SYSTEMS RESEARCH INSTITUTE
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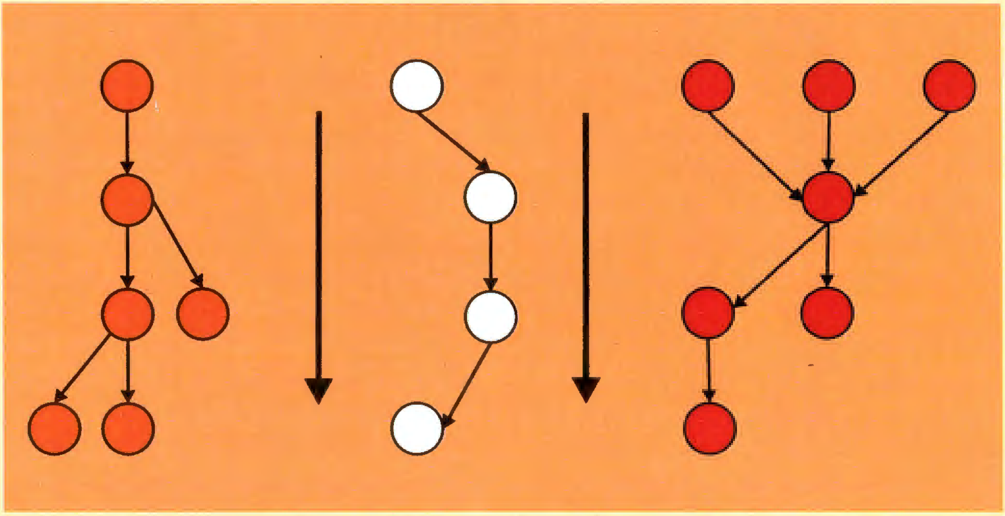
MULTICRITERIA ORDERING AND RANKING:
PARTIAL ORDERS, AMBIGUITIES AND APPLIED ISSUES



Jan W. Owsiński and Rainer Brüggemann
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Applications and Comparisons

Partial Order Ranking Methodologies in CSR Driven Innovation

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Corporate Social Responsibility (CSR) driven innovation is typically highly complex as a wide range of parameters (descriptors) needs to be taken into account. The paper introduces a simple method to aggregate descriptor values prior to partial order ranking is suggested as a decision support tool that advantageously may be applied by companies being engaged in CSR driven innovation processes, the eventual objective being the selection of one or more products or services for further development that, e.g., fulfil the requirements of being of a) value to the company and at the same time strengthening or consolidating the company's image, b) value to the target customer and at same time being safe to use, and c) value to the society and at the same time being environmentally friendly. Thus, in cases where pairs of descriptors are closely relation, like value and importance factors, the descriptors may advantageously be aggregated by multiplying the single corresponding values, the aggregated values subsequently being used for conventional partial order ranking. The aggregation is an alternative in situations where the ration between the total number of descriptors and the number of elements to be ranking is unfavourable possibly leading to non-robust models. The rationale behind the descriptor aggregation is discussed.

Keywords: partial order ranking, descriptor aggregation, decision support, Corporate Social Responsibility, CSR, innovation

1. Introduction

In recent years an increasing demand for products and services that are beneficial for the society as well as for the environment. At the same time organizations obviously focus on their turn over, contribution margin and profit. Hence, CSR (Corporate Social Responsibility) driven innovation is an innovation process where social and environmental considerations are in onset for the development of services and products in a profitable way. By applying CSR driven innovation as part of the overall strategy organizations can increase their growth and competitiveness through products or services that are beneficial for the surrounding environment and society.

In the present paper we want to illustrate the application of partial order methodologies in the innovation process in an arbitrary company that focus it development work on products and services in order to comply with the CSR requirements. Hence, let us look at a company that on the one hand will focus on a) products or services that has a value to the company, e.g. ensuring a proper contribution margin but on the other hand it is important to the company to ensure and consolidate its image as a green producer/supplier. Further the company wants to focus b) on products or services of value to the target customers but at the same time do not jeopardize the human health, and c) at products and services of value to the society but at the same time is environmentally friendly. The basic of this study is to develop a method, based on partial order ranking that simultaneous takes into account both the "Value" and the "Importance" factors. Thus, the paper introduces the VI-POR concept.

Examples could be taken from the cosmetics industry where the products may well have value for the target customer and the society as a whole, like cleaning agents, but these may well cause damage to the human health, e.g., through allergic reactions, and to the environments through discharge of environmentally hazardous substances. Obviously, examples are legio and the single Value – Importance descriptions should be disclosed according to the specific industry in question.

2. Methodologies

In the following partial order ranking (POR) including linear extensions (LE) and average rank will be shortly presented.

2.1. Partial order ranking

The theory of partial order ranking is presented elsewhere (Brüggemann. et al., 2001; Brüggemann and Carlsen, 2006).

In brief, Partial Order Ranking is a simple principle, which a priori includes "≤" as the only mathematical relation among the objects. If a system is considered, which can be described by a series of descriptors p_i, a given object *A*, characterized by the a set of descriptors $p_i(A)$ can be compared to another object B, characterized by the descriptors $p_i(B)$, through comparison of the single descriptors, respectively. Thus, object A will be ranked higher than object B, i.e., $B \leq A$, if at least one descriptor for A is higher than the corresponding descriptor for B and no descriptor for A is lower than the corresponding descriptor for B. If, on the other hand, $p_i(A) > p_i(B)$ for some descriptor i and $p_j(A) < p_j(B)$ for some other descriptor j, A and B will be denoted incomparable. Obviously, if all descriptors for A are equal to the corresponding descriptors for B, i.e., $p_i(B) = p_i(A)$ for all i, the two objects will have identical rank and will be considered as equivalent, i.e., $A \sim B$.

In mathematical terms the basic of partial order can be expressed as

$$B \leq A \Leftrightarrow p_i(B) \leq p_i(A) \text{ for all } i \tag{1}$$

In the following we consider the partial order not only of objects, but also of possible equivalence classes, i.e. the order of the quotient set, under the equivalence relation "equality" (see Brüggemann, Bartel, 1999; and a recent application: Brüggemann et al., 2006). In practice this means representatives of equivalence classes were taken to perform the partial order analysis. The other equivalent elements are considered only if necessary. Besides the axioms of reflexivity and anti-symmetry it further follows that the axiom of transitivity applies, namely if $A \geq B$ and $B \geq C$ then $A \geq C$.

In partial order ranking – in contrast to standard multidimensional statistical analysis – neither any assumptions about linearity nor any assumptions about distribution properties are made. In this way the partial order ranking can be considered as a non-parametric method. There is in general no need for quantifying a preference among the descriptors. However, due to the simple mathematics outlined above, it must be emphasized that the method a priori is rather sensitive to noise. Hence, even minor fluctuations in the descriptor values may lead to non-comparability or reversed ordering (Carlsen, 2004, 2005a, b). A possible improvement is to apply weights within a step-by-step procedure (Simon et al., 2005), or to apply fuzzy partial order concepts (van de Walle et al., 1995; De Baets and De Meyer, 2003; Brüggemann et al., 2008). The present study applies a binary classification scheme (a discussion on classification and posets can be found in Brüggemann and Bartel, 1999).

The graphical representation of the partial ordering is often given in a so-called Hasse diagram (Brüggemann et al., 2001; Halfon, Reggiani, 1986; Hasse, 1952; Brüggemann et al., 1995).

In practice the partial order rankings are performed using the WHASSE software (Brüggemann et al., 1995). The software is freely available from the respective author upon request (brg_home@web.de).

2.2. Descriptors

The single descriptor values (DvT_j and DiT_j, T ranging from 1 to M) may, depending on the actual problem studied be obtained as absolute values. However, in the type of studies being the basis for the present paper the values will typically be a result of a consensus decision among a series of experts involved in the innovation process. Thus, the individual experts judge the both the values and the corresponding importance on a scale from, e.g. 0-5, 0 being the lowest and 5 the highest score meaning that if a given product is assumed to have a significant value to the company it my scored 4 or 5 by an expert but at the same time the expert may judge that may only have limited importance in relation to the image of the company and score it only 2 or 3. The actual descriptor value used for the ranking are a result of an aggregation, by summing the single expert scores followed by normalization to the 0-5 range as previously described in a study on accumulating partial order ranking, APOR (Carlsen and Brüggemann, 2008):

$$DvT_j = \frac{\sum_{q=1}^{q=Q} DvT_j^q}{Q} \tag{2}$$

$$DiT_j = \frac{\sum_{q=1}^{q=Q} DiT_j^q}{Q} \tag{3}$$

where Q is the number of experts participating in the consensus meeting.

2.3. Value-Importance Partial Order Ranking

The Value-Importance Partial Order Ranking (VI-POR) is a simple ordinary partial order ranking of the Value-Importance aggregated data. Thus, a set of Value descriptors (Dv) are developed for the number of element studied and subsequently the corresponding set of Importance descriptors (Di) is developed and the eventual Value-Importance descriptors (Dvi) are generated by multiplying the corresponding Dv and Di matrices followed by normalization to the 0-5 scale. Hence, the Dvi descriptors are given by

$$DviT_j = \frac{DvT_j \: x \: DiT_j}{Q}. \tag{4}$$

The rationale of the descriptor aggregation is discussed in Section 3 (*vide infra*). In Fig. 1 a graphical representation of the VI-POR approach is depicted.

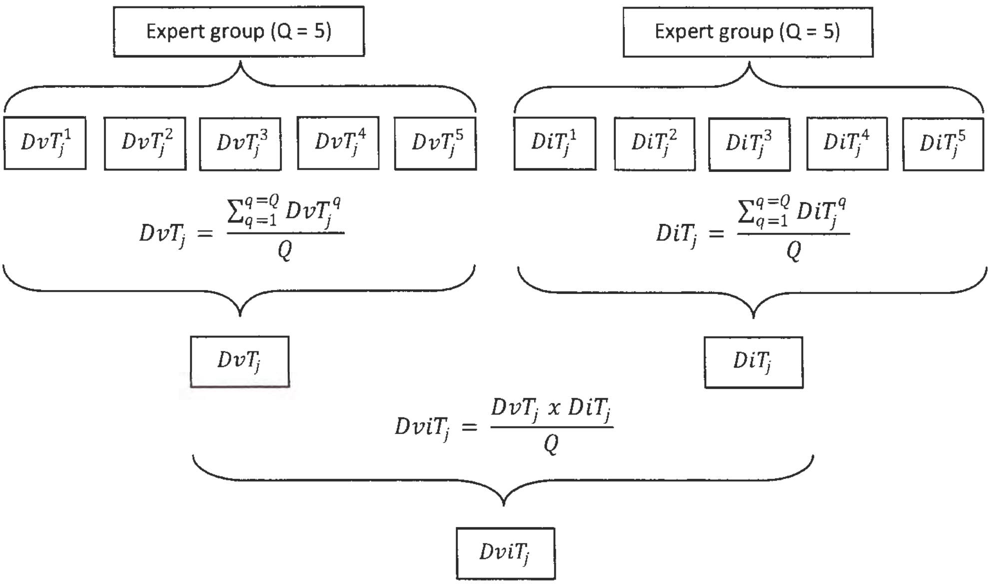

Figure 1. Graphical representation of the VI-POR approach

The partial order ranking is performed both on the individual *Dv* and *Di* descriptor set as well as on the *Dvi* set. Subsequently, ranking probabilities as well as an "absolute" rank may be obtained based on the principles described below (see Sections 2.4 and 2.5).

2.4. Linear extensions

The number of incomparable objects in the partial ordering constitutes a limitation in the attempt to rank e.g. a series of chemical substances based on their potential environmental or human health hazard. To a certain extent this problem can be remedied through the application of the so-called linear extensions of the partial order ranking (Fishburn, 1974; Graham, 1982). A linear extension is a total order, where all comparabilities of the partial order are reproduced (Brüggemann et al., 2001). Due to the incomparisons in the partial order ranking, a number of possible linear extensions corresponds to one partial order. If all possible linear extensions are found, a ranking probability can be calculated. Hence, based on the linear extensions the probability that a certain object, O, has a certain absolute rank can be derived. Further, it is possible to calculate the average ranks of the single objects in a partially ordered set (Winkler, 1982, 1983).

2.5. Average rank

The generation of the average rank of the single object in the Hasse diagram can be obtained through deriving a large number of randomly generated linear extensions (Bubley and Dyer, 1998, Sørensen et al., 2001; Lerche et al., 2002, 2003). Recently, an improved version of the random linear extension approach has been suggested by Sørensen et al (2007) taking into account that not all descriptors may be equally important.

Alternatively an approximate generation of the average rank of the single objects in the Hasse diagram is obtained applying the simple relation recently reported by Brüggemann et al. (2004). The average rank of a specific object, O, can be obtained by

$$Rk_{av} = (N + 1) - (S + 1) \times (N + 1)/(N + 1 - U) \quad (5)$$

where N is the number of objects in the diagram, S the number of successors, i.e., comparable object located below, to O and U the number of objects being incomparable to O (Brüggemann et al., 2004, further developments, see Brüggemann et al., 2005). Eqn. 4 is based on a local partial order model, i.e., every single object is selected and the remaining objects are approximately classified into comparable or isolated objects. It should be noted that in the ranking according to eqn. 4 the lower the number the higher the levels. Thus, the highest level will be "1". This is reversed compared to the original approach (Brüggemann et al., 2004).

3. Results and discussion

Let us assume a company that is in the process of developing new products and/or services that should live up to the conditions outlined in the introductory section of this paper. Thus the three value descriptors are a) value to the company, b) value to the target customer and c) value to the society, respectively, whereas the three corresponding importance descriptors are a) the company image, b) the target customer health and c) the environmental friendliness, respectively. Imagine a consensus meeting with an expert group that is to decide which of the possible products or services proposed by the development department should be further developed in accordance with the corporate social responsibility program of the company.

In Table 1 the descriptor values developed by the aggregation methods described in sections 2.2 and 2.3, respectively, in the present arbitrary example the members of the expert group participating in the consensus meeting being assumed to be 5 (the single scores being given in appendix 1 and 2, respectively).

Whereas the aggregation of the single descriptor values by addition and subsequent normalization appears straight forward (Carlsen and Brüggemann, 2008) the aggregation leading to the *Dvi* descriptor values may a priori appear somewhat more dubious. However, the rationale for this type of aggregation is discussed in the following.

Table 1: Descriptor value applied for partial order ranking, the single *Dv* and *Di* values being a result of 5 individual scoring, the single values being aggregated according to eqn. 2 and 3, the *Dvi* values being generated as according to eqn. 4

ID	*Dv1*	*Dv2*	*Dv3*	*Di1*	*Di2*	*Di3*	*Dvi1*	*Dvi2*	*Dvi3*
1	2.4	1.8	3.2	3.4	1.8	4.0	1.63	0.65	2.56
2	0.8	4.2	3.2	3.0	3.0	2.6	0.48	2.52	1.66
3	3.2	1.0	0.4	2.0	2.8	1.6	1.28	0.56	0.13
4	2.4	3.0	1.8	2.4	1.8	3.8	1.15	1.08	1.37
5	2.2	1.2	3.2	1.6	3.0	1.2	0.70	0.72	0.77
6	2.8	4.4	1.8	3.4	2.4	2.6	1.90	2.11	0.94
7	1.6	1.6	3.4	2.4	4.0	2.8	0.77	1.28	1.90
8	4.4	3.2	3.2	2.8	3.4	3.2	2.46	2.18	2.05
9	4.2	2.0	2.2	2.4	2.4	2.8	2.02	0.96	1.23
10	3.2	2.6	2.4	2.4	2.6	2.8	1.54	1.35	1.34
11	1.6	4.6	2.4	2.2	3.6	3.6	0.70	3.31	1.73
12	1.0	0.6	3.2	2.2	2.6	2.2	0.44	0.31	1.41
13	1.8	3.4	3.4	3.0	3.2	2.8	1.08	2.18	1.90
14	4.8	1.0	2.4	2.0	2.6	2.2	1.92	0.52	1.06
15	3.0	2.6	3.2	2.8	2.6	1.0	1.68	1.35	0.64
16	3.2	1.6	3.2	2.0	3.2	3.0	1.28	1.02	1.92
17	0.8	2.2	0.4	1.8	2.2	3.4	0.29	0.97	0.27
18	3.2	0.8	4.6	2.6	3.0	3.4	1.66	0.48	3.13
19	3.0	1.2	1.0	1.8	3.4	2.2	1.08	0.82	0.44
20	1.6	2.2	1.2	3.2	3.8	2.8	1.02	1.67	0.67
21	4.0	1.8	1.0	2.4	3.4	1.8	1.92	1.22	0.36
22	2.8	1.8	4.8	3.2	3.0	1.6	1.79	1.08	1.54
23	2.6	0.8	2.6	2.4	1.6	3.4	1.25	0.26	1.77
24	3.6	1.8	4.8	2.2	2.0	1.8	1.58	0.72	1.73
25	2.6	3.0	4.8	3.0	2.4	2.4	1.56	1.44	2.30

Obviously it is *a priori* possible to rank the products and services under discussion based on all 6 (3 *Dv* and 3 *Di*) descriptors simultaneously. However, this may well lead to an unfavourable combination of the number of descriptors and the number of elements (Sørensen et al., 2000) and consequently the ranking will appear of minor value. Thus, in order to obtain a more robust ranking the suggested aggregation may be brought into play.

Combining value and importance descriptors by aggregation by multiplication of the single descriptor values (eqn. 4) may immediately it may look like combining incomparable element. However, in the present case the aggregation by multiplication appears justified by the fact that the company on one hand want to develop products and service that are of value to the company AND at the same time contribute to a positive image of the company. Likewise products and services that are of value to the target customer and the society, respectively AND at the same time is safe to the customer and environmentally friendly, respectively. In other words, the company obviously does not want to market products and services that are of low or no value to the business even if it contribute significantly to the company image and, on the other hand, products and services that are of value to the company should not be marketed if it does contributes to the company's image in a positive way. Obviously the same type of arguments holds in the case of target customer value vs. customer safety and society value vs. environmental friendliness, respectively.

The initial expert judgement apparently is their individual estimates of the probabilities for the value and importance of the single descriptors. Thus, taken element ID #14 as an example, the score for *Dv1* equal to 4.8 indicates a belief that there is an 96 % chance that the product or service will be of value to the company.

On the other hand a score of 2.0 for, e.g., *Di1* indicates the probability that the product will contribute in a positive way to the company's image is only 40 %. Hence, the normalized *Dvi* descriptors can be regarded as a combined probability measure for the associated value and importance. This means that the resulting, normalized *Dvi1* equals 1.92 corresponding to a probability of ca. 38 % for the products being both of value to the company and contributing to the company's image in a positive way.

In Fig. 2 the partial order rankings based on the *Dv* and *Di* descriptors, respectively are depicted.

Immediately, element ID #8 calls for attention as it appears to be ranked high both based on the *Dv* and the *Di* descriptors and as such it would also be expected to be among the top ranks in ranking based on the *Dvi* descriptors or alternatively based on simultaneous inclusion of the *Dv* and *Di* descriptors, respectively. Looking only at the *Dv* descriptors element ID #25 should receive some attention whereas elements ID #18 and ID #20 *a priori* appear as being of interest and possible also elements Id #13 and ID #2 from level 2 and 3, respectively.

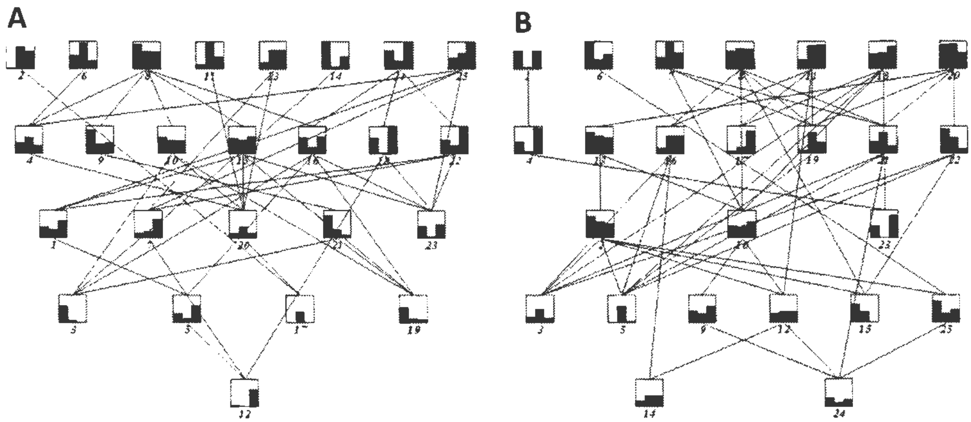

Figure 2. Ranking of the 25 element given in Table 1 based on the *Dv* descriptors (A) and *Di* descriptors (B), respectively. The heights of the bars reflect the mutual size of the single descriptors (*Dv1*, *Dv2*, *Dv3* and *Di1*, *Di2*, *Di3*, respectively).

In Figs. 3 and 4 the rankings of the 25 element (see Table 1) based on the *Dv+Di* and the *Dvi* descriptors, respectively are depicted.

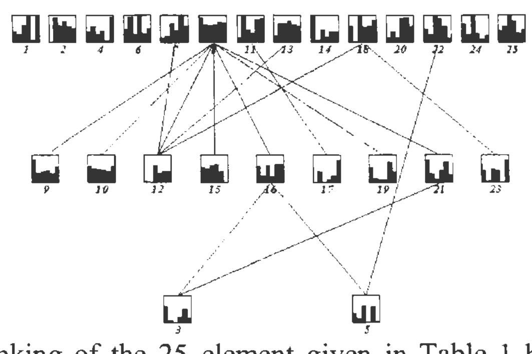

Figure 3. Ranking of the 25 element given in Table 1 based on the *Dv* descriptors and *Di* descriptors. The heights of the bars reflect the mutual size of the single descriptors (*Dv1*, *Dv2*, *Dv3*, *Di1*, *Di2*, *Di3*).

It is immediately seen that the ranking based on the combined descriptor set (*Dv+Di*) (Fig. 3) is inappropriate due to a too high number of descriptors compared to the number of elements as discussed by Sørensen et al. (2000). However, as expected element ID #8 is displayed as being of interest and to some extent element ID #13. However, due to the rather poor ranking it appears difficult to apply the results based on the latter ranking approach.

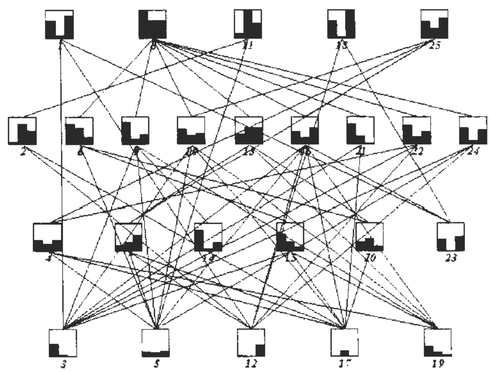

Figure 4. Ranking of the 25 element given in Table 1 based on the *Dvi* descriptors. The heights of the bars reflect the mutual size of the single descriptors (*Dvi1*, *Dvi2*, *Dvi3*).

On the other hand it is obvious that applying the aggregated descriptors (Fig. 4) an increased number of connections, and thus a significantly improved ranking has developed. In this context it is worthwhile to note that the multiplication operation is order preserving and consequently the graph depicted in Fig. 4 is enriched by a number of new relations. Thus, it is seen that in Fig. 3 ID #5 < ID #22 and in Fig. 4 ID #5 < ID #4 < ID #22. Analogously in Fig. 3 ID #12 < ID #8, which in Fig. 4 is enriched to ID #12 < ID #24 < ID #8.

Not surprisingly element ID #8 appears as the top candidate for further studies, but also element ID #25 deserves to be looked further upon. More interesting is the element ID #18 that in the final ranking is found in the top level. However, on the other hand this element ID #18 appears as being of minor or no interest due to the very low score of descriptor *Dvi2* as a consequence of a low *Dv2* score (Table 1). This element would most probably be rejected as a candidate in a final expert judgement.

Returning to the above discussion regarding the *Dvi* descriptors as probability measures it is of course interesting to look closer at element that are located high based on, e.g., the importance descriptors whereas it is located significantly lower based on the associated value descriptors. Element ID #20 may serve as an example. Thus, ID #20 is located in the top level based on the importance descriptors (Table 1 and Fig. 2B); whereas it is located only at the level 3 (of 5) based on the associated value descriptors (Table 1 and Fig. 2A). Consequently, the combined probabilities, amounting to ca. 20, 33 and 13 %, respectively (Table 1) are fairly low resulting in a low ranking in the eventual ranking based on the aggregated *Dvi* descriptors, the ID #20 being found in level 3 (of 4) (Fig. 4).

In Table 2 the average ranks of the 25 elements (Table 1) are given. As already indicated elements ID #8 and ID #25 appear as the top ranked elements.

Table 2. Average ranks of the 25 elements as calculated based on eqn. 1 (left hand side) and calculated based on 20.000 randomly selected linear extensions (right hand side)

ID	*S*	*U*	*Rkav*		*RLE*	*ID*
8	19	5	1.2		1.4	8
25	10	14	2.2		4.2	25
11	4	20	4.3		7.2	6
1	3	21	5.2		7.3	13
6	6	17	5.8		8.0	11
13	6	17	5.8		8.2	22
22	6	17	5.8		8.7	9
18	2	22	6.5		9.6	1
9	4	19	7.4		10.1	18
16	6	16	7.8		10.4	16
24	3	20	8.7		10.9	24
10	4	18	9.8		11.6	10
2	2	21	10.4		11.7	21
21	2	21	10.4		13.0	15
15	3	19	11.1		13.7	7
4	3	18	13.0		13.9	2
7	3	18	13.0		13.9	4
20	1	20	17.3		15.8	20
14	0	22	19.5		15.9	14
23	0	19	22.3		19.1	23
19	0	14	23.8		21.4	19
3	0	13	24.0		21.9	3
12	0	13	24.0		22.0	5
5	0	12	24.1		22.0	12
17	0	10	24.4		23.1	17

The ranking (Fig. 4) is further substantiated by estimating the average rank according to eqn. 5 (Brüggemann et al., 2004) as well as calculated based on randomly selected linear extensions (Bubley and Dyer, 1998, Sørensen et al., 2001; Lerche et al., 2002, 2003).

It is noted that some minor differences prevail between the two methods of calculating average ranks. This is not surprising as has been seen and discussed previously (Brüggemann et al., 2004). However, both approaches unequivocally disclose the elements ID #8 and ID #25 as the 2 top candidates for further development. Further it is immediately seen that the element ID #20 being discussed above also based on the average rank is ranked fairly low (*Rkav* = 17.3, *RLE* = 15.8).

It should of course be noted that the average rank not by itself disclose the actual uncertainty of the rank determination. For that the ranking probability need to be estimated. In Fig. 5 the probability distribution of ranks of elements ID #8 and ID #25 is given based on the calculation of 20,000 randomly selected linear extensions.

It is immediately clear that whereas the element ID #8 for sure is ranked high the element ID #25 display a much broader probability distribution, a fact that has to be discussed before a final decision concerning further development of this product is initiated. Similar analyses may obviously be carried out for further elements to the extent needed for the ongoing discussion and eventual expert judgement within the company on product development.

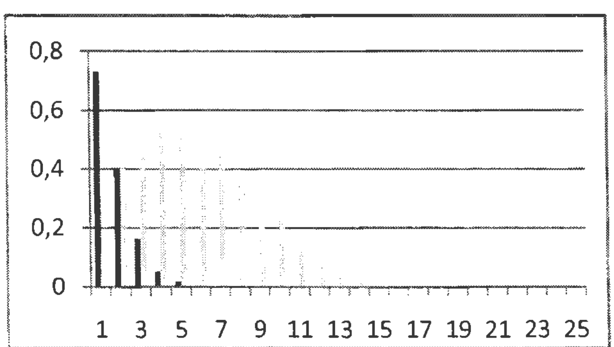

Figure 5. Probability distribution of possible ranks of element ID #8 (dark grey) and ID #25 (light grey) based on 20,000 randomly selected linear extensions

4. Conclusions and outlook

The above results and discussions have demonstrated that the aggregation of closely associated descriptor values constitute an alternative to the application of all descriptors in cases where the ratio between the number of descriptors and the number of elements to be ranked does not allow the development of a robust model. Thus, aggregated descriptor values, generated by simple multiplication of correspondingly associated values may be used for a subsequent partial order ranking and constitute as a combined probability measure for the associated value and importance.

The paper discusses this in term of a decision support tool to be applied by companies in connection with, e.g., their CSR driven innovation work and hereby leading to the development of products and services that, e.g., are of a) value to the company and at the same time strengthening or consolidating the company's image, b) value to the target customer and at same time being safe to use, and c) value to the society and at the same time being environmentally friendly

An arbitrary example is given with reference to the cosmetic industry. Hence, based on the above analyses it appears obvious that the aggregated *Dvi* descriptors advantageous may be applied in an attempt to verify potential products and /or services that the company should further develop in order to live up to the requirements for CSR. Going back to the cosmetic industry as mentioned in the introduction as an example the further development phase should comprise that the company.

- classifies, packs, labels and stores chemicals according to prevailing rules.
- does not use chemicals that are prohibited according to international conventions on hazardous substances.
- does not use ozone layer depleting substances.
- currently evaluates if the use of environmental and heath damaging substances can be substituted by less hazardous substances.

It should be stressed that the above described partial order approach should be regarded as a decision support tool, the eventual decision concerning the selection of products or services for further development should be based on an individual expert judgement.

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Appendix 1. Single scores for the value descriptors *Dv*

ID	Expert 1			Expert 2			Expert 3			Expert 4			Expert 5		
	Dv1	Dv2	Dv3	Dv1	Dv2	Dv3	Dv1	Dv2	Dv3	Dv1	Dv2	Dv3	Dv1	Dv2	Dv3
1	4	1	4	2	2	3	2	2	3	2	2	3	2	2	3
2	0	1	4	1	5	3	1	5	3	1	5	3	1	5	3
3	4	1	2	3	1	0	3	1	0	3	1	0	3	1	0
4	4	3	1	2	3	2	2	3	2	2	3	2	2	3	2
5	3	2	4	2	1	3	2	1	3	2	1	3	2	1	3
6	2	2	5	3	5	1	3	5	1	3	5	1	3	5	1
7	4	4	1	1	1	4	1	1	4	1	1	4	1	1	4
8	2	4	4	5	3	3	5	3	3	5	3	3	5	3	3
9	1	2	3	5	2	2	5	2	2	5	2	2	5	2	2
10	4	5	4	3	2	2	3	2	2	3	2	2	3	2	2
11	4	3	0	1	5	3	1	5	3	1	5	3	1	5	3
12	1	3	0	1	0	4	1	0	4	1	0	4	1	0	4
13	1	1	1	2	4	4	2	4	4	2	4	4	2	4	4
14	4	1	4	5	1	2	5	1	2	5	1	2	5	1	2
15	3	1	4	3	3	3	3	3	3	3	3	3	3	3	3
16	4	4	4	3	1	3	3	1	3	3	1	3	3	1	3
17	0	3	2	1	2	0	1	2	0	1	2	0	1	2	0
18	4	0	3	3	1	5	3	1	5	3	1	5	3	1	5
19	3	2	5	3	1	0	3	1	0	3	1	0	3	1	0
20	4	3	2	1	2	1	1	2	1	1	2	1	1	2	1
21	4	1	5	4	2	0	4	2	0	4	2	0	4	2	0
22	2	1	4	3	2	5	3	2	5	3	2	5	3	2	5
23	1	0	1	3	1	3	3	1	3	3	1	3	3	1	3
24	2	5	4	4	1	5	4	1	5	4	1	5	4	1	5
25	1	3	4	3	3	5	3	3	5	3	3	5	3	3	5

Appendix 2. Single scores for the value descriptors *Di*

ID	Expert 1			Expert 2			Expert 3			Expert 4			Expert 5		
	Di1	Di2	Di3	Di1	Di2	Di3	Di1	Di2	Di3	Di1	Di2	Di3	Di1	Di2	Di3
1	3	3	5	5	1	3	3	1	3	4	3	5	2	1	4
2	5	0	4	2	5	4	2	4	4	1	2	0	5	4	1
3	2	2	4	4	3	1	1	1	2	0	4	1	3	4	0
4	1	0	4	3	4	4	4	1	3	1	1	3	3	3	5
5	0	5	2	1	4	0	1	1	0	3	3	1	3	2	3
6	4	2	1	4	2	5	5	1	3	0	5	3	4	2	1
7	3	3	2	1	4	4	3	4	3	2	4	2	3	5	3
8	5	1	5	3	4	4	2	2	3	4	5	1	0	5	3
9	2	4	4	4	1	4	2	4	0	1	0	2	3	3	4
10	2	4	1	3	4	5	1	1	4	5	2	3	1	2	1
11	1	1	3	2	4	5	1	4	4	5	4	1	2	5	5
12	3	3	1	0	4	3	3	3	3	5	2	3	0	1	1
13	0	5	1	5	3	2	4	5	4	4	2	3	2	1	4
14	1	2	1	4	3	3	1	4	2	0	2	1	4	2	4
15	2	3	0	2	2	4	2	3	0	3	4	0	5	1	1
16	1	2	5	3	3	4	1	5	1	3	1	2	2	5	3
17	3	0	4	1	4	5	2	3	0	2	2	4	1	2	4
18	3	3	5	3	2	0	1	4	3	4	3	5	2	3	4
19	1	2	2	2	1	3	1	4	3	4	5	3	1	5	0
20	4	5	2	2	4	5	2	2	1	4	5	3	4	3	3
21	5	3	2	1	3	4	1	3	2	1	5	0	4	3	1
22	4	5	0	3	3	3	1	3	1	5	3	3	3	1	1
23	2	3	4	1	0	4	3	0	4	3	1	4	3	4	1
24	2	3	0	1	2	2	1	0	3	4	1	1	3	4	3
25	3	1	3	3	2	2	2	2	4	3	4	0	4	3	3

This book is a collection of papers, prepared in connection with the 8th International Workshop on partial orders, their theoretical and applied developments, which took place in Warsaw, at the Systems Research Institute, in October 2008. The papers deal with software developments (PYHASSE and other existing software), theoretical problems of ranking and ordering under various assumed analytic and decision-making-oriented conditions, as well as experimental studies and down-to-earth pragmatic questions.

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