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Tensile strength and extensibility of concrete and mortar idealized as elastic- or plastic-brittle

D. C. DRUCKER (URBANA)

A COMPARISON is made between the predicted behavior and the actual performance of concretes and mortars based upon the extreme idealizations of perfectly elastic-brittle and perfectly plastic, pseudo-brittle behavior of a governing cement-based phase. Despite the enormous difference between the idealizations both can explain the appreciably higher tensile strength computed from flexural tests in comparison with direct tension or splitting tests. Both are consistent with the stable growth of microcracks and the applicability of a Griffith criterion for unstable fracture. Yet the differences are significant both conceptually and in practice. Reasons are given for preferring the plastic to the elastic idealization in partial analogy to the apparently brittle response of tungsten carbide-cobalt systems and the very low average tensile elongation of some aluminum alloys.

Przeprowadzono porównanie między teoretycznie przewidywanym a rzeczywistym zachowaniem się betonów i zapraw w oparciu o wysoce wyidealizowany model idealnie krucho-sprężystych lub idealnie plastycznych i pseudokruchych własności wiodącej fazy, opartej na cemencie. Mimo ogromnych różnic między tymi idealizacjami obie są w stanie wytłumaczyć fakt, że wytrzymałość na rozciąganie obliczona w próbie zginania daje wartości znacznie większe aniżeli bezpośrednie próby rozciągania. Obie metody uwzględniają stateczny wzrost mikronapięć i kryterium Griffitha dla nieustalonego procesu zniszczenia. Niemniej przeto różnice między nimi są istotne tak pod względem koncepcyjnym jak i praktycznym. Podano przyczyny, dla których należy jednak przyznać pierwszeństwo idealizacji plastycznej nad sprężystą, wobec częściowej analogii do kruchych właściwości ukłdaów wolframowo-karbidowo-kobaltowych oraz wobec bardzo niskiej średniej wydłużalności pewnych stopów aluminiowych.

Проведено сравнение между теоретически предвиденным и реальным поведением бетонов и растворов опираясь на высокой степени идеальную модель идеально хрупко-упругих или идеально пластических и псевдо-хрупких свойств ведущей фазы, опирающейся на цемент. Несмотря на огромные разницы между этими идеализациями обе они в состоянии объяснить факт, что прочность на растяжение вычисленная в испытании на изгиб дает значения значительно большие, чем непосредственные испытания на растяжение. Оба метода учитывают статический рост микротрещины и критерий Грифита для неустановившегося процесса разрушения. Все же разницы между ними существенны, так в идейном, как и в практическом отношениях. Даются причины, по которым следует однако признать превосходство пластической идеализации над упругой идеализацией ввиду частичной аналогии с хрупкими свойствами вольфрамо-карбидо-кобальтовых систем, а также ввиду очень низкого среднего удлинения некоторых алюминиевых сплавов.

1. Introduction

A VERY useful and interesting discussion on the tensile strength of concrete (or rock) and the proper experimental technique for its determination continues in the technical literature [1–5]. Comparison of special results in direct tension, in flexure, in ring tests, and in splitting or Brazilian tests has proved puzzling enough to lead to a variety of successful correlations [3–5]. Explanations include a statistical size effect due to flaw distribution, a simple or elaborate tensile strain or stress criterion, and a modified Griffith type of fracture hypothesis. Almost all of the comparisons have been based on the assumption of the validity of a linear or elastic theory for the calculation of stress and of strain. Yet W. F. CHEN [5] has been most successful with predictions for concrete and rock based upon plastic limit analysis.

The assumption of elastic behavior is understandable in view of what appears to be an almost linear response of concrete in tension or flexure to the very point of fracture [6–9]. However, it is all too similar to the same assumption which impeded understanding of the fracture of the sintered tungsten carbide-cobalt system and which so often confuses the analysis of brittle fracture in metals [10–11]. Highly localized plastic deformation on the microscale, as in the cobalt, and even on the macroscale, as in large metal structures, often does not show up appreciably in load-deflection measurements or in nominal stress-strain curves.

The almost hidden but highly effective plastic behavior of a binder constituent, such as cobalt in a sintered carbide, smooths out major stress concentrations around the particles bound together. It manifests itself in an otherwise mysterious factor close to 1.5 in the ratio of flexural to tensile strength when the linear assumption is made, Fig. 1. Such a factor

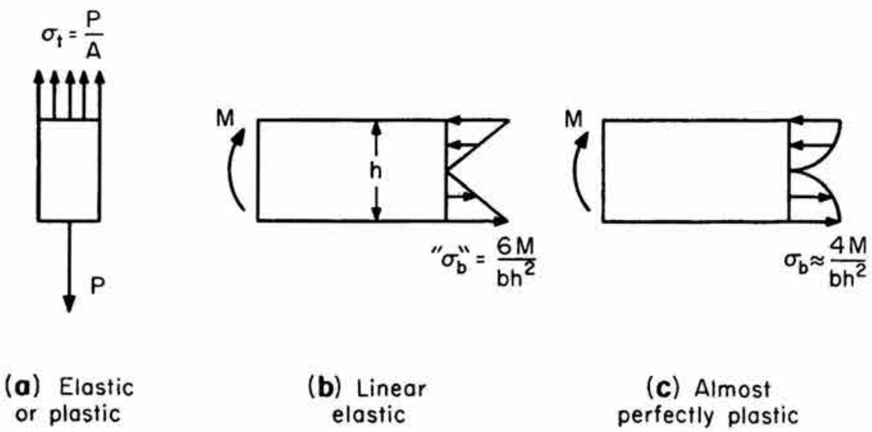

FIG. 1. "σ_b"$/\sigma_t \approx 1.5$.
A simple illustration with equivalent properties for tension and compression.

has been reported for mortars and for some concretes by KAPLAN [6], WELCH [9] and others. Therefore, it is well worth exploring the possibility that the cement phase in concrete (or the binder phase in rock) plays a similar although less effective role than the cobalt. The proposal, in fact, is made that greater understanding will be achieved and better although lower values of tensile and adhesive bond strength will be obtained by idealizing the cement phase in concrete as perfectly plastic and pseudo-brittle, Fig. 2a, rather than linear- or elastic-brittle, Fig. 2b. This is not to imply that the mortar or concrete really is perfectly plastic prior to fracture but only that this extreme approximation may be more realistic than the assumption of elastic behavior.

Of course, there is wide variability in the properties of the constituents from point to point in the mortar or concrete. Heterogeneity is all-pervasive in the real world [12]. Microcracks in large number do exist before any load is applied [8]. These cracks extend and many new ones develop as load is applied. The simplest realistic model would contain load carrying elements of a wide range of strength and deformability arranged both in series and parallel. Figure 3 shows that the apparent stress-strain curve in tension for a parallel

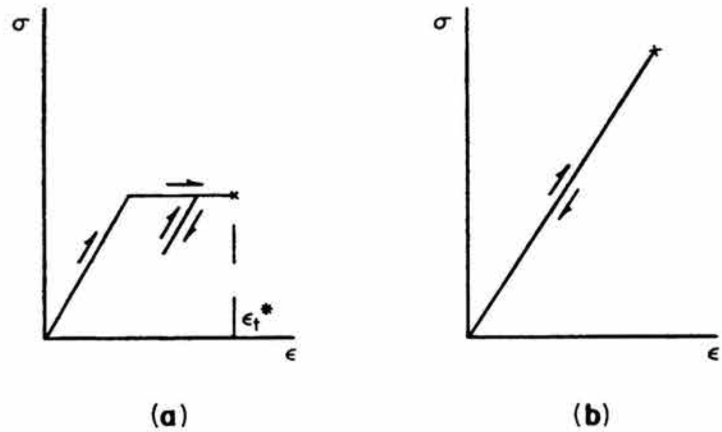

FIG. 2. Two extreme idealizations for cement phase. (a) Perfectly plastic — pseudo brittle. (b) Elastic-brittle.

arrangement of linear elastic elements of differing strengths in tension does resemble a plastic stress-strain curve and would equally well account for a marked overestimate of the tensile strength computed from a flexural test in which linear response is assumed in the calculations for stress. If Fig. 3 represents schematically the behavior of the weakest set of parallel elements in a chain of many such sets in series, the overall response of the

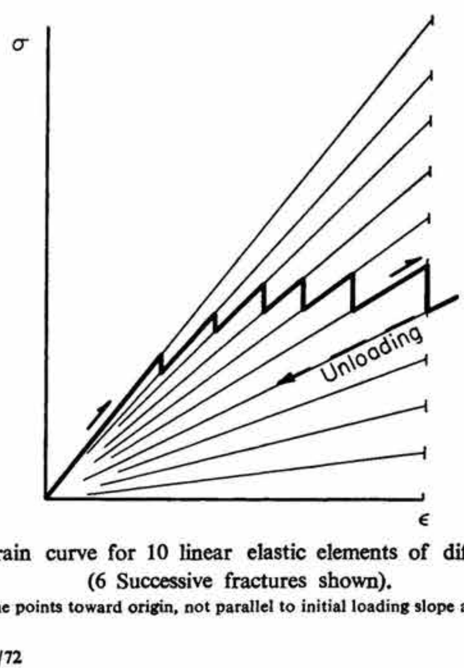

FIG. 3. Apparent stress-strain curve for 10 linear elastic elements of differing strengths in parallel (6 Successive fractures shown).

Note: Unloading line points toward origin, not parallel to initial loading slope as for plastic response.

test specimen would still be close to linear up to failure. The marked local difference between the unloading response of the two idealizations, elastic and plastic, also would not be that easy to differentiate. This would be especially true in a flexure test where elastic recovery on the compression side favors full return to the initial configuration.

Several highly idealized plastic models

Figure 4 is a guide to several crude plastic models for the behavior of concrete in tension. An elastic-plastic mortar surrounds the elastic aggregate to which it is or is not reasonably well bonded. In turn the mortar is a composite of an elastic-plastic cement

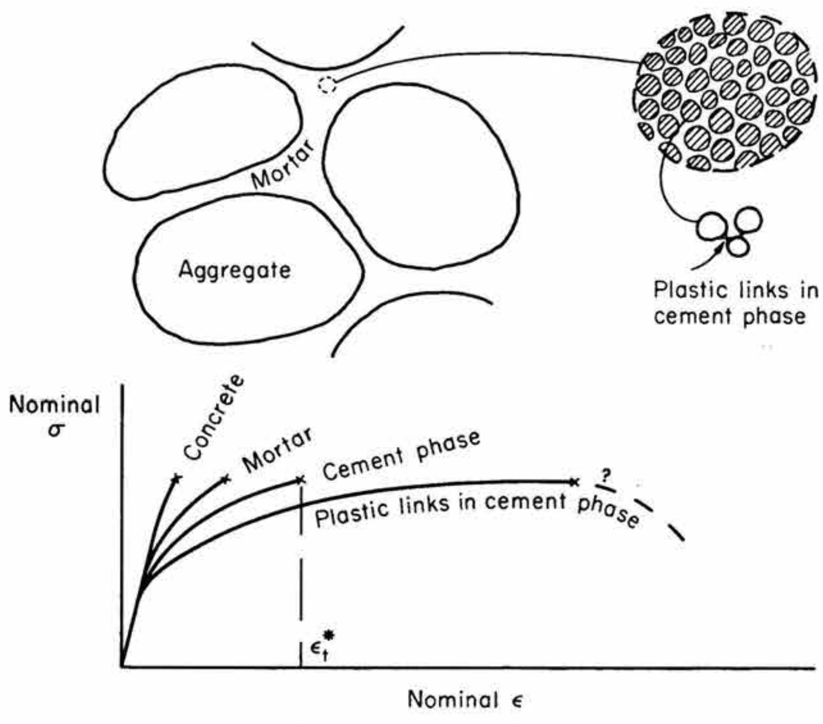

FIG. 4. A possible crude first approximation for mortar and concrete.

phase with empty or water-filled voids which surrounds the sand particles. The bond to the sand may be strong, weak, or absent. To add to the considerable complexity of what is at best a greatly oversimplified picture, voids are present on the scale of the sand particles or on a much smaller scale within the cement phase. Voids on the scale of the aggregate may be employed for architectural texture but otherwise would indicate poor concrete practice.

The behavior of the elastic-plastic models of mortar or concrete depend critically on the void size and distribution as well as the presence or absence of bond. Some indication of the appropriateness of one model as compared with another can be found through comparison of experimental or test data and the predictions of each model.

Some prediction of the plastic models

One extreme possibility for the cement phase is that it is almost void free, another that the voids are numerous and comparable in volume to the solid phase. The second leads to the simpler answers represented pictorially in Fig. 4 and will be examined first.

A sufficiently high and uniform distribution of voids in an elastic-plastic phase nullifies any plastic constraint. Closely enough for the purpose of this discussion, yielding, flow, and fracture will follow for an almost perfectly plastic response when the average stress on the solid material reaches the yield strength in simple tension, Fig. 2. Figure 4 shows a more realistic gradual approach to fracture at ε_t^* in the cement phase. The large local deformation of the plastic links in the cement phase, just like the infinite local ductility of aluminum in some "brittle" aluminum alloys, is reflected as a small nominal strain to fracture because the deforming region is so limited [10].

If the strength of the bond between the sand particles and such a cement phase exceeded the flow stress of the cement phase, then the tensile strength of the mortar would be equal to the tensile strength of the cement phase. Nominal strain to failure, however, would be but a fraction of ε_t^* as shown schematically in Fig. 4. Similarly, if the bond between the mortar and the aggregate exceeded the flow stress of the mortar, the tensile strength of the concrete would be equal to the tensile strength of the mortar. With sufficient bond to the sand, this in turn would be the tensile strength of the cement phase. The nominal strain to fracture in either case would be little greater than the elastic response, Fig. 4; almost all the enormous but highly localized plastic response would be masked whether or not the sand was well bonded.

If, however, the bond to the aggregate failed early over a large fraction of the contact area [13], the tensile strength of the concrete would be far less than that of the mortar. In the limit of zero bond strength, the strength of the concrete would be less than the strength of the mortar multiplied by the volume fraction of mortar in the concrete. In a random distribution, the cross-sectional area fraction and the volume fraction are equal. The fracture path through the mortar, however, could and would meander to involve far less mortar than for a random path. Similarly, if the bond to the sand were zero or negligible, the tensile strength of the mortar would be less than the tensile strength of the cement phase multiplied by the volume fraction of that phase.

Nonlinear viscous behavior or moderate work-hardening leads to the same qualitative answer as for perfectly plastic behavior. In the absence of significant constraint to the flow in the binder phase, and with adequate bond strength, the flow and therefore the fracture tensile stress for the assemblage is the nominal fracture stress of the binder phase alone.

If, on the contrary, the cement phase were reasonably void free along with the mortar and the concrete, a quite different result would emerge, provided *all* bond strengths were adequate. The picture now would be like the tungsten carbide-cobalt system [11] with the aggregate replacing the carbide and the mortar replacing the cobalt. Sand in volume fractions of 1/2 or more would plastically constrain the cement phase and raise its flow stress appreciably. Thin layers of mortar between the aggregate would also be strongly constrained plastically so that the flow strength would be far greater than the flow stress of the cement phase itself.

However, just as in the case of sintered carbides, local bond strength would become inadequate as the volume fraction of inclusions and the constraint factor rises. At sufficiently high volume fractions, limit loads could not be reached, local stresses would become high, and bond failure would lead to decreasing tensile strength with increasing constraint. The special and ingenious briquet type tensile test reported in Ref. [13] may well lie in this range of high plastic constraint and so lead to earlier bond failure than in less restrained concrete.

If the bond between the sand and the cement phase were a weak link, then the tensile strength of the concrete would again equal the strength of the mortar, but the fracture stress for the mortar would be somewhat less than the fracture stress of the cement phase multiplied by the volume fraction of that phase in the mortar.

Comparison with experiments by WELCH [9]

Table 1 is a direct copy of Table 1 of Ref. [9] with only the "Special" Concrete entries 18, 19, 20 omitted. Mortar strength and concrete strength agree very well for the pavement quality concrete and the high strength concrete with crushed granite aggregate. The strength reported for the lean concrete exceeds that of the mortar.

At first glance, the plastic model with bonded aggregate and unbonded sand particles would seem in principle to be verified beyond doubt, because the ratios of mortar strengths in flexure are 1055:695:170 for proportions by weight of cement to sand of 1:1.0, 1:2.1, 1:6.0. In agreement with that model, the volume fraction of the cement phase does seem to give the strength of the mortar, while the volume fraction of aggregate makes little if any consistent difference. Unbonded aggregate prior to fracture would lead to far lower values for the concretes.

However, quite another explanation might be advanced. Perhaps the correlation of mortar strength with cement content might be just a coincidence. The water/cement ratio might govern, a concept well accepted since the work of ABRAMS. Table 1 does not permit this question to be examined because each grade of concrete has just one water/cement ratio and one sand/cement ratio. However, the data of Ref. [13] demonstrate that the water/cement ratio has only a small effect on the tensile strength of cement paste and mortar despite the large influence on compressive strength.

What Table 1 does establish clearly is the absence of a significant constraint factor due to the aggregate; the same strength is obtained for markedly different volume fractions. Stress concentration effects also seem minor because the rounded gravel aggregates produced lower, not higher, strengths than the angular granite. The data do indicate inelastic amelioration of stress concentrations along with some governing weakness of bond between the cement phase of the mortar and the gravel, as in Ref. [13], but not the granite.

An elastic-brittle model

As described earlier [7], an elastic-brittle model of the mortar could give a fairly flat nominal stress-strain curve, Fig. 3, and the 1.5 factor of Fig. 1 as well as the greater extensi-

Table 1. (From Welch, Ref. [9])

Mix	Class of concrete	Ratio: free water / cement	Size and type of coarse aggregate	Proportions by weight cement:sand:stone	Workability: Slump (in.)	Workability: Compacting factor	Workability: Vebe (degrees)	28 day strengths: Flexural strength (lb/in²)	28 day strengths: Compressive strength (lb/in²)
1	High strength	0.35	¾ in. round gravel B	1:1:2.5	2	0.85	4.2	755	8.820
2			¾ in. irreg. gravel C		½	0.78	13.0	695	9.070
3			¾ in. crushed granite		0	0.65	30.0	1.040	11.170
4			nil	1:1:0	9	0.99	—	1.055	7.800
5	Pavement quality	0.50	1½ in. round gravel B	1:2.1:6	¾	0.88	5.0	500	4.810
6			1½ in. irreg. gravel C	1:2.1:4.7	½	0.85	7.0	510	6.160
7			1½ in. crushed granite		½	0.84	7.5	650	6.800
8			¾ in. round gravel B	1:2.1:5.3	½	0.86	5.2	565	6.180
9				1:2.1:4.1	3½	0.95	3.0	575	5.490
10			¾ in. irreg. gravel C		1	0.87	5.9	505	5.620
11			¾ in. crushed granite		¾	0.84	7.0	715	6.600
12				1:2.1:4.7	0	0.82	14.8	695	7.090
13			nil	1:2.1:0	9½	0.99	—	695	5.370
14	Lean base	1.00	1½ in. round gravel B	1:6:11	¼	0.87	8.0	220	1.490
15			1½ in. irreg. gravel C		0	0.81	18.0	220	1.850
16			1½ in. crushed granite		0	0.78	20.0	270	2.060
17			nil	1:6:0	1½	0.93	2.0	170	830

bility of mortar than concrete. It would also predict the equality of mortar strength and concrete strength in tension for adequate adhesion of mortar and aggregate. Stable cracking and absence of notch sensitivity also could be explained by the presence of small voids everywhere and by multiple cracking. Together, the voids and multiple cracks would lower the stress at the tip of a crack from the enormous values for a homogeneous solid with a single sharp crack to a far smaller but still very high value.

Although the curved stress-strain relation for simple loading could be either elastic-brittle or elastic-plastic at the very local level, the unloading curve, if it could be observed, would be very different for each type of behavior. A more important difference still is the implication for elastic-brittle material of very high local stresses in the mortar and at the surface of the aggregate over dimensions of the order of a moderate fraction of the size of a small sand particle. Local concentrations of more than 10 times the applied stress would occur over such small but still macroscopic regions in the vicinity of the ends of cracks or zones of separation between mortar and aggregate when the response is elastic. The unconstrained plastic picture which does not have such concentrations is far more comforting. Theoretical calculations of bond strengths might well be of help here.

Conclusion

A comparison from the literature of the tensile and flexural strengths of mortar and concrete over a range of water/cement, sand/cement, and aggregate/sand ratios lends credence to the suggestion that the cement based phase behaves in an elastic-plastic-pseudo-brittle, not an elastic-brittle, manner. The rough equality which can be achieved between the tensile strength of concrete and the mortar it contains shows that adequate bond to the aggregate can be maintained most everywhere up to the fracture stress. However, an examination of the variation of the tensile strength of concrete or mortar with the water/cement and sand/cement ratio suggests that the usual bond between the sand particles and the cement phase is too weak in tension to do more than permit the load to be carried by the cement phase in proportion to its volume fraction. The tensile strength of concrete, to a crude first approximation, then is given by the tensile strength of the mortar. In turn the tensile strength of the mortar is the strength of the cement phase multiplied by its volume fraction in the mortar. If the reasoning presented is correct and if adequate bond could be achieved between the cement phase and the sand, the tensile strength of the concrete would be the tensile strength of the cement phase itself, a very much higher value than is found in lean concretes.

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COLLEGE OF ENGINEERING,
UNIVERSITY OF ILLINOIS, URBANA

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