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**MODELLING CONCEPTS  
AND DECISION SUPPORT  
IN ENVIRONMENTAL SYSTEMS**

**Editors:**

**Jan Studzinski  
Olgierd Hryniewicz**

Polish Academy of Sciences • Systems Research Institute

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The purpose of the present publication is to popularize information tools and applications of informatics in environmental engineering and environment protection that have been investigated and developed in Poland and Germany for the last few years. The papers published in this book were presented during the workshop organized by the Leibniz-Institute of Freshwater Ecology and Inland Fisheries in Berlin in February 2006. The problems described in the papers concern the mathematical modeling, development and application of computer aided decision making systems in such environmental areas as groundwater and soils, rivers and lakes, water management and regional pollution. The editors of the book hope that it will support the closer research cooperation between Poland and Germany and when this intend succeeds then also next publications of the similar kind will be published.

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## CHAPTER 3

# **Water management and Decision support**



## DESIGN AND USE OF DECISION ORIENTED APPLICATIONS IN THE TRANSCAT PROJECT DSS

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***Abstract:** The Transcat DSS, or TDSS, designed and developed within the framework of the 5 FP project TRANSCAT, contains in particular, the layer of software applications concerning the aspect of direct decision analysis and support. These decision analytic (DA) applications are meant to serve a range of functions related to decision elaboration, analysis and support under various circumstances. Of particular importance is the possibility of enabling in the framework of the range of DA applications the societal discourse (public participation), accommodation of various opinions, evaluation of diverse options, as well as design and implementation of decision-making procedures for definite routine or emergency situations. The present paper addresses the DA layer, by describing its prerequisites and design, contents and functions of individual DAs, as well as their implementation. An ampler documentation of the DAs is provided together with each of them in their implemented forms. It is shown how the DAs designed, developed and implemented within the framework of the TRANSCAT Project fulfil the functions required of this layer of the TDSS, in conformity, in particular, with the Water Framework Directive and its stipulations. The paper indicates also how the DAs can and should be used in real-life situations. The present paper does not describe the mDSS or Mulino application, except for a short note, as developed within a different project, and modified in TRANSCAT by the developers (FEEM). Appropriate references are made to that application whenever required.*

**Keywords:** decision support, web-based applications, water resource management, participative management.

### 1. Introduction – the rationale

The TRANSCAT project deals with integrated water management according to individual water catchments, particularly the transboundary catchments, where the usual problems associated with adequate management of water resource systems tend to be severely compounded by the existence of state boundaries.

The main objective of the project is the development and implementation of an operational, possibly integrated and comprehensive Decision Support System (DSS), allowing for optimal water management of catchments stretching across state boundaries. The development of the DSS is primarily the responsibility of the Polish

team of the Project, hosted by the Systems Research Institute of the Polish Academy of Sciences in Warsaw.

The proposed DSS is able to cope with the complexity of the water resources systems and the uncertainty of decision-making. The DSS is built around the core, dealing mainly with data management and processing, as well as communication, and its functions that allow for effective consideration of the range of different climatic, topographic, environmental and socio-economic conditions found in various EU and candidate countries' transboundary catchment areas. Section 2 describes the overall system structure and its basic functions.

The concrete DSS realisations, along with the complementary elements and modules, have been implemented in the Pilot Areas of the Project, so as to provide the verification ground for the principles and technical solutions selected for the DSS development, and to gather the experience from actual use.

In particular, the Transcat DSS, or TDSS, contains the layer of software applications concerning the aspect of direct decision analysis and support. These decision analytic (DA) applications are meant to serve a range of functions related to decision elaboration, analysis and support under various circumstances. Section 3 describes the basis for design of the set of DAs and the individual applications, as well as the principles of their use. Individual DAs are described in somewhat deeper detail in Section 4.

In this context, of particular importance is the possibility of enabling in the framework of the range of DA applications the societal discourse (public participation), accommodation of various opinions, evaluation of diverse options, as well as design and implementation of decision-making procedures for definite routine or emergency situations.

Thus, we address the DA application layer, by describing its prerequisites and design, contents and functions of individual DA applications, as well as their implementation. An ampler documentation of the DAs is provided with each of them in their implemented forms.

It is shown here how the DAs designed, developed and implemented within the framework of the TRANSCAT Project fulfil the functions required of this layer of the TDSS, in conformity, in particular, with the Water Framework Directive and its stipulations.

The paper indicates also how the DAs can and should be used in real-life situations. Section 5 is devoted to this aspect.



## 2. The structure and the functions of the TDSS

The overall structure of the DSS system designed and developed within the TRANSCAT Project, the TDSS, is shown in the scheme on the next page.

The system, as outlined on the scheme, is composed of three essential kinds of elements, from the top to the bottom of the scheme:

- The top layer is composed of the *decision analytic applications* (DAs), including mDSS (MULINO), Mediator, ProDec, Bargain and ArgWar, most of them associated with strategic type of decisions (policies, large projects, etc.), but also with the design of decision procedures (ProDec), including the immediate (emergency) decision procedures,
- The middle layer contains the *core system*, split into the server (cTD) and client (xTD) components, being the main data provision, processing and interfacing tool, oriented mainly at operational functions, but, naturally, providing also the basis for strategic analyses and designs, and
- The bottom layer, consisting of *models*, and the related applications, serving to represent individual components of the natural, technical and socio-economic object system (e.g. the HEC-HMS surface flow model, the MODFLOW underground flow model, etc.).

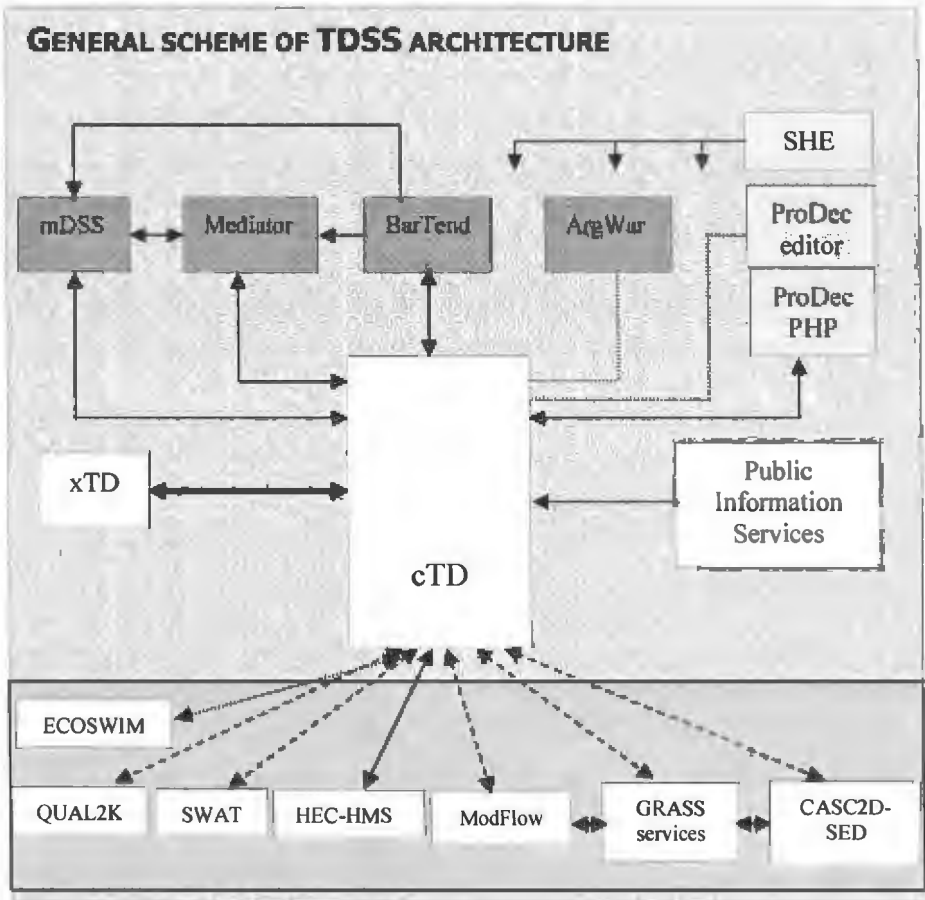
Additionally, there are some auxiliary applications, like SHE, the editing tool facilitating parallel translations of documents, and the use of Public Web Services, accessible through the interfaces provided by the cTD.

The general character of the functions of these three kinds of elements is as follows:

- the decision analytic applications, DA, serve primarily to represent and analyse the *value* layer of the processes and phenomena considered; they are meant to provide support for the multifaceted evaluation of the various choices that exist or may exist within the object system (evaluations guiding the choice of decisions, actions, projects, policies, etc.); the DAs included in the TDSS range from very simple ones, like Bargain, allowing for the establishment of bargaining output in well-structured uncomplicated situations, to relatively complex, like mDSS, which includes several stages and options in the process of evaluation within a broadly defined projects; resulting from the value-based assessments and the relations between particular objects in the object system (here: water resource system) there may be different actions, and in case of definite action-oriented decision procedures they can be designed and edited with the help of ProDec;
- the core system, the cTD together with the client xTD, handle the *information* layer regarding the object system, including basic processing, retrieval, visualisation, composition, and interfacing between the (other) elements providing information, processing it and receiving; the cTD serves as the main interfacing tool

for the elements of the system; it also provides authorisation and authentication services, integrated database acting as data warehouse, map services and data access services by means of the web technology; a smart web client application is an entry point for typical end-users;

- the models represent the layer of *relations* among data items, leading to the possibility of *prediction*, even if at a qualitative level, scenario or policy construction, or identification (e.g. through “simulation/derivation”) of data items not observed directly.



The TDSS is not an integrated system in the sense of “stiff” connections between the system elements. First, all of its elements can be used as self-standing entities (this being, in particular, obvious for the commonly available model applications). The connections are actually established according to needs, though the (interfacing) facilities for them are made available.

A concrete TDSS implementation can be configured according to the needs of a specific object system. Thus, out of the architecture outlined before, a subsystem would be carved out for purposes of a given object system.

### 3. The functional design of the DAs

The range of decision applications designed and developed for the purpose of TDSS was meant to cover a definite scope of functions associated with the analysis, preparation and supporting of decisions regarding definite problems, project options, etc.

These functions can be deployed according to the general scheme, consisting of the essential functions, which ought to be incorporated into a DSS of a similar kind, presented in the following. This scheme is here complemented by the comments related to the (potential) realisation and meaning of the respective functions for the broader decision-making process.

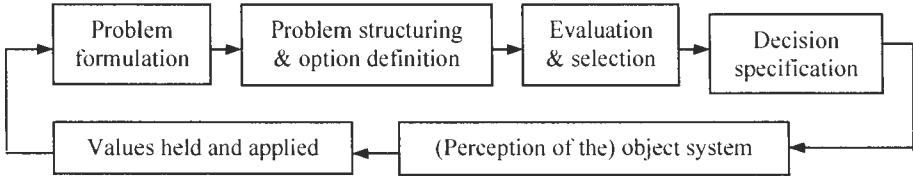
Note that the functions as here proposed and commented upon are understood in the sense of potential computer-based applications.

**Table 1.** Decision-related functions and their significance

Function	Domain & role	Standard realisation(s)
Opinion gathering	Public participation	Voting-like web-based procedures
Debate	Public participation Problem formulation	Web-based discussion groups
Problem structuring	Identification of problem structure: - basic entities & notions - relations among them - solution options - evaluation aspects	Brainstorming & cognitive mapping applications
Evaluation	Definition of values, objectives, criteria, etc., and their yardsticks Assessment of options	Multicriteria appraisal, review and improvement techniques Group decision rules
Choice	Ultimate selection	Aggregation and/or selection measures
Decision procedure design	Design and running of decision procedures (e.g. for emergency management)	<i>Scarcely any (mainly editing applications or, e.g. decision tree identification)</i>
Editing support	Support for the other functions	Standard editing applications

The above functions are in reality deployed in accordance with certain logic of decision process and the institutional realisation of this process. This process

takes on the form of a loop, which works in an iterative (or recursive) manner along a number of dimensions.



**Figure 1.** The scheme of the decision-making process loop.

The most important dimensions, along which the decision process loop is deployed over time, include

- the advance from the initial idea (problem perception) to a precise decision content, through the stages of initial assessments, followed by more and more accurate formulations of the particular elements of the process, shown in the scheme;
- the institutional dimension, in which either different organisations or more complex institutional frameworks enter in a certain sequence, defined by the managerial procedures, and by the appropriate social processes.

Of course, these two dimensions are tightly interconnected. It is the way, in which this interconnection is shaped, that defines the actual decision elaboration process. In any case, this process has of necessity to involve the functions outlined in Table 1.

It is desirable to ensure the appropriate fitting between, on the one hand, the functions from Table 1 and the corresponding software applications, which realise them, and, on the other hand, the elements and the deployment of the decision making process of Fig. 1. Thereby, the respective software applications may constitute an adequate decision support system or at least its essential component (the remaining potential component(s) being associated with the data layer and information processing).

This exactly was the prerequisite behind the design and development of the set of decision applications (DAs) for the TRANSCAT Project.

Table 2, below, provides a slightly ampler explanation of the roles fulfilled by these DAs against the functions of Table 1.

Thus, while the decision applications offered in the framework of the TDSS do by no means define any strict procedure of decision making (e.g. the necessity of public participation) they can be used in a sufficiently wide variety of such procedures, with exception, perhaps, of very specific technical, legal or organisational requirements, imposed on such procedures.

**Table 2.** Roles played by the DAs designed and developed for the TRANSCAT Project

Function	TRANSCAT DAs	Way of applying
Opinion gathering	ArgWar	Installation on a public web site of an involved stakeholder with an essential ("burning") question
Debate	Arg War	Web-based acquisition and processing of arguments within a definite group
Problem structuring	mDSS ArgWar	Cognitive mapping function within mDSS Argument structures from ArgWar
Evaluation	Bargain mDSS Mediator	Various manners of specification of preferences, value assignments, rankings etc.
Choice	Bargain mDSS Mediator	All these applications provide a method for supporting selection of decision among options
Decision procedure design	ProDec	Design, testing and running of decision procedures for definite problems
Editing support	cTD SHE	Special editing applications, facilitating, in particular, translation and comparison of texts

#### 4. The individual Decision Analytic Applications

This section presents particular DAs, with somewhat more extensive descriptions of the ones designed and developed by the team associated with the Systems Research Institute of the Polish Academy of Sciences. For completeness, a short presentation is provided also of the mDSS, as an integral part of the range of DAs offered within the TDSS.

Throughout the presentation of the decision applications some notions will be used in a consistent manner: a *project* (or a *problem*), meaning a broadly defined issue, which has to find a way of realisation (e.g. flood protection, or drinking water supply); the relatively well-defined ways of its realisation will be called *options* (the options being characterised, in particular, by their cost); in an ultimate case the options may be dichotomous: to do something or not to do it. This simple language will only be different in the case of ProDec application.

##### 4.1. ArgWar

ArgWar is a decision-oriented application, whose original purpose was web-based polling. Using ArgWar enables:

- formulation of a question to be subject to public scrutiny (e.g. "Are you FOR or AGAINST construction of a reservoir on the river, stretching between Downstream Village and Upstream City"), to which answers FOR or AGAINST are expected.

- publishing this question, along with additional information (like maps and diagrams, data etc.) on the web site of the interested party (e.g. local self-governmental authority or water resources management body),
- tracking the results in terms of FOR and AGAINST statistics (is the public opinion, as expressed by the web clients, favourable or opposed to the given idea); *it is up to this point that the ArgWar application resembles, or mimics the usual web-based polling instruments, yet, it is equipped with some additional capacities:*
  - the key question, to be answered FOR or AGAINST, is accompanied by the short lists of arguments, supporting the FOR or the AGAINST opinions (“I am FOR/AGAINST because...”): the respondent, when “voting”, indicates the argument that is most convincing for her/him (in the implementation that can be seen and used at [www.ibspan.waw.l/transcat](http://www.ibspan.waw.l/transcat) these lists, as seen at the first glance, contain three arguments each),
  - on the top of this, the respondent can add an argument (FOR or AGAINST) of her/his own, writing it into the appropriate place on the web site,
  - in the course of the use of application not only the ultimate FOR and AGAINST statistics are shown, but also the popularity of particular arguments; the arguments appearing at a given moment on the web site (“by default”) are the ones having scored the best until that moment among the respondents, except for the very start, when initial arguments are posted, for which no one has voted yet, and except for the newly introduced arguments, which are granted an “indemnity period”, before they are ranked automatically along with all the other (older) ones,
  - the application includes the mechanism for moderating the “arguments war”, meaning the possibility of operating on the set of arguments, especially in view of the possibility of appearance of aggressive and insulting arguments, but, more importantly, there is a mechanism for automatic ranking and elimination of arguments.

Thus, ArgWar provides the tool not only for opinion polling, but also for probing of the motivations behind the opinions expressed, and structuring them.

The application was implemented in some pilot areas (PAs) of the TRAN-SCAT Project, for definite current issues of the respective locations, but the responses obtained were discouraging. The reasons were seen in the inadequate promotion of the polling action through other media than the web, and inadequate tracking (prompting) during posting.

It is planned in some PAs that the tool will be used rather as a promotional instrument within the framework of definite campaigns related to water resource economy and environmental protection.

Yet, it is obvious that ArgWar can be used as a debate-supporting tool, not necessarily in the public domain, but rather within a group of involved persons, specialists or otherwise, who are trying to initially structure a problem at hand. The output from running of a procedure would consist in a list of argument formulations, with corresponding rankings. The “voting” involved does not have, of course, to be FOR and AGAINST, but also FOR one or another option, or a subset of options. In this manner the use of a relatively simple tool can be significantly broadened.

#### **4.2. mDSS**

The mDSS is a Decision Support System, developed initially within the MULINO Project, aimed at supporting the choice from a set of discrete policy options. The DSS bases on a set of multicriteria decision methods (MCA) to choose from according to the specific characteristics of the problem at hand and according to the background of the policy makers. The MCA methods are complemented by sensitivity (robustness) analysis and by the analysis of sustainability of the management regime. Group decision making (i.e. decision involved a group of policy makers and stakeholders) is supported. The mDSS is composed by the computerised part (coded in a single stand-alone software for Windows operating system) and set of methodologies guiding the problem structuring, identification of relevant actors/stakeholders and recognising salient features of the problem considered. These methodologies include social network analysis, mental model elicitation and cognitive map building.

Input data to mDSS may come from (environmental or socioeconomic) models, monitoring systems, expert judgement, etc. (either from the cTD or from other sources). The output is constituted by the selection of preferred option / rank order.

Within the TDSS structure, mDSS has connections

- structured with cTD (get map, get value, get function, catalogue), and
- (un)structured with Mediator (TBD).

The application is available from:

[www.feem.it/mulino](http://www.feem.it/mulino) or <http://siti.feem.it/mulino/index1.htm>

#### **4.3. Bargain**

The Bargain or BarTend application realises a simple bargaining or tender situation, serving to determine the relative (or perhaps even absolute) value of an option for two or more participating (bargaining) parties.

The application, which is also web-based and web-operated, encompasses the editing functions related to problem formulation and management (project definition, specification of options meant to realise a given project), as well as management of a particular exercise (e.g. user definition).

The technique used works as follows:

- given a set of options among which to choose for a particular, possibly well-defined purpose, these options being characterised, in particular, by their realisation cost,
- and given the upper limit on the budget that can be used to implement one or more of these options (the options need not be mutually exclusive),
- the participants assign, independently one of another, the funds for potential realisation of particular options.

Once the participants decide as to the assignment (they are offered the stage of “deliberation” and “final decision”), the option is selected for which the maximum total sum from all the participants has been assigned.

In this manner the (relatively) most preferred option is selected and, in addition, in many cases (depending upon the limit of funds and its relation to the option realisation cost, the participants would pay for the realisation of this option less than they have offered (i.e. when the total sum assigned exceeds the cost of realisation of the option selected).

Thus, if we denote by  $f_{ik}$  the fund assignment of the participant  $i$  for option  $k$ , the option selected is the one, for which we have

$$f^* = \max_k \sum_i f_{ik},$$

and the costs to be actually borne by the participants,  $c_i^*$ , for the realisation of the selected option  $k^*$ , are determined as

$$c_i^* = c_k^* f_{ik^*} / f^*,$$

where  $c_k$  is the assumed cost of realisation of an option  $k$ . Hence, for  $f^* > c_k^*$ , we have  $c_i^* < f_{ik^*}$ .

This simple and intuitive solution could, of course, be replaced by some other one, which would take into account, e.g., the relative valuations of the particular participants (e.g. the sum of shares of total assignments of particular participants), which might seem to be more “just” in certain situations. Such a rule of option selection would be based on

$$\max_k \sum_i (f_{ik} / \sum_j f_{jk}),$$

which, however, would be much less intuitive and less easily followed.

Indeed, a number of game theoretic solutions and conditions could be applied and imposed, but the assumption behind the simple technique was that it be clear and transparent to the participants. Thus, even though “forcing” of certain options by



“wealthier” parties may occur, this is a valid outcome in view of the objectives of the application.

Given that the use of Bargain should lead to realistic estimates of the valuations of options by the respective parties, it is strongly advised that the fund limits and the assignments specified corresponded to some actual circumstances (funds at disposal of a local authority, funds applied for in the framework of a broader programme, funds envisaged in some definite plan, etc.).

The evaluations obtained from Bargain can be used as such for the selection of options, or can constitute input to mDSS or to Mediator-like tools.

#### 4.4. Mediator

This application, also web-based, provides support for group decision making. Each case of use of Mediator can be addressed as a “session” or a “project”, the latter meaning that the participants consider a definite issue, which can be resolved, or a vision that can be realised, through a certain set of options.

As in all other DAs, distinction is made between the administrator of the application (or a session), who can manage the contents and the course of the “session”, and the users, who contribute their opinions.

Thus, in many real life situations it is needed to make decisions when the criteria of choice cannot be formulated precisely or are subject to some informal constraints. In such a case one can make use of expert judgements. One of the common tasks solved by groups of experts consists in defining the order of elements (options) or choosing the best element (option), or elements, with respect to a chosen criterion or set of criteria. On the basis of expert judgements the system determines the group opinion. The definitions of the group decisions are provided in the Annex.

To facilitate the group work it is worth to use some tools of group decision making, e.g. a system that makes it possible to organize a session with invited experts.

Organizing a session means the following:

- preparing an agenda for the meeting,
- managing the logging of the experts to the session,
- preparing list of projects and options to be considered,
- asking for experts’ opinions,
- determining group judgement on the basis of experts’ opinions acquired,
- displaying the results of the session,
- preparing final report.

The MEDIATOR system consists of two parts – the administrator (moderator) and the users' (experts') application.

#### The administrator application

It is a supervisory application, which controls the work of the system, i.e. it performs all the actions necessary to perform a session except of entering experts' opinions which is the essence of the expert application.

By means of administrator interface it is possible to manage the following functionalities:

- *preparing the agenda for the sessions to be introduced,*
- preparing the list of projects and the options of realisation of these projects to be considered (there is also a possibility of annexing additional information on the project(s) and the options, such as extended descriptions, maps, diagrams etc.),
- preparing the list of experts invited,
- managing the accessibility to the sessions for the users,
- gathering experts' opinions,
- choosing a method – from the set of methods being implemented - of group decision determining and therefore also of calculating the result (more than one method can be used for the same set of expert opinions),
- displaying the group opinion and the graphic presentation of some additional statistical measures of the experts' opinions.

#### *Choosing the method of group decision determining.*

There are several algorithms to be used for generating a group opinion (see the Annex). The reason of developing still new methods are the deficiencies of the existing ones, observed specially in the form of the so called paradoxes of voting, when a method effectively fails in determining the group opinion. This means, in particular, that the method indicates an outcome that is inconsistent with the more general assumptions adopted.

The most known algorithms implemented in the MEDIATOR system basically belong to two groups. The first group consists of algorithms derived from pairwise comparisons represented by the methods of Condorcet, Copeland and the Kemeny's median. The second group includes the positional algorithms represented by the Borda, plurality and max-min methods. The Hare and Coombs algorithms are multi-stage methods in which some properties of the two mentioned groups of algorithms have been utilised.

The choice of the method depends on the character of the problem examined and on the experience of the moderator. It is also possible for the moderator to ob-

serve the results of all the methods implemented and to choose the method considered the best.

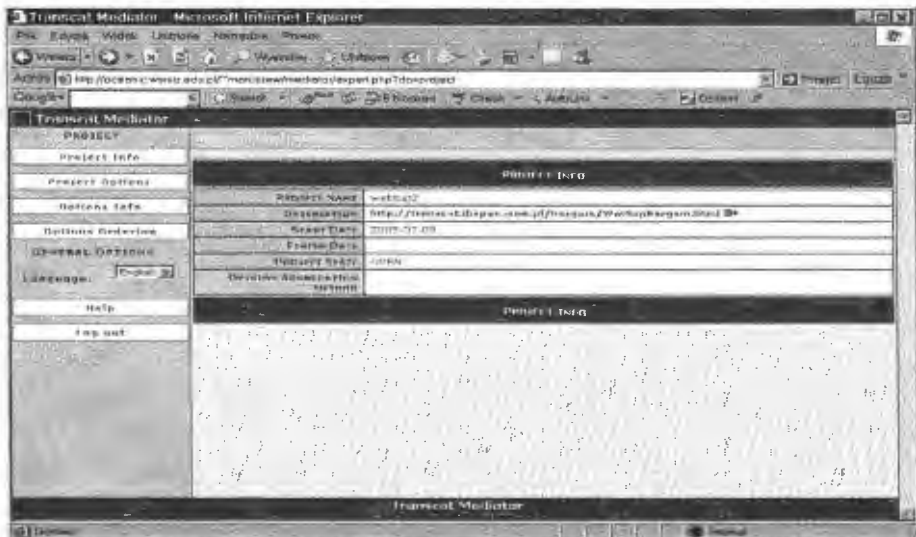
It should be noted that some of the methods of group decision determination are quite simple and intuitive (like the Borda score, which is equivalent to the sum of “points” assigned, explicitly, or implicitly – depending upon the way in which preferences are specified – by the participating experts). The question then arises why to refer to other, in some cases quite non-intuitive methods (like, e.g., Kemeny’s median).

The need of disposing of a bigger number of methods of opinion aggregation results from, first, the (theoretical) shortcomings of the methods available, expressed through the known paradoxes. Even though these paradoxes do not occur in many situations, there must exist a possibility of avoiding them, should the very formulation of the problem entail an increased probability of such an occurrence. Secondly, the different methods represent different principles of opinion aggregation, which may correspond to various kinds of situations and to various convictions as to how the opinions should be aggregated. The final argument is that in a lot of cases most of methods yield the same results (ordering), and hence the existence of differences in the results is an indication of a definite specificity of the given situation, requiring a deeper insight, which can be provided exactly by the application of different methods of opinion aggregation.

An Annex to the present paper contains a more detailed description of the general formal aspects and the actual methods used in Mediator.

### The expert application

After logging to the session the main screen of the expert application opens, namely



in which the project name and other information, as well as some useful links are displayed.

At the left side of the screen there is a control panel with a set of horizontal bars. Moving through these bars makes it possible to examine the available functionalities of the system and to enter expert's opinion.

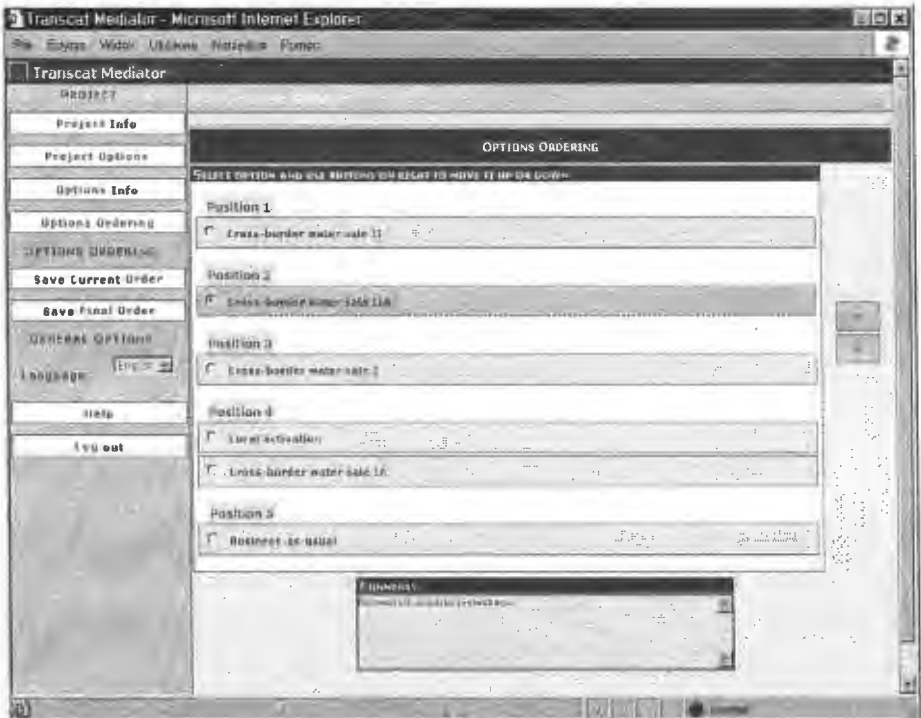
By pressing consecutive bars the user can display information about the project and the list of options to be ordered. Being familiar with the problem the user may order the options considered.

It is assumed that an expert can observe only his/her own opinion as well as, after terminating the session, the determined group opinion.

By pressing the "Options Ordering" bar we open a new window for entering the order of options, shown in next page.

Defining the proper order of options is the most important feature of the MEDIATOR system.

It is assumed that experts' opinions are expressed as preference orders of given elements, i.e. options (equivalence, i.e. equal ranking of options can occur in experts' opinions). The experts are asked to analyse the problem and to order options considered.



The option regarded by a given expert as the best one (in the sense of criteria assumed) is put in the first position, the option regarded as the worst one is put in the last position. It is possible that more than one option is put in the same position - such options are regarded as equivalent. The ordering of options is completed when an expert has put all the options considered in proper positions.

The user may change the order of options by moving a selected option with **Move Up** (icon with up arrow) and **Move Down** (icon with down arrow) buttons. Single click moves the option one level up or down depending on which button has been pressed.

A context help is available any time it is needed.

#### 4.5. ProDec

This application has a different character from the previously outlined three, or even all four. It does not, namely, support decision making in the sense of selecting an option for some future project or undertaking among a set of options that are meant in principle to secure the realisation of this project, but, instead, it supports operational decision making in situations, when appropriate selection process has already been concluded.

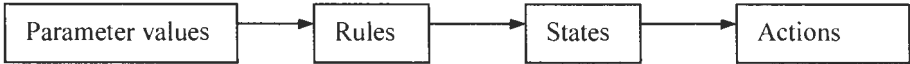
ProDec serves, first, as an editing application, which enables formulation of a decision procedure in the form of a system of IF... THEN... rules, leading from the values of certain observations or measurements, entering the conditions (IF...) to definite actions, that should be undertaken in such a situation (THEN...). The rules admitted by the application may involve standard logical operations of AND, OR and NEGATION, linking respective values of observations or measurements, defining the conditions.

More importantly, however, the rules can involve in the conditional part intuitively understandable linguistic expression of the form, e.g., "low level", "high intensity" etc., corresponding to the respective measurements (e.g. river level, precipitation intensity etc.). Thus, a rule admitted by ProDec in case of preparation of a decision procedure for flood protection purposes, might sound:

IF (*river level* high AND *today's precipitation* high AND *precipitation forecast* very high OR *river level* very high AND *today's precipitation* medium AND *precipitation forecast* high) THEN *flood alarm of degree 2*.

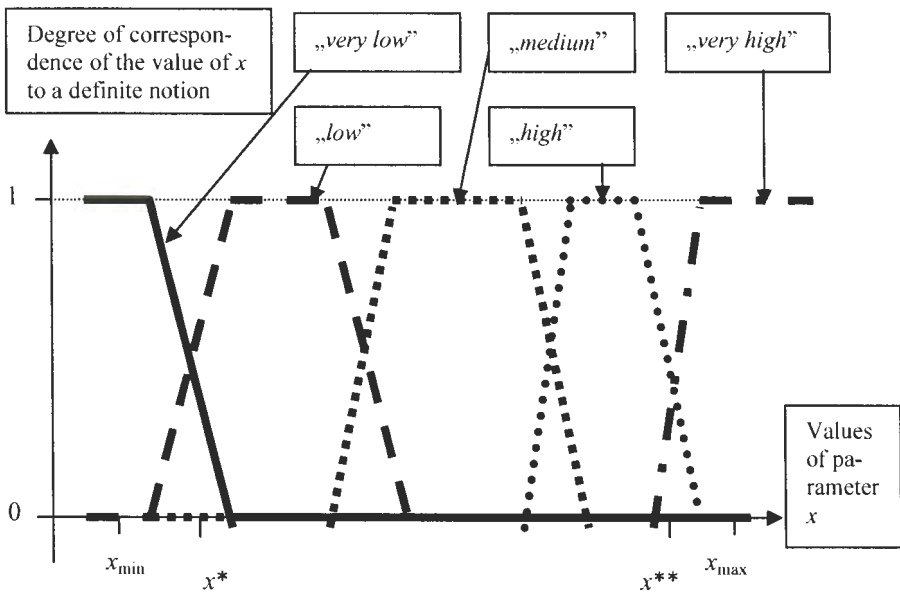
The above example shows the essential components of the intended decision procedure system: the observed variables, entering the conditional part, like, here, *river level*, *today's precipitation* and *precipitation forecast* are called **parameters**. The current values of the parameters are measured or observed and after the decision procedure has been designed and edited, they can be entered into the respective rules. The concluding part contains the specification of **states**. Each state has defi-

nite **actions** assigned, which do not appear explicitly in the formulation of the rules, since they are assigned in a stiff manner to the states.



Although the values of parameters – notwithstanding the way, in which they are obtained, whether through measurement, appraisal, estimation, output from a model, or just a guess – enter the system “as they are”, i.e. most often as numbers (e.g. precipitation in millimetres or water level in centimetres), they are treated in the rules through the intermediary of predefined linguistic values (low, medium, high,...).

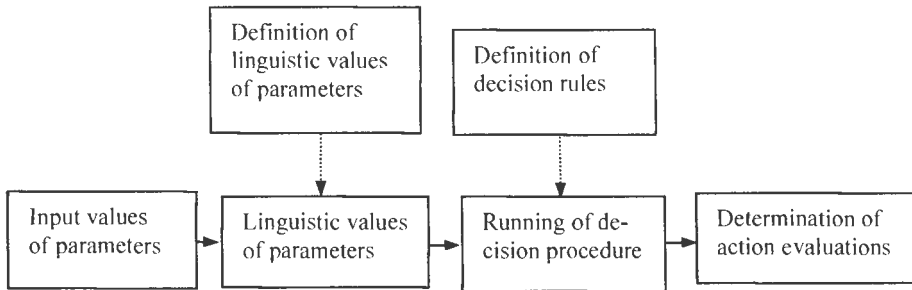
Thus, assume a parameter takes the values from the interval  $[x_{\min}, x_{\max}]$ , like in the following figure (e.g. river level between 0 cm and 280 cm, the all-time high).



The ProDec application enables introducing the “definitions” of the linguistic values corresponding to the fuzzy subsets of the interval  $[x_{\min}, x_{\max}]$ , like, for instance, in the case here illustrated: “very low”, “low”, “medium”, “high” and “very high”. These definitions do not just specify the sub-intervals, but also the fuzzy edges of the respective –sub-intervals. And so, e.g., the value  $x^*$  of the diagram, should it occur as actually measured, is “very low” in degree 0.4 (or in 40%) and at the same time it is “low” in degree 0.6 (or in 60%). (Note that the requirement that the total of degrees of membership of all the values of parameters be equal 1.0 or

100% is in fact not formally required in ProDec as putting too strict a constraint on the generally user-friendly way of proceeding.)

Once the linguistic values of the parameters defined through the specification of corresponding fuzzy sets, and the IF... THEN... rules established, both with the help of ProDec, then the procedure obtained can be tested, validated and finally used in practice.



The need of testing and validating the entire system, composed of rules and definitions of linguistic values of parameters, results from the fact that functioning of ProDec is based upon human input, related to individual elements of the whole (fuzzy sets, single rules). Thus, there are two issues – first, the correctness of each of these elements specified, and second – the consequences for the entire system. The function of testing is embedded in ProDec in the form of easy and traceable changes in parameter values.

The diagram above shows as the final the stage “determination of action evaluations”, associated, of course, with the “firing” of rules for respective inputs. Given that the parameter values appear through their linguistic expressions in the rules, defining states, and therefore also actions, and that these parameter values can belong to various linguistically defined values to differing degrees, the resulting procedure determines the degrees, to which particular actions should be undertaken. Thus, on the basis of linguistic values and the rules, in which they intervene, one obtains ultimately actions along with the evaluations of the degree of certainty, with which they should be undertaken, again on the scale of 0 to 1 (or 0 to 100). If an action is assigned the value close to 1 (or 100) then it should be undertaken (or at least seriously considered), while when the evaluation of the degree of its certainty is closer to 0 (e.g. 0.2, or 20), it may at the given moment be disregarded.

#### Why use fuzzy sets and certainty evaluations?

At this point it is worthwhile to explain the reasons for introducing the linguistic values, based on fuzzy sets, and processing them through the logical rules towards graduated evaluations of action certainty. There are several reasons for such an approach:

1. *The uncertainty associated with the values of parameters.* Even if the values of parameters result from formal measurements, they can be charged with an error, to say nothing of the assessments made by humans (especially when they are based on information from several sources). Similarly, if we use models, their (forecasting) output is usually subject to a wide margin of uncertainty. If so, appropriate definition of the respective fuzzy values lowers the ultimate effect of errors (by decreasing the sensitivity to these errors).
2. *The uncertainty associated with the rules.* Even if the rules, as this could also be the case, are the results of functioning of some formal models, they are valid within a certain “corridor” of values, both at the input and at the output. Such “corridors” might have been determined through appropriate sensitivity analysis. Yet, even such an analysis is biased in view of the inherent errors and uncertainties, especially close to the “outer limits” of the “corridor”. The use of fuzzy values, again, decreases vulnerability with respect to the uncertainty involved.
3. *Additional information.* In case of a “strict” rule of the form, say, “IF water level in the river exceeds 220 cm THEN flood alarm” there is no information what to do when the level is at 219 cm, unless a set of special rules are defined for particular levels. These rules would then be associated with definite, differing actions, and will have to correspond to a range of conditioning circumstances. In case of ProDec, there will be a natural, human language consistent indication of the degrees of certainty with respect to application of particular actions (e.g. flood alarm – 0.95, or 95; emergency preparations – 1, or 100). Thereby, information provided the user is much ampler and much more useful.

#### 4.6. SHE

SHE is an auxiliary application, supporting transborder functionality, and is meant to help in editing, and especially in translation. It is, actually, an editor for simultaneous work in two languages with exchange features for the document under development. It is used, in particular, as a part of the ProDec editor.

The input to the application is constituted by the original document in the initial form, while the output is provided by the possibility of editing the translation and the original document.

SHE constitutes an addition to the capabilities offered by the cTD, within which there exists a possibility of translation, though of somewhat limited volume and scope. SHE was developed for handling one-time translation of larger pieces of text.



## 5. Conclusions and recommendations

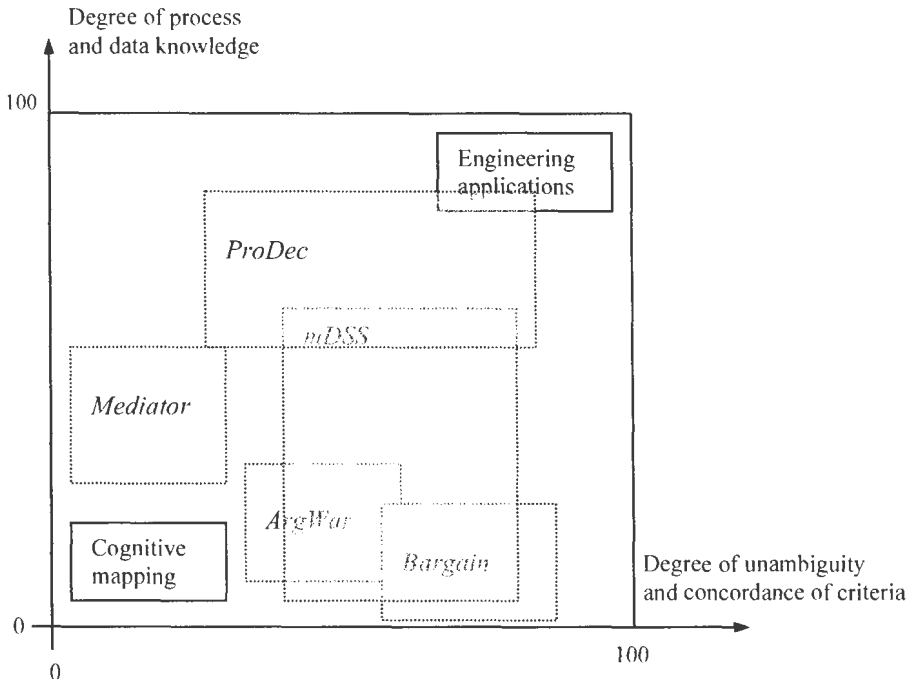
The work on development of decision-oriented applications within the TRANSCAT DSS was aimed at creation of a possibly complete set of relatively simple tools that could be used by non-experienced users in various situations and under different contexts. Given that it was known from the start that the mDSS, developed within the MULINO project, would make a part of the range of DAs in TRANSCAT, it was assumed that the other applications have, on the one hand, to span the decision functions that either are not included in mDSS or are not sufficiently pronounced there, and, on the other hand, to provide much more easily operated instruments for individual kinds of issues. So, for instance, questions and approaches such as cognitive mapping or cross-impact analysis were devoted much less attention.

The above set of prerequisites was at least partly satisfied by the developed and upgraded DAs, outlined in this deliverable report. Yet, definitely, the tools here presented by no means do constitute a complete and exhaustive set that can respond to any sort of decision situation. What is offered, instead, is a toolbox, which can be handy in most of the different cases when a computer-based support might or even should be called for.

A couple of observations, forwarded in what follows, concern both the use of computer-based decision support tools in general and the use of the applications here described in particular.

1. In cases, when well-calibrated and sufficiently precise (verified) instruments exist, encompassing the decision problem considered, whether computer-based (models, calculation schemes) or otherwise (data, trade rules and standards), they should be the primary, or just the sole basis for making of decisions.
2. It is true that in numerous situations the reliability of such quantitative instruments and/or data is limited, even to the extent that some special approaches have to be applied. Yet, as long as the uncertainty or vagueness concern the technical subject matter of the problems (e.g. cost of a project or impact on groundwater head), the additional instruments and approaches used should explicitly aim only at decreasing the imprecision and/or the negative effects of making (technically) wrong decisions (designs). *This is, in particular, the case for use of ProDec application. It should be used, namely, when some precise model cannot determine the course of action to be undertaken for the information that is available. Such a situation may result from the insufficient precision of input data and/or from the insufficient technical knowledge of the processes involved. When, however, on the one hand, we dispose of definite experience-based domain knowledge, and on the other – there is a necessity of taking definite decisions, a procedure like produced by ProDec can be designed.*

3. The possibility of using quantitative tools, with perhaps extensions, like those alluded to under 2 above, results also from the fact that in a given situation our perception of the respective objectives or quality criteria are not subject to doubt nor discussion (e.g. trying to satisfy environmental standards). It should be noted that this does not mean the existence of only one objective or criterion, but rather the possibility of accommodating the objectives or criteria intervening within a well-defined and unambiguous procedure. This may mean, e.g., simultaneous satisfaction of certain predefined "minimum requirements" with respect to the objectives considered, or assignment of agreed upon weights of values representing these objectives. *So, here, as well, a tool like ProDec might be applied. The objectives or criteria involved would appear indirectly as driving the rules established.*
4. Within the same paradigm of using decision supporting tools we may include in the appropriate procedure for accommodating different objectives and criteria the tools for expressing them, in order then to be included in some broader framework. *This is the situation, in which Bargain might be used, provided its output is fed into a well-defined procedure (a model or another scheme).*
5. The subsequent level of application of computer-based tools corresponds to the situation, in which objectives and criteria are (relatively or sufficiently) well known, and can be expressed either in terms of results from models or other formal tools, or as subjective, but valid statements, yet they cannot be a priori put together in a consistent manner. In such a situation, one is obliged to recur to less structured so-called "multicriteria" methods, in which various objectives and criteria are somehow accommodated. *This is the proper place for applying mDSS, Mediator and similar techniques.* It should be remembered, though, that if the objectives and/or criteria are not well-founded in some concrete measures, and if the procedure, in which they are involved gets complicated (number and complexity of operations performed on them), we should be very careful in relying on the output obtained. That is why DSS are in such situations just the "support" systems, whose purpose is to provide additional information and assist in analysis rather than to determine the decisions.
6. Finally, we may have problems with the very expression of objectives and criteria. This is usually the very initial stage of analysing the problem(s) of a socio-economic-resource system. *ArgWar, with its specification of arguments and their ranking, may be one of the proper tools at this stage, especially when we wish to consult broader public. It can also be used for structuring the perception of the problem area ("cognitive mapping", whatever this may mean). Actually, mDSS offers methodologies for dealing with cognitive mapping, as well as with generally less structured situations with respect to objectives and criteria.* Like in the previous point, though, the outputs from such approaches ought to be considered with utmost care for their validity. These outputs should also be regarded as a piece of knowledge helping in cognition and resolution of the problems faced.



Complementing the list of six points provided before, the simplified scheme above indicates roughly the areas of application of the particular TRANSCAT DSS with respect to precision of “process & data knowledge” as well as “unambiguity and concordance of criteria”. Notwithstanding, however, this rough indication, it should be remembered that the DAs available are to be used depending upon the specific needs associated with a concrete decision situation.

## 6. References

The domain of decision support has a very vast literature, including several international journals, entirely or partly devoted to this domain. That is why cited here are only the references associated directly with the work on TRANSCAT Project. The further pertinent references are provided in the ones here cited. On the top of this, worth visiting are the web sites presenting mDSS and the DAs here outlined, namely:

[www.feem.it/mulino](http://www.feem.it/mulino) or <http://siti.feem.it/mulino/index1.htm> and <http://transcat.ibspan.waw.pl>.

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## 7. Annex: The Methodological Background of Mediator

### GROUP OPINION DETERMINATION – AN INTRODUCTION

One of the common tasks solved by groups of experts consists in defining the order of elements (options) or choosing the best element (or elements) with respect to a predefined criterion or set of criteria. If the criterion (or a set of criteria) can be defined in precise mathematical terms, then the tasks mentioned can be formulated as mathematical programming problems. Unfortunately, in many real life situations such formalization is not possible. In these cases one can make use of expert judgements. The result of such a process of expertise is called the group opinion (group judgement).

There exist a lot of algorithms making it possible to determine group opinion on the basis of expert judgements. However, it is very difficult to define conditions, under which the application of a given algorithm is the most advantageous. The use of different methods to determine group opinion may in general produce different results. The choice of the algorithm applied depends on the task to be solved as well as on user's experience.

Expert judgements may have various forms. They can be given on the order as well as on the number scales. Moreover, they may have the form of preference orders as well as of pairwise comparisons. For the situations considered in the system it is assumed that expert judgements have the form of preference orders, i.e. an expert is asked to point out the preferred order of options with the assumption that an option regarded as the best one is placed in the first position, an option regarded as the worst is placed in the last position and options regarded as equivalent take the same position.

There are several algorithms to be used for generating a group opinion in the case mentioned. The reason of developing still new methods are the deficiencies of the existing ones observed specially in the form of so called paradoxes of voting when a method fails in determining the group opinion.

The most known algorithms are those of Condorcet, Borda, plurality, max-min, Hare, Coombs, Copeland and Kemeny's median. In general, they are derived from pairwise comparisons or they make use of information on the position taken by a given option in the order of options. The first group is represented by the Condorcet, Copeland and Kemeny median algorithms, while the second one is represented by the Borda, plurality and max-min methods. The Hare and Coombs algorithms are multistage methods in which some properties of the former algorithms have been made use of.

#### SOME THEORETICAL REMARKS

We are given a set of  $n$  elements,  $O = \{O_1, \dots, O_n\}$ , which is to be ordered by a group of  $K$  experts. Let us assume that experts' opinions are given on the order scale. Hence the  $k$ -th expert opinion has the form of a preference order  $P^k$ , where the element regarded by this expert as the best one – in the sense of some accepted criterion (criteria) - is located in the first position and the last position is occupied by the element regarded as the worst one.

If in experts' judgements no equivalent elements can occur, the number of positions in the preference order is equal to  $n$ , otherwise it is less than  $n$ .

So, we dispose of the preference order of  $n$  alternatives presented by the  $k$ -th expert ( $k=1, \dots, K$ )  $P^k = \{O_{i_1^k}, \dots, O_{i_n^k}\}$ ,  $k = 1, \dots, K$ , where  $O_{i_1^k}$  denotes the element placed by the  $k$ -th expert in the  $j$ -th position in the preference order.

For this preference order the following matrix of pairwise comparisons can be constructed

$$A^k = \begin{bmatrix} a_{11}^k & \dots & a_{1n}^k \\ \vdots & \ddots & \vdots \\ a_{n1}^k & \dots & a_{nn}^k \end{bmatrix} \text{ where } a_{ij}^k = \begin{cases} 1 & \text{for } O_i \succ O_j \\ 0 & \text{for } O_i \approx O_j \\ -1 & \text{for } O_i \prec O_j \end{cases}$$

The notation  $O_i \succ O_j$  should be read as follows: the  $i$ -th alternative (option)  $O_i$  is better than the  $j$ -th alternative (option)  $O_j$  with respect to a chosen criterion (a set of criteria);  $O_i \approx O_j$  denotes that the  $i$ -th alternative  $O_i$  is equivalent to the  $j$ -th alternative  $O_j$  with respect to a criterion considered.

By making use of all the matrices  $A^k$  ( $k=1, \dots, K$ ) one can construct the so called outranking matrix  $L=[l_{ij}]$  describing the distribution of expert opinions. Elements of such a matrix are defined as follows:

Let  $l_{ij}$  denote the number of experts regarding the element  $O_i$  as better than  $O_j$  (briefly written as  $O_i \succ O_j$ ). Hence, the number of experts having opposite opinion is equal to  $l_{ji} = K - l_{ij}$ ,  $i, j = 1, \dots, n$ .

	$O_1$	$O_2$	$\dots$	$O_{n-1}$	$O_n$	$WB_i$
$O_1$	-	$l_{12}$	$\dots$	$l_{1n-1}$	$l_{1n}$	$\sum_{j=1}^n l_{1j}$
$O_2$	$l_{21}$	-	$\dots$	$l_{2n-1}$	$l_{2n}$	$\sum_{j=1}^n l_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$O_{n-1}$	$l_{n-11}$	$l_{n-12}$	$\dots$	-	$l_{n-1n}$	$\sum_{j=1}^n l_{n-1j}$
$O_n$	$l_{n1}$	$l_{n2}$	$\dots$	$l_{nn-1}$	-	$\sum_{j=1}^n l_{nj}$

The above matrix can be formed due to the assumption mentioned above that experts' judgements are given as preference orders.

**DESCRIPTION OF THE ALGORITHMS**

**1. The Condorcet method**

The Condorcet algorithm is, historically, the first method of determining group decision, based on pairwise comparisons.

Let us assume, without discussing details of this definition, that the Condorcet winner is such an element  $O_{i_c}$  that the following condition is fulfilled

$$l_{i_c, j} \geq K_{pl} \quad \text{where} \quad K_{pl} = \begin{cases} \frac{K + 1}{2} & \text{when } K \text{ is an odd number} \\ \frac{K}{2} + 1 & \text{otherwise} \end{cases},$$

for each  $j \neq i$  ( $j = 1, \dots, n$ ).

In other words, this element, according to the opinion of the so defined plurality of experts – is better than the other ones. The problem with the Condorcet winner is that it may happen that such a winner does not exist.

*If it is not possible to determine the Condorcet winner on the basis of experts' judgements or application of this method is limited (which is the case when so called paradoxes occur) then other methods of group opinion determining should be used.*

**2. The Copeland method**

The Copeland winner is an element that precedes the biggest number of elements in the sense of pairwise comparisons. If there exists a Condorcet winner, it is also the Copeland winner. To determine the Copeland winner one has to rewrite the outranking matrix in the following form:

	$O_1$	$O_2$	...	$O_{n-1}$	$O_n$	$WCP_i$
$O_1$	–	$\delta_{12}$	...	$\delta_{1n-1}$	$\delta_{1n}$	$\sum_{j=1}^n \delta_{1,j}   \delta_{1,j} = 1$
$O_2$	$\delta_{21}$	–	...	$\delta_{2n-1}$	$\delta_{2n}$	$\sum_{j=1}^n \delta_{2,j}   \delta_{2,j} = 1$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$O_{n-1}$	$\delta_{n-1,1}$	$\delta_{n-1,2}$	...	–	$\delta_{n-1,n}$	$\sum_{j=1}^n \delta_{n-1,j}   \delta_{n-1,j} = 1$
$O_n$	$\delta_{n,1}$	$\delta_{n,2}$	...	$\delta_{n,n-1}$	–	$\sum_{j=1}^n \delta_{n,j}   \delta_{n,j} = 1$

$$\text{where } \delta_{ij} = \begin{cases} 1 & \text{for } l_{ij} \geq K_{pl} \\ -1 & \text{otherwise} \end{cases}$$

The element  $i^*$  with the highest  $WCP_i$  score is the Copeland winner.

### 3. The Borda method

This method, similarly as the Condorcet method, was proposed in the 18<sup>th</sup> century to determine the winners of elections in the French National Assembly. As it was mentioned before, the choice of the winner is determined on the basis of the position of an element in the preference order. It is assumed that the most preferred element is placed in the first position and the least preferred is placed in the last position. The gist of the Borda method lies in determination of the so-called Borda score, which is defined as follows:

$$WB_i = \sum_{j=1}^n (n-j)l_i^j,$$

where  $i$  – denotes the number of an element,  $j$  – denotes the number of the position of this element,  $l_i^j$  – denotes the number of experts who put element  $O_i$  in the  $j$ -th position.

The Borda score can be also determined from the outranking matrix.  $WB_i$  is defined as the sum of elements  $l_{ij}$  in the rows of the outranking matrix. Both results are equivalent.

*The Borda winner is an element (option)  $O_i$  with the highest Borda score, that is:  $WB_i = WB_{max}$ .*

*The Borda winner, in distinction from the Condorcet winner, can always be determined.*

*For the Borda method the set of elements  $O = \{O_1, O_2, \dots, O_n\}$  is ordered with respect to the decreasing values of the  $WB_i$  scores.*

### 4. The plurality method

The winner is an element placed in the first position by the greatest number of experts.

### 5. The max-min method

The max-min method consists in determining the minimal element  $l_{ij}$  in each row of the outranking matrix, i.e. in determining an element  $O_j$  such that the judgement  $O_i \succ O_j$  is supported by the minimal number of experts. This element is denoted as  $l_{i \min}$ . Then an element for which the number  $l_{i \min}$  is the largest one is chosen. This rule may be written down as follows:



$$\max_i \min_j l_{ij}, i, j = 1, \dots, n.$$

## 6. The Hare method

To determine the Hare winner one has to determine the elements placed in the first position (simple plurality) by the greatest and the least number of experts. If there is an element placed in the first position by the majority (more than 50%) of experts then such an element is the winner.

If this is not the case then an element or elements placed in the first position by the least number of experts are deleted from all the preference orders. The elements placed in the first position by the greatest and the least number of experts are then determined again.

The procedure of deleting options from the set of options initially considered is repeated until only one or more than one element but with the same number of votes remain.

## 7. The Coombs method

This method was developed as the modification of the method devised by T. Hare.

In order to find the Coombs winner one has to determine the elements (options) placed in the first position (simple plurality) and in the last position by the greatest number of experts. First, if there is an element (option), which was placed in the first position by the majority (more than 50%) of experts, then this option is the Coombs winner.

If this is not the case, then an element or elements placed in the last position by the greatest number of experts is deleted from all the preference orders. The procedure of deleting elements from the set of elements is repeated until only one or more than one element with the same number of votes remain.

## 8. The Kemeny's median method

This method consists in determining a preference order that is simultaneously the "closest" to all the opinions of the voters. The measure of the "closeness" is the minimum of some distance defined with the use of the pairwise comparison matrix.

The distance between two preference orders  $P^{k_1}$  and  $P^{k_2}$  can be expressed as follows:

$$d(P^{k_1}, P^{k_2}) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n |a_{ij}^{k_1} - a_{ij}^{k_2}|$$
, where  $a_{ij}^{k_1}, a_{ij}^{k_2}$  are the coefficients of respective pair-wise comparisons matrices.

We are given a set of preference orders  $\{P^{(k)}\} = \{P^1, \dots, P^K\}$ .

The distance of some preference order  $P$  from this set is defined as follows

$$d(P, P^{(k)}) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^K d_{ij}(P, P^k) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^K d_{ij}^k = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^K |a_{ij}^k - a_{ij}^P|.$$

Assume that in the order  $P$  one has  $O_i \succ O_j$  i.e.  $a_{ij}^P = 1$  and no equivalent elements occur. To determine the distance of that order from the given set of orders  $\{P^{(k)}\}$  let us define some coefficients  $r_{ij}$  given as follows

$$r_{ij} = \sum_{k=1}^K d_{ij}(P, P^{(k)}) = \sum_{k=1}^K |a_{ij}^k - a_{ij}^P| = \sum_{k=1}^K |a_{ij}^k - 1|, \quad r_{ij} = 0 \text{ for all } i = j.$$

The values of  $r_{ij}$  coefficients (also called loss coefficients) depend solely on the orders  $P^k, k=1, \dots, K$ .

A preference order  $P^{KM}$  such that

$$P^{KM}(P^1, \dots, P^K) = \arg \min_P d(P, P^{(k)}), \text{ where } d = \sum_{i=1}^n \sum_{j=i+1}^n r_{ij}.$$

is called the median of a set  $\{P^1, \dots, P^K\}$ .

In other words it is such a preference order that in sense of the distance here defined is the "closest" one to all the preference orders of the set  $\{P^{(k)}\}$ .

### Example

Let us assume that the set  $\{P^{(k)}\}$  consists of four preference orders and they are as follows:

$$\begin{aligned} P^1: & O_2, O_4, O_1, O_3; \\ P^2: & O_1, (O_3, O_4), O_2; \\ P^3: & (O_2, O_3), O_4, O_1; \\ P^4: & O_3, O_2, (O_1, O_4) \end{aligned}$$

Pair-wise comparison matrices corresponding to these preference orders are as follows:

$$A^1 = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ -1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \quad A^4 = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

The outranking matrix L is

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	WB <sub>i</sub>
O <sub>1</sub>	-	1	2	1	4
O <sub>2</sub>	3	-	1	3	7
O <sub>3</sub>	2	3	-	2	7
O <sub>4</sub>	3	1	2	-	6

$K_{pl}=3$

**Conclusions:**

1. There is no Condorcet winner
2. The Copeland winner is O<sub>2</sub>:

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	WCP <sub>i</sub>
O <sub>1</sub>	-	-1	-1	-1	0
O <sub>2</sub>	1	-	-1	1	2
O <sub>3</sub>	-1	1	-	-1	1
O <sub>4</sub>	1	-1	-1	-	1

3. The Borda winners are O<sub>2</sub> and O<sub>3</sub> (WB<sub>max</sub> = 7) and the order of element (subject to the decreasing values of WB<sub>i</sub>) is as follows: (O<sub>2</sub>, O<sub>3</sub>), O<sub>4</sub>, O<sub>1</sub>.
4. The plurality winners are O<sub>2</sub> and O<sub>3</sub>:

Number of experts that placed an element in the first position	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>
	1	2	2	0

5. The max-min winner is O<sub>3</sub>:

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	min <sub>j</sub> l <sub>ij</sub>
O <sub>1</sub>	-	1	2	1	1
O <sub>2</sub>	3	-	1	3	1
O <sub>3</sub>	2	3	-	2	2
O <sub>4</sub>	3	1	2	-	1

$\max_i \min_j l_{ij} = 2$  and it corresponds to O<sub>3</sub>.

6. The Hare winner

There is no element placed in the first position by the plurality of experts. The biggest and the least number of experts that placed given element in the first position are to be determined:

Number of experts that placed an option in the first position	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>
	1	2	2	0

Element (option) O<sub>4</sub> is to be removed and one gets:

Number of experts that placed an element in the first position	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>
	1	2	2

Now, the option  $O_1$  is to be removed and one gets:

Number of experts that placed an element in the first position	$O_2$	$O_3$
	2	3

Element  $O_3$  is the Hare winner (it is now placed in the first position by the plurality of experts).

### 7. The Coombs winner

There is no element placed in the first position by the plurality of experts. The greatest number of experts that placed a given option in the first and in the last position is to be determined.

Number of experts that placed an element	$O_1$	$O_2$	$O_3$	$O_4$
in the first position	1	2	2	0
in the last position	2	1	1	1

So, option  $O_1$  is to be removed and one gets:

Number of experts that placed an element	$O_2$	$O_3$	$O_4$
in the first position	2	3	0
in the last position	1	1	2

Now, option  $O_4$  is to be removed and one gets:

Number of experts that placed an element	$O_2$	$O_3$
in the first position	2	3
in the last position	1	1

Element (option)  $O_3$  is the Coombs winner (it is now placed in the first position by the plurality of experts).

### 8. The Kemeny's median

The loss matrix associated with these preference orders  $R = \begin{bmatrix} 0 & 6 & 4 & 5 \\ 2 & 0 & 5 & 2 \\ 4 & 3 & 0 & 3 \\ 3 & 6 & 5 & 0 \end{bmatrix}$ .

The order of options that is the "closest" to the given set of orders is  $O_3, O_2, O_4, O_1$  and its total distance from the given set of orders is equal 17.

**Jan Studzinski, Olgierd Hryniewicz (Editors)**

**MODELLING CONCEPTS AND DECISION  
SUPPORT IN ENVIRONMENTAL SYSTEMS**

This book presents the papers that describe the most interesting results of the research that have been obtained during the last few years in the area of environmental engineering and environment protection at the Systems Research Institute of the Polish Academy of Sciences in Warsaw and the Leibniz-Institute of Freshwater Ecology and Inland Fisheries in Berlin (IGB). The papers were presented during the First Joint Workshop organized at the IGB in February 2006. They deal with mathematical modeling, development and application of computer aided decision making systems in the areas of the environmental engineering concerning groundwater and soil, rivers and lakes, water management and regional pollution.

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