Delta C + \Delta C^2)(C - \Delta C - b) + 2C^2(C - b) + \Delta C(C_{\max} - C)(C - b) =$$

$$= -2C_{\max}C\Delta C - 2C^2(C-b) + 2C^2\Delta C + 2\Delta C(2C-\Delta C)(C-\Delta C-b) +$$
$$+ 2C^2(C-b) + \Delta C(C_{\max} - C)(C-b) =$$
$$= -2C_{\max}C\Delta C + 2C^2\Delta C + 2\Delta C(2C-\Delta C)(C-\Delta C-b) +$$
$$+ \Delta C\left(C_{\max}C - C^2 - C_{\max}b + Cb\right) =$$
$$= -2C_{\max}C\Delta C + 2C^2\Delta C + 2\Delta C(2C-\Delta C)(C-\Delta C-b) + \Delta C C_{\max}C - C^2\Delta C - \Delta Cb(C_{\max} - C) =$$
$$= -C_{\max}C\Delta C + C^2\Delta C + 2\Delta C(2C-\Delta C)(C-\Delta C-b) - \Delta Cb(C_{\max} - C)$$

Since, by virtue of the assumption, $\Delta C > 0$, then

$$\left[-C_{\max}C + C^2 + 2(2C-\Delta C)(C-\Delta C-b) - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[-C_{\max}C + C^2 + 2(2C-\Delta C)(C-b) - 2\Delta C(2C-\Delta C) - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[-C_{\max}C + C^2 + 2\left(2C^2 - \Delta CC - 2Cb + \Delta Cb\right) - 4\Delta CC + 2\Delta C^2 - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[-C_{\max}C + C^2 + 4C^2 - 2\Delta CC - 4Cb + 2\Delta Cb - 4\Delta CC + 2\Delta C^2 - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[-C_{\max}C + 5C^2 - 4Cb + 2\Delta Cb - 6\Delta CC + 2\Delta C^2 - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[2\Delta C^2 + \Delta C(2b-6C) + 5C^2 - 4Cb - C_{\max}C - b(C_{\max} - C)\right]\Delta C =$$
$$= \left[2\Delta C^2 - \Delta C(6C-2b) + 5C^2 - 4Cb - C_{\max}C - bC_{\max} + bC\right]\Delta C =$$
$$= \left[2\Delta C^2 - 2\Delta C(3C-b) + C(5C-3b) - C_{\max}(C+b)\right]\Delta C .$$

After substitution, we obtain

$$\Delta Z_A = \frac{\lambda_{\max}}{2C_{\max}}\left[\frac{2\Delta C^2 - 2\Delta C(3C-b) + C(5C-3b) - C_{\max}(C+b)}{2C-\Delta C}\right] * \Delta C .$$

ΔZ_A shall be greater than zero, when

$$W_2 = 2\Delta C^2 - 2\Delta C(3C-b) + C(5C-3b) - C_{\max}(C+b) > 0 ,$$

and $\Delta C < 2C$, or even $\Delta C \le C - b$.

We now determine the roots of this parabola

$\Delta = b^2 - 4ac$

b=-2(3C-b)

a=2

$c = C(5C - 3b) - C_{\max}(C + b)$

$$\begin{aligned}
\Delta &= 4(3C - b)^2 - 4 * 2[C(5C - 3b) - C_{\max}(C + b)] = \\
&= 4(3C - b)^2 + 8C_{\max}(C + b) - 8C(5C - 3b) = \\
&= 4(9C^2 - 6Cb + b^2) - 40C^2 + 24Cb + 8C_{\max}(C + b) = \\
&= 36C^2 - 24Cb + 4b^2 - 40C^2 + 24Cb + 8C_{\max}(C + b) = \\
&= -4C^2 + 4b^2 + 8C_{\max}(C + b) = \\
&= -4C^2 + 8C_{\max} * C + 4b^2 + 8C_{\max} b = \\
&= 4 * [-C^2 + 2C_{\max} C + b(2C_{\max} + b)]
\end{aligned}$$

Δ is bigger than zero, when

$$\frac{1}{4}\Delta = -C^2 + 2C_{\max} C + b(2C_{\max} + b) \geq 0$$

As we now deal again with a parabola, we must, like before, determine the roots of the respective binomial:

$\Delta = b^2 - 4ac$

a=-1

$b = 2C_{\max} C$

$C = b(2C_{\max} + b)$

$$\Delta = 4C_{max}^{2} - 4*(-1)b(2C_{max} + b) =$$
$$= 4C_{max}^{2} + 4b(2C_{max} + b) =$$
$$= 4C_{max}^{2}\left[1 + \frac{b}{C_{max}}\left(2 + \frac{b}{C_{max}}\right)\right]$$

The here obtained expression for Δ is always positive, and so we have two roots, namely:

$$C_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2C_{max} \pm 2C_{max}\sqrt{1 + 2\frac{b}{C_{max}} + \left(\frac{b}{C_{max}}\right)^{2}}}{-2} =$$
$$= C_{max} \pm C_{max}\sqrt{\left(1 + \frac{b}{C_{max}}\right)^{2}} = C_{max}\left[1 \pm \left(1 + \frac{b}{C_{max}}\right)\right]$$
$$C_{1} = C_{max}\left[1 + \left(1 + \frac{b}{C_{max}}\right)\right] =$$
$$= C_{max}\left(2 + \frac{b}{C_{max}}\right) > 0$$
$$C_{2} = C_{max}\left[1 - \left(1 + \frac{b}{C_{max}}\right)\right] =$$
$$= C_{max} * \frac{b}{C_{max}} = b > 0$$

* * *

This ends the analysis, leading to the determination of the values of prices, which are meant to secure not only an extension of the market share, but also an increase of profit value for Company A, the one that lowered its sales price.

CONCLUDING REMARKS

This book, like the previous one, constituting Volume I of the introduction to a theory of market competition, contains considerations that involve a number of approximations and simplifications. We, that is – the authors of both these volumes – would like to draw the attention of the Reader to them.

In general, we do not explicitly consider the vague, uncertain or "fuzzy" character of some of the quantities we refer to. This concerns, in particular, such quantities as the limit value of d in the determination of the demand function, $\lambda(C)$.

Likewise, the uncertain, or specific character of some relations has not been treated in an explicit manner. Thus, for instance, we state in the book that the situation when the market shares of two competitors are equal, 50% each, constitutes indeed a kind of equilibrium, but this is an unstable equilibrium point, for any disturbance to this situation shall drive it far away from the equilibrium (assuming, of course, that this disturbance, due to behaviour of one of the competitors, does not find any "appropriate" reaction from the side of the other competitor).

In reality, though, this equilibrium point is not that unstable, i.e. it is not that sensitive to the very small disturbances. Actually, an interval of insensitivity always exists, due to various reasons, such as delays, information shortage, lack of reaction of customers to very small price changes etc. It is even possible that the "hysteresis" effect may appear. In terms of the notions introduced in this book, the magnitude of the zone of insensitivity depends upon the slope of the production characteristics (the value of the derivative $dk/d\mu$).

Independently of the above remarks the considerations here presented neglect the effect of the change in the number of potential customers due to the change in product price. Namely, along with the change in product price, there is also change in the value of the difference *C-b*, exerting the decisive influence on the magnitude of the optimum radius R^* of the area, over which the company effectively caters to its potential customers. This radius R^* defines, in turn, the number of such potential customers, i.e. the ones, to whom the products are effectively supplied. This number, in turn, together with the income structure of the customers, defines the value of Λ_{mx} (see also Volume I). Yet, in the book, for both companies selling their products for different prices, the very same value of Λ_{mx} was adopted.

Neglecting this particular aspect is justified by the following circumstances:

- a company that just enters the market (as well as the one, which tries to expand its market share) can hardly afford the advertising saying that its product is not worse than the one of the competitor, even though it is cheaper – and this not for all the potential customers, exception being constituted by the farthest ones;
- on the other hand, the company defending its market share and for this purpose decreasing the sales price of its products, ought not get rid of its more distant customers, since this would make a very disadvantageous effect on the remaining customers and would actually accelerate elimination of such a company from the market.

Of course, the fact that we neglected the influence, exerted by the changes in the reach, *R*, was also largely due to the wish of simplifying the complicated interrelations, constituting the description of the market process, the mechanism of functioning of the "invisible hand of the market".

Considerations, contained in the book, do not account, either, for the influence of advertising, although certain remarks on this subject are forwarded in Chapter I.

Likewise, we did not forward the estimates for the cost of entry onto an alien market, which could be formulated with the use of the formulae for the sales magnitudes (Λ_A and Λ_B).

When considering the (initial) shares of two competing companies, we analysed the case, when they start from equal market shares. For modelling and analysing other possible situations, we could use the coefficient u.

In the case when more than two companies have (non-negligible) shares in a market, the struggle for the market share ought to be started with the weakest company, avoiding the appearance of a hostile coalition of the remaining companies on the market. Otherwise, it would become necessary to establish an own coalition that would be able to withstand the competition of the other coalition. In such a case the struggle for the market shares would reduce to the case of two competitors, that is – to the situation described in the book.

In view of these and, indeed, many other aspects that remain to be accounted for, it is obvious that the description of the mechanism behind the functioning of the "invisible hand" is far from complete.

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In view, on the one hand, of the vast body of literature on the mechanisms of the market, mainly related to the micro-economic models and analyses, and, on the other, of the quite self-contained character of the considerations here forwarded, the list of references provided is quite short. It contains the publications of the authors, containing a similar or related content to the one here presented, and the essential positions, known internationally, which deal with similar problems.

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This book is the second part of an exposition of a coherent and far-reaching theory of market competition. The theory is based on simple precepts, does not require deep knowledge of either economics or mathematics, and is therefore aimed primarily at undergraduate students and all those trying to put in order their vision of how the essential market mechanisms might work. The present Volume II constitutes a complement to the considerations, contained in Volume I.

The logic of the presentation is straightforward; it associates the easily grasped microeconomic elements of quantitative character in order to arrive at both more general conclusions and at concrete formulae defining the way the market mechanisms work under definite assumed conditions.

Some may consider this exposition too simplistic. In fact, it is deliberately kept very simple, for heuristic purposes, as well as in order to make the conclusions more clear. Adding a lot of details that make theory more realistic – these details, indeed, changing from country to country, and from sector to sector – is mainly left to the Reader, who is supposed to be able to design the more accurate image on the basis of the foundations, provided in the book.

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ISBN 9788389475343