Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

SRI PAS IBS PAN

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Systems Research Institute
Polish Academy of Sciences

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

IBS PAN SRI PAS

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 83-894-7540-5

Dedicated to Professor Beloslav Riečan on his 75th anniversary

Decision making problem on intuitionistic fuzzy relations

Paweł Drygaś and Barbara Pękala
Institute of Mathematics, University of Rzeszów
Rejtana 16A, Rzeszów, Poland
paweldr@univ.rzeszow.pl, bpekala@univ.rzeszow.pl

Abstract

We propose the use of intuitionistic fuzzy sets as a tool for reasoning under imperfect facts and imprecise knowledge.

We consider two issues for solution group decision problem in medical diagnosis. We suggest application of diverse types of composition of intuitionistic fuzzy relations to this problem and we analyze their connection with the reasoning scheme which is based on a distance between Atanassov's intuitionistic fuzzy sets.

Keywords: Atanassov's intuitionistic fuzzy sets, decision making problem, medical diagnosis.

1 Introduction

In this paper we propose the use of inuitionistic fuzzy sets as a tool for reasoning under imperfect facts and imprecise knowledge.

We consider two issues for solution group decision problem in medical diagnosis. Since the connections between patients and symptoms, and symptoms and diagnosis are given by intuitionistic fuzzy relations, a composition of fuzzy relations is a natural way which allow to obtain connection between patients and diagnosis. However, using the method based on a distance between Atanassov's

New Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2012.*

intuitionistic fuzzy sets we obtain more satisfactory answer. The idea of this paper is to use diverse types of composition of intuitionistic fuzzy relations to this problem, analyze their connection with the reasoning scheme which is based on distance between Atanassov's intuitionistic fuzzy sets and find a compromise between these two methods by proposing the assumption that has all (or given) properties obtained by the use of a method based on a distance between Atanassov's intuitionistic fuzzy sets. At the end of this paper we put open problems which are connected with considered problem.

2 Basic definitions

Now we recall some definitions which will be helpful in our investigations. One of the possible generalizations of a fuzzy set in X and a fuzzy relation in $X \times Y$ is an Atanassov's intuitionistic fuzzy sets or relations assign to each element of the universe not only a membership degree but also non-membership degree.

Definition 1 ([1])**.** *Let* $X, Y \neq \emptyset$*,* $A,\ A^d : X \to [0,1]$*,* $R,\ R^d : X \times Y \to [0,1]$ *be fuzzy sets fulfilling the condition*

$$A(x) + A^d(x) \leq 1,\ \ x \in X$$

and fuzzy relations fulfilling the condition

$$R(x,y) + R^d(x,y) \leq 1,\ \ (x,y) \in X \times Y.$$

A pair $\mathcal{A} = (A, A^d)$ *is called an Atanassov's intuitionistic fuzzy set and* $\rho = (R, R^d)$ *is called an Atanassov's intuitionistic fuzzy relation. The family of all Atanassov's intuitionistic fuzzy relations (sets) described in the given sets* X, Y *is denoted by* $AIFR(X,Y)$ $(AIFS(X))$*. In the case* $X = Y$ *we will use the notation* $AIFR(X)$*.*

The boundary elements in *AIFR*(X,Y) are

$$\mathbf{1} = (1,0) \textit{ and } \mathbf{0} = (0,1),$$

where $0, 1$ are the constant fuzzy relations.
Moreover, basic operations for $\rho = (R, R^d)$, $\sigma = (P, P^d) \in AIFR(X,Y)$ (see [2], Definition 1.4) are the union and the intersection build with fuzzy operations, respectively

$$\rho \vee \sigma = (R \vee P, R^d \wedge P^d),\ \ \rho \wedge \sigma = (R \wedge P, R^d \vee P^d).$$

These operations can be presented more generaly by supremum and infimum. So for arbitrary set $K \neq \emptyset$

$$(\bigvee_{k\in K} \rho_k)(x,y) = (\bigvee_{k\in K} R_k(x,y), \bigwedge_{k\in K} R_k^d(x,y)),$$

$$(\bigwedge_{k\in K} \rho_k)(x,y) = (\bigwedge_{k\in K} R_k(x,y), \bigvee_{k\in K} R_k^d(x,y)).$$

Moreover, we can compare elements of $AIFR(X,Y)$. The order is defined by

$$\rho \leq \sigma \ \Leftrightarrow \ (R \leq P,\ P^d \leq R^d).$$

$(AIFR(X,Y), \leq)$ is a partially ordered set.
Operations $\vee, \wedge$ are the binary supremum and infimum in the family *AIFR*(X,Y), respectively and (*AIFR*$(X,Y), \vee, \wedge)$ is a complete, distributive lattice.

The fuzzy relation $\pi_\rho : X \times Y \to [0,1]$ is associated with each Atanassov's intuitionistic fuzzy relation $\rho = (R, R^d)$, where

$$\pi_\rho(x,y) = 1 - R(x,y) - R^d(x,y), \ \ x \in X,\ y \in Y.$$

The number $\pi_\rho(x,y)$ is called an index of an element (x,y) in an Atanassov's intuitionistic fuzzy relation ρ. It is described as an index (a degree) of hesitation whether x and y are in the relation ρ or not. This value is also regarded as a measure of non-determinacy or uncertainty and is useful in applications.

Primarily we will use special classes of operations which are useful in our considerations for *FR*(X,Y), i.e. triangular norms (t-norms) and triangular conorms (t-conorms), which are useful in approximate reasoning, e.g. for medical diagnosis or information retrieval.

Definition 2 (cf. [6])**.** *A t-norm T is an increasing, commutative, associative operation* $T : [0,1]^2 \to [0,1]$ *with neutral element* 1.
A t-conorm S is an increasing, commutative, associative operation $S : [0,1]^2 \to [0,1]$ *with neutral element* 0.

Example 1. *Well-known t-norms and t-conorms are:*

$$T_M(x,y) = \min(x,y), \qquad S_M(x,y) = \max(x,y), \tag{1}$$

$$T_P(x,y) = x \cdot y, \qquad S_P(x,y) = x + y - xy, \tag{2}$$

These operations are associative and as a result they have n-argument representations.

$$T_M(x_1,...,x_n) = \bigwedge_{i=1}^{n} x_i, \qquad S_M(x_1,...,x_n) = \bigvee_{i=1}^{n} x_i, \tag{3}$$

$$T_P(x_1,...,x_n) = \prod_{i=1}^{n} x_i, \qquad S_P(x_1,...,x_n) = 1 - \prod_{i=1}^{n} (1-x_i). \tag{4}$$

Now, let us recall the notion of composition in its standard form

Definition 3 (cf. [5], [3]). *Let $\sigma \in AIFR(X \times Y)$, $\rho \in AIFR(Y, Z)$. Let denote $\sigma = (P, P^d)$, $\rho = (R, R^d)$. By the composition of relations σ and ρ we call the relation $\sigma \circ \rho \in$ AIFR$(X \times Z)$,*

$$(\sigma \circ \rho)(x,z) = ((P \circ R)(x,z), (P^d \circ' R^d)(x,z)), \tag{5}$$

where

$$(P \circ R)(x,z) = S_{y \in Y} T(P(x,y), R(y,z)),$$
$$(P^d \circ' R^d)(x,z) = T_{y \in Y} S(P^d(x,y), R^d(y,z)).$$

According to [3], Proposition 1 we obtain that the following condition holds

$$0 \leq (P \circ R)(x,z) + (P^d \circ' R^d)(x,z) \leq 1, \quad (x,z) \in X \times Z.$$

3 Medical diagnosis

We should pay attention to the fact that intuitionistic fuzzy relations we can use in medical diagnosis. As we know, the approach presented in [4] involves the following three steps:

1) Determination of symptoms.

A set of n patients is considered. For each patient p_i, $i = 1, 2, ..., n$, a set of symptoms $\mathcal{S}$ is given. As a result, an intuitionistic fuzzy relation $\mathcal{Q}$ is between the set of patients and the set of symptoms $\mathcal{S}$.

2) Formulation of medical knowledge expressed by intuitionistic fuzzy relations.

It is assumed that another intuitionistic fuzzy relation $\mathcal{R}$ is given - from a set of symptoms $\mathcal{S}$ to the set of diagnoses $\mathcal{D}$ (medical knowledge) and

3) Determination of diagnosis on the basic of a max-min composition of intuitionistic fuzzy relations.

The composition $\mathcal{C}$ of intuitionistic fuzzy relations $\mathcal{R}$ and $\mathcal{Q}$ describes the state of

a patient given in terms of a membership function $C(p_i, d_k)$ and non-membership function $C^d(p_i, d_k)$, for each patient p_i and diagnosis d_k.
There exist Atanassov's intuitionistic fuzzy relations which are not comparable. So we consider a fuzzy relation which will allow to obtain a diagnosis.

We generalize this method to diverse compositions and we compare the result with the method based on distance between two Atanassov's intuitionistic fuzzy relations.

Example 2. *Let there be set of patients:* $\mathcal{P} = \{A, B, C, D\}$. *The set of considered symptoms is* $\mathcal{S}$ *= {Temperature, Headache, Stomach pain, Cough, Chest pain}. The intuitionistic fuzzy relation* $\mathcal{Q} = (q, q^d)$ *is given in table below that contains descripture of symptoms for a group of patients.*

Table 1.

$\mathcal{Q}$	*Temperature*	*Headache*	*Stomach pain*	*Cough*	*Chest pain*
A	*(0.3, 0)*	*(0.9, 0)*	*(0.7, 0.3)*	*(0.7, 0)*	*(0, 1)*
B	*(0.3, 0.4)*	*(0.7, 0.3)*	*(0.5, 0.2)*	*(0.3, 0.7)*	*(0.5, 0)*
C	*(0.3, 0.7)*	*(0.4, 0.1)*	*(0.5, 0.5)*	*(0.3, 0.6)*	*(0.4, 0.5)*
D	*(0.3, 0.5)*	*(0.4, 0.2)*	*(0.2, 0.3)*	*(0.2, 0.6)*	*(0.4, 0.2)*

Let us consider the set of diagnosis $\mathcal{D}$ *={Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}.*

The intuitionistic fuzzy relation $\mathcal{R} = (r, r^d)$ *(the medical data describing dependencies between the symptoms and a diagnosis, as expert knowledge [4] is given in the table*

Table 2.

$\mathcal{R}$	*Viral fever*	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Temperature	*(0.4, 0)*	*(0.7, 0)*	*(0.3, 0.3)*	*(0.1, 0.7)*	*(0.1, 0.8)*
Headache	*(0.3, 0.5)*	*(0.2, 0.6)*	*(0.6, 0.1)*	*(0.2, 0.4)*	*(0, 0.8)*
Stomach pain	*(0.1, 0.7)*	*(0, 0.9)*	*(0.2, 0.7)*	*(0.8, 0)*	*(0.2, 0.8)*
Cough	*(0.4, 0.3)*	*(0.7, 0)*	*(0.2, 0.6)*	*(0.2, 0.7)*	*(0.2, 0.8)*
Chest pain	*(0.1, 0.7)*	*(0.1, 0.8)*	*(0.1, 0.9)*	*(0.2, 0.7)*	*(0.8, 0.1)*

Similarly as the mentioned authors we use the (max-min, min-max) composition and we obtain the relation $\mathcal{C} = (c, c^d)$ between the patients and the diagnosis given in the Table 3, where for all $i \in \mathcal{P},\ j \in \mathcal{D}$ (see (5), (1), (3)*)*

$$\mathcal{C}(i,j) = (\mathcal{Q} \circ \mathcal{R})(i,j) = \left(\bigvee_{k \in \mathcal{S}} (q(i,k) \wedge r(k,j)),\ \bigwedge_{k \in \mathcal{S}} (q^d(i,k) \vee r^d(k,j)) \right).$$

Table 3.

$\mathcal{C}$	*Viral fever*	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
A	*(0.4, 0)*	*(0.7, 0)*	*(0.6, 0.1)*	*(0.7, 0.3)*	*(0.2, 0.8)*
B	*(0.3, 0.4)*	*(0.3, 0.4)*	*(0.6, 0.3)*	*(0.5, 0.2)*	*(0.5, 0.1)*
C	*(0.3, 0.5)*	*(0.3, 0.6)*	*(0.4, 0.2)*	*(0.5, 0.4)*	*(0.4, 0.5)*
D	*(0.3, 0.5)*	*(0.3, 0.5)*	*(0.4, 0.2)*	*(0.2, 0.3)*	*(0.4, 0.2)*

Moreover, to compare the obtained results we construct the relation

$$S_{\mathcal{C}} = c - c^d \pi_{\mathcal{C}}. \tag{6}$$

Table 4.

$S_{\mathcal{C}}$	*Viral fever*	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
A	*0.4*	*0.7*	*0.57*	*0.7*	*0.2*
B	*0.18*	*0.18*	*0.57*	*0.44*	*0.46*
C	*0.2*	*0.24*	*0.32*	*0.46*	*0.35*
D	*0.2*	*0.2*	*0.32*	*0.05*	*0.32*

The greatest element in each row points out the diagnosis for a given patient. For example, we can see that patient B has typhoid, but for A we obtain the same value in two columns. Thus, we observe that this method doesn't give always the diagnosis.

This is why in the next paragraph we consider another method which was introduced by Szmidt and Kacprzyk.

3.1 Distance between any two intuitionistic fuzzy sets

The method uses distance between any two intuitionistic fuzzy sets $\mathcal{A}$ and $\mathcal{B}$ containing n elements [7], e.g. the normalized Hamming distance:

$$l_{IFS}(\mathcal{A},\mathcal{B}) = \frac{1}{2n}\sum_{i=1}^{n}(|A(x_i)-B(x_i)|+|A^d(x_i)-B^d(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|),$$

where $0 \leq l_{IFS}(\mathcal{A},\mathcal{B}) \leq 1$.
To find a proper diagnosis we minimize the distance between intuitionistic fuzzy relations $\mathcal{R}$ and Q.

Table 5.

$l_{IFS}(\mathcal{R},\mathcal{Q})$	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
A	0.32	0.33	0.29	0.41	0.67
B	0.36	0.51	0.27	0.27	0.33
C	0.27	0.42	0.2	0.2	0.3
D	0.28	0.43	0.19	0.25	0.27

So we calculate for each patient the distance between her symptoms and the symptoms characteristic for each diagnoses using the following formula

$$l_{IFS}(\mathcal{R},\mathcal{Q})(i,j) =$$

$$\frac{1}{10}\sum_{k=1}^{5}(|R(k,i)-Q(j,k)|+|R^d(k,i)-Q^d(j,k)|+|\pi_R(k,i)-\pi_Q(j,k)|),$$

where $i \in \mathcal{D}$, $j \in \mathcal{P}$.
To find a proper diagnosis we minimize the obtained distance. We observe that obtained result is different from the one obtained in the first method (patient D) and we do not have clear diagnosis for B and C.

This is why, we consider other types of compositions in order to find the optimal diagnosis which will coincide with the distance method and this new method also give clear solution.

First we apply (max-T_P, T_P-max) composition and we obtain the relation $\mathcal{C} = (c, c^d)$ between the patients and the diagnosis given in the Table 6, where for all $i \in \mathcal{P}$, $j \in \mathcal{D}$ (see (5), (1), (3), (2), (4))

$$\mathcal{C}(i,j) = (\mathcal{Q}\circ\mathcal{R})(i,j) = \left(\bigvee_{k\in\mathcal{S}} T_P(q(i,k),r(k,j)),\ \underset{k\in\mathcal{S}}{T_P}(q^d(i,k)\vee r^d(k,j))\right).$$

Table 6.

$\mathcal{C}$	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
A	(0.28, 0)	(0.49, 0)	(0.54, 0.1)	(0.56, 0.3)	(0.14, 0.8)
B	(0.21, 0.4)	(0.21, 0.4)	(0.42, 0.37)	(0.4, 0.2)	(0.4, 0.1)
C	(0.12, 0.6)	(0.21, 0.6)	(0.24, 0.28)	(0.4, 0.5)	(0.32, 0.55)
D	(0.12, 0.5)	(0.21, 0.5)	(0.24, 0.28)	(0.16, 0.3)	(0.32, 0.28)

Using the formula (6) to Table 6 we obtain the diagnosis different from the ones in the distance method:

Table 7.

$S_{\mathcal{C}}$	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
A	0.28	0.49	0.504	0.518	0.092
B	0.054	0.054	0.3423	0.32	0.35
C	-0.048	0.096	0.1056	0.35	0.2485
D	-0.07	0.065	0.1056	-0.002	0.208

So, next we apply $(S_P - T_P, T_P - S_P)$ composition and we obtain the relation $\mathcal{C} = (c, c^d)$ between the patients and the diagnosis given in the Table 6, where for all $i \in \mathcal{P},\ j \in \mathcal{D}$ (see (5), (2), (4))

$$\mathcal{C}(i,j) = \left(\underset{k \in \mathcal{S}}{S_P} (T_P(q(i,k), r(k,j))), \underset{k \in \mathcal{S}}{T_P} (S_P(q^d(i,k), r^d(k,j))) \right).$$

Table 8.

$\mathcal{C}$	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
A	(0.569, 0)	(0.66, 0)	(0.69, 0.01)	(0.69, 0.06)	(0.28, 0.44)
B	(0.45, 0.11)	(0.49, 0.15)	(0.57, 0.13)	(0.57, 0.06)	(0.51, 0.06)
C	(0.38, 0.22)	(0.45, 0.24)	(0.44, 0.15)	(0.53, 0.17)	(0.44, 0.36)
D	(0.33, 0.13)	(0.39, 0.16)	(0.39, 0.11)	(0.34, 0.09)	(0.39, 0.16)

Using the formula (6) to Table 8 we obtain the same diagnosis as in the distance method and also we obtain the solutions to the previously not solved case (B and C).

Table 9.

S_C	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
A	0.569849	0.669622	0.686203	0.68478	0.160579
B	0.39948	0.43647	0.537699	0.554575	0.481776
C	0.290444	0.373825	0.376546	0.486323	0.370623
D	0.259622	0.329725	0.332474	0.28709	0.318353

As a result $(S_P - T_P, T_P - S_P)$ composition seems to be the best one.

4 Conclusions

On the basis of our observations we want to put the following problems:

• which properties of applied composition and operations are essential for obtaining the proper and clear diagnosis,

• how can we replace the relation S_C with another transformation for which there will be no such situation as for $\mathcal{C}$

$$(0.3, 0.7, 0) \quad and \quad (0.3, 0, 0.7)$$

where the second triple seems to be better (because the non-membership value is zero) but for both triples we obtain the same value of S_C,

• consider the composition according to the Goguen's definition but we don't like to use two separate compositions for membership and non-membership functions, which allow us to take into account also the value of hestitation degree (as it is considered in distance method used in paper [7]). It should be noted here that by the application the Definition 3 the hestitation function is not considered, only membership and nonmembership functions are taking into account.

Acknowledgments

This paper is partially supported by the Ministry of Science and Higher Education Grant Nr N N519 384936.

References

[1] Atanassov K.T. (1986) Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87–96.

[2] Atanassov K. (1999) Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag.

[3] Burillo P. and Bustince H. (1995) Intuitionistic Fuzzy Relations. Effect of Atanassov's Operators on the Properties of the Intuitionistic Fuzzy Relations. Mathware and Soft Computing, 2, 117–148.

[4] De S.K., Biswas R., Roy A.R. (2001) An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets and Systems, 117, 209-213.

[5] Goguen A. (1967) L-fuzzy sets. Journal of Mathematical Analysis and Applications, 18, 145–174.

[6] Klement E. P., Mesiar R., Pap E., (2000) Triangular norms. Kluwer Acad. Publ., Dordrecht.

[7] Szmidt E., Kacprzyk J. (2005) Distances between intuitionistic fuzzy sets. Fuzzy Sets and Systems, 114, 505-518.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) organized in Warsaw on September 30, 2011 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2011

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Tenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2011) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475404
ISBN 838947540-5
9 788389 475404