SEMI-ACTIVE CONTROL OF MECHANICAL ENERGY TRANSFER BETWEEN VIBRATIONAL MODES

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1. Introduction

The vibration attenuation problem has been solved using many different methods, some of which involve the use of advanced control algorithms [1]. The topic of harvesting the energy of structural vibrations is less explored [2]. For that reason, this contribution studies the problem of conversion of mechanical energy of vibrations. The paper presents a method of semi-active control, which is applied to dynamically transfer the vibration energy into a selected vibration mode. The target mode is selected in such a way that the amount of energy that can be recovered during the vibration process is maximized. In other words, switching between two modes is not intended to dissipate the energy of vibrations, but rather to maximize the energy-harvesting potential of the overall system. The concept will be illustrated using an example of a simple frame structure, in which semi-actively controlled lockable joints modify the modal properties of the structure [3, 4].

2. Mathematical formulation

We consider a linear discretized undamped system with the following equation of motion:

(1)
$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \quad ,$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, and $\mathbf{q}$ denotes the vector of configuration coordinates (displacements and rotations of nodes). In the above equation all nodes are unlocked. When a selected node is locked then the corresponding degree of freedom (Dof) becomes constrained and effectively lost, and the matrices $\mathbf{M}$ and $\mathbf{K}$ change. Instead of adjusting the number of Dofs, we introduce a damping matrix $\mathbf{C}(\mathbf{u})$ with very large damping factors. Here $\mathbf{u}$ is the vector of the control signals (one signal for each lockable node). The damping factors and the zeros are distributed in the matrix in such a way that there is a negligible difference of angular velocities of beam ends connected at the locked node (as in a rigid connection). Each node can be controlled independently of other nodes. The state equation derived from equation (1), including the controlled damping matrix, is as follows:

(2)
$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}(\mathbf{u}) \end{bmatrix} \mathbf{x} \quad ,$$

where $\mathbf{x}$ is the vector of state variables, $\mathbf{I}$ denotes the identity matrix.

Our control aim is to stabilize the system (2) while transferring the energy to selected modes. For that purpose, we define the energy of the p-th mode corresponding to its amplitude

(3)
$$V_p = a_p^2 + \frac{1}{\omega_p^2}\dot{a}_p^2 \quad .$$

Here a_p is the amplitude of the p-th mode and ω_p is the p-th natural frequency. The total energy V of the system is given by $\sum_p V_p$. In order to transfer the system's energy to a selected mode p, we aim to increase the corresponding energy V_p. The proposed control maximizes the derivative of V_p

(4)
$$\mathbf{u}^* = \arg\max_{\mathbf{u}\in U} \dot{V}_p \quad .$$

The existence of the solution to (4) is ensured by the compactness of the set of admissible controls $U = [0, u_{max}]^m$ (m – number of control inputs). Note that for $u = 0$ the node is unlocked and for u_{max} it is jammed. In addition to (4) we require $\dot{V} \leq 0$ which for (2) guarantees stability.

3. Numerical example

As an example we present a simple frame structure with two lockable nodes and five degrees of freedom. There are five modes of vibration when all nodes are unlocked. The example initial condition is that the third, fourth and fifth modal amplitudes have respectively the values of {1, -1, 1}, while the initial amplitudes of the first and the second modes are equal to zero. The first mode is chosen for maximization of its participation in vibrations. This is shown in figure 1a). The results of numerical simulations are presented in figures 1b) and c) which correspond to, respectively, the modal amplitudes and the energies. The mode shapes are not normalized with respect to the mass matrix **M**.

a)

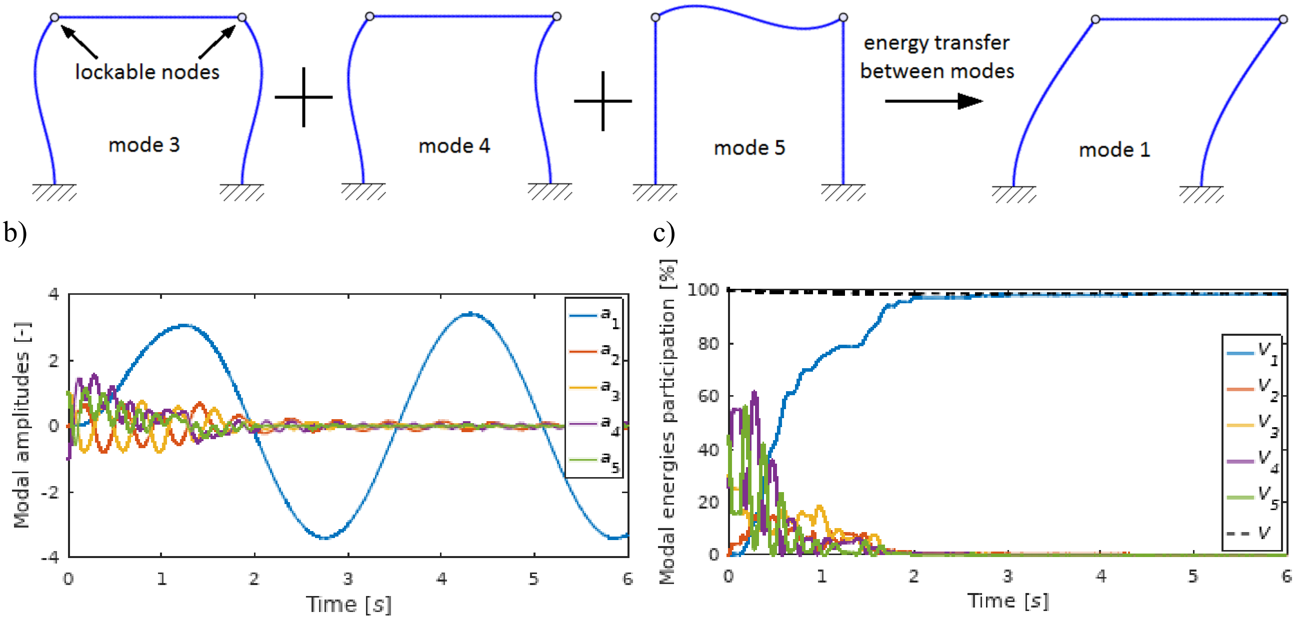

Fig. 1 Numerical example: a) visualization of initial modes and modes chosen for maximization, b) Time history of modal amplitudes, c) Time history of mode mechanical energies

4. Conclusion and further research

It is possible to dynamically switch between vibrational modes while keeping the system non-asymptotically stable. Non asymptotic stability is desired here, because it results in conservation of energy for an energy-harvester. The presented approach can be applied to structures excited by a short force impulse. After transferring the energy to a mode suitable for the installed energy-harvester, the mechanical energy can be extracted. In further research we will analyse more complex structures, use a model of the energy harvester, and test other control algorithms.

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