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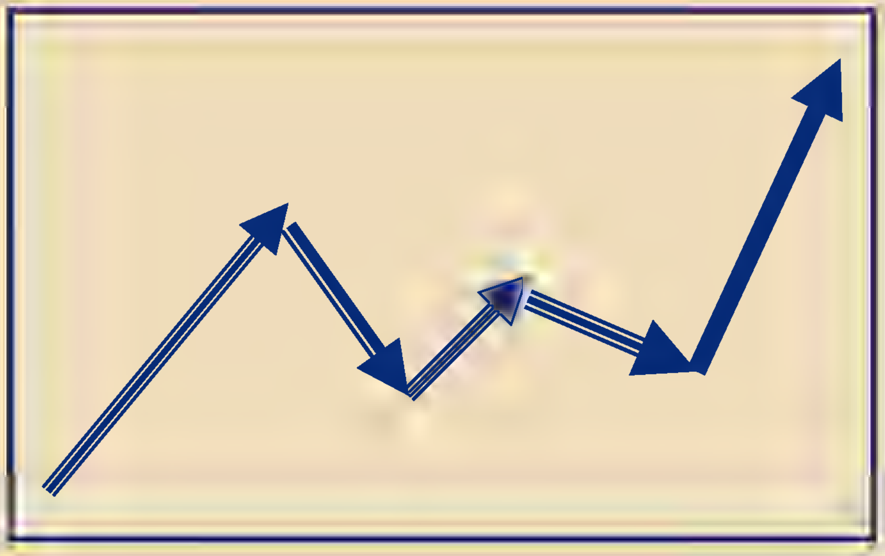
AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

Volume I



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S. PIASECKI, J.W. OWSIŃSKI, AN INTRODUCTION TO A THEORY OF MARKET COMPETITION

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Chapter I

SOME NOTIONS AND ASSUMPTIONS

In this chapter we present the fundamental assumptions and definitions, concerning:

- dependence of demand Λ upon the price C of a product,
- dependence of the costs κ of production upon the scale of production, μ,
- the notion of the "product", and, in particular, the "equivalent competitive product",
- the notion of profitability of productive (economic) activity, and
- the influence of fashion and opinion on demand.

The above notions and relations are broadly described in the literature, as well as in the earlier works of the first author. This applies, in a particular manner, to the model of demand for a newly introduced product, along with a forecast for the "lifecycle" of a product.[1] In order to secure the completeness of the reasoning, though, these aspects are also presented in Chapter I in the context of market competition processes.

Let us only remind at this point that, as mentioned in the Introduction, we consider the case, when competitors dispose of the same technologies of production, transport and sales.

1. Dependence of demand upon product price

We shall, in general, call demand for a given product on a given market the expected number, or volume, of products sold per unit time.

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[1] See Piasecki (2002-2004).

In terms of illustration we shall be using in further course of the book year as the time unit and shall measure production and sales in the number of products, which has no bearing whatsoever on the generality of considerations.

Demand for a product is a function of many variables, often non measurable ones, and we shall not list them all here. Yet, among these variables there are two, exerting the biggest influence on the volume of demand.

These two are the magnitude of the (potential) market, determined by the number of customers that would buy the product in question at the price C, and the exploitation characteristic of the product.

Before we determine the magnitude of the market, we must explain whom we mean when using the notions of buyer, customer or potential customer. In particular, each of these notions can address a natural person (of a definite age, gender, denomination, etc.), a social group (family, professional or socio-economic groups, etc.), or legal persons (enterprises, corporations, holdings, etc.).

Establishment of the kind of potential customers (buyers) results from the purpose of the product, defined by the designer and the producer, in terms of the target group. Hence, establishment of the kind of customers should not constitute a problem.

Since demand shall always be associated with a definite product, we shall first of all distinguish the group of potential customers (of whatever kind, as mentioned above), who – as we expect – would like to own the product, and for whom it is meant. The number of these potential customers shall be denoted by L_{mx}. Then, among these potential customers we shall distinguish the ones, who can afford purchasing (and using) the product at the given price C. This group of potential customers shall be referred to as customers or buyers. We shall denote their number by L, with, of course, $L \leq L_{mx}$.

Naturally, the value of L is the function of price, C, $L(C)$, with $L(\infty) = 0 \leq L(C) \leq L(0) = L_{mx}$. In order to determine this function we must establish when a customer shall not be able to afford the product, in view of its too high price (and cost of use), given the customer's income. We shall denote this income (per time unit chosen, here: one year) by d.

So, we start with the notion of too high price, which itself depends upon a number of factors, and is naturally a fuzzy notion. We shall approach this issue in a simplified manner, assuming that the value of "too high price" results from the comparison of two quantities:

- what is the fraction, γ, of the income d, that a potential customer can spend on purchase – and use – of the product (with, of course, $0 < \gamma < 1$?; and
- what is the cost (or benefit) resulting from the use of the product?

In order to determine these quantities, two kinds of products ought to be distinguished: those consumed at one time, and the durable ones. In both cases, though, we have to define the period of use of the product, T. In case of a one-time consumption product, like a loaf of bread, the period T is the time of consumption. If, on the other hand, it is a TV set, the time T is equal to the period of its use. Of course, in any case the respective time periods are realisations of random variables, and so the values of T, referred to here, are taken as the expected values of the respective variables.

So, in case of one-time consumption products the cost of using the product shall be defined as C/T.

The case of durable products (like the TV set, mentioned before) the situation is more complex. To the quotient, defined above, we must then add the running cost of using the product (in the case of a TV set – the cost of electricity per unit time). Hence, the overall cost of the product shall be expressed as $C/T + e$, where e is the running cost of use per unit time. This cost, when we consider, for instance, a car, would encompass fuel, oils and lubricants, tires etc., as well as repairs, registration, insurance and the like costs. If a product serves in conduct of a business activity, then the expression $C/T + e$ should be decreased by the value of benefit, brought by this product, again per unit time. We shall yet treat this issue in some more detail in the present book.[2]

In order, however, to keep the problem appropriately simple, we shall refer to the further course of the book to the ratio C/T as describing the cost of using the product.

Attention should also be paid to the fact that the quantity γd has a similarly broad and fuzzily defined interpretation. The simplest one is to treat it – in case of more expensive durable goods – as the limit value of the credit repayment, per unit time, when considering purchase of a definite product. (Yet, in such an interpretation one would have to still add the cost of credit service.)

In the case of one-time consumption products we can treat the expression γd as corresponding to the amount a person (a household, a company) is ready to spend on buying (and consuming) a given product in unit time. Actually, the value of fraction γ is often explicitly used by the persons managing the household finances. It is namely common to split up the monthly family income into the fractions, devoted to purchasing of definite kinds of consumption products, including those that are used in a continuous manner (like electricity, water supply etc.).

After this introduction we can now try to define the notion of "too high price". And so, we shall assume that a price C is too high

[2] See also Piasecki (1972, 2002-2004).

for a given potential customer when the inequality $\gamma d > C/T$ is not satisfied.

In particular, using this inequality, we can determine the limit income, d_{lim}, of the potential customers, such that the customers with (annual) income lower than d_{lim} shall not be able to afford the product at price C and time period of use T, with, therefore, $d_{lim} = (C/T)/\gamma$.

Consequently, demand generated by an individual customer (buyer) using a single product, is expressed as $\lambda_0 = 1/T$.

Hence, in order to determine the overall demand for a given product on a given market, we have to establish the number of customers, L, whose income is not lower than d_{lim}. This means that we should know the normalised cumulative income distribution function of the potential customers, $F(d)$:

$$F(d) = \frac{1}{L_{mx}} \cdot L(d)$$

where the function $L(d)$ defines the number of potential customers, whose income is not higher than d, while the function $F(d)$ tells us what part of the entire population of potential customers disposes of incomes not greater than d. These two functions are, generally, not negative, and they satisfy the following conditions:

$$L(0) = 0 \ , \ L(d_{mx}) = L_{mx} \quad ; \quad F(0) = 0 \ , \ F(d_{mx}) = 1,$$

where d_{mx} is the highest income of a potential customer on the market considered. An instance of the respective function is shown in Fig. 1.1.

In the sociological research, though, another function is used most often, the income distribution function, defined as

$$l(d) = \lim_{\Delta d \to 0} \frac{F(d + \Delta d) - F(d)}{\Delta d}.$$

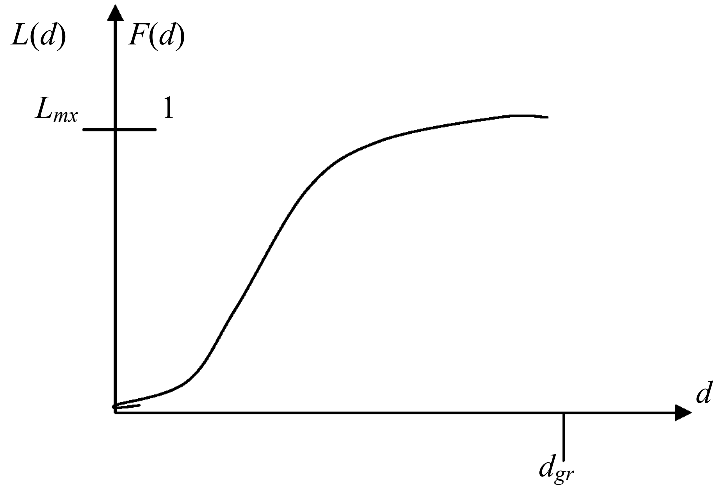

Figure 1.1. An illustration for the cumulative income distribution function, absolute and normalised

Formally, therefore, the income distribution function $l(d)$ is the derivative of the cumulative function $F(d)$. An instance of such $l(d)$ is sketched in Fig. 1.2.

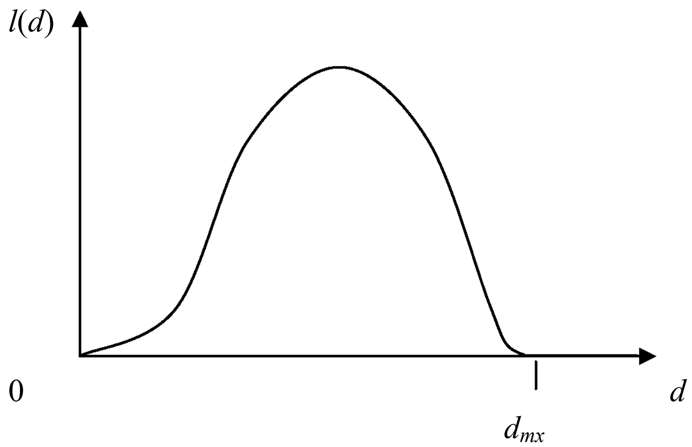

Figure 1.2. An example of the income distribution function

The values of the income distribution function fulfil the following obvious conditions: $l(0)=0$, $l(d)>0$ for $0<d<d_{mx}$, $l(d_{mx})=0$.

In particular, knowing the income distribution function, we can determine the average income of the potential customers:

$$\overline{d} = \int_0^{d_{mx}} x \cdot l(x) \cdot dx$$

and the number of buyers of the product, that is – these potential customers, whose incomes are not less than d_{lim}. This number equals $L_{mx} \int_{d_{\lim}}^{d_{mx}} l(x)dx = L_{mx}q(d_{lim})$, where $q(d_{lim}) = \int_{d_{\lim}}^{d_{mx}} l(x)dx = 1 - F(d_{lim})$.

The here introduced function $q(d)$ describes the income structure of the population of potential customers.

It is quite common in the sociological studies to show the distribution function $l(d)$ as a bar diagram, illustrated in Fig. 1.3.

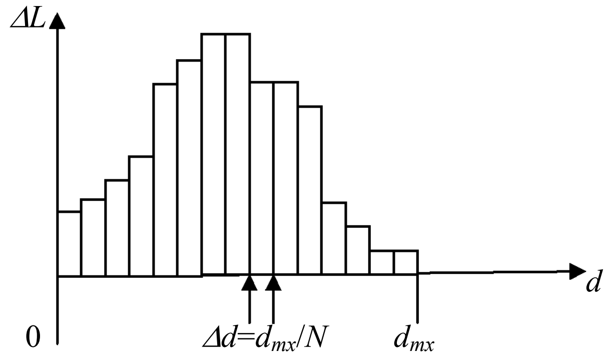

Figure 1.3. Presentation of the customer number income distribution in the form of the bar diagram

The bar diagram is constructed by splitting the interval of values of d between 0 and d_{mx} into N equal segments, each of the length $\Delta d=d_{mx}/N$, and calculating, on the basis of statistical data, the values of ΔL, corresponding to these segments, with

$$\Delta L(d_n) = L(d_{n+1}) - L(d_n) \qquad n = 1, 2, ...N, \text{ where } \sum_{n=1}^{N} \Delta L(d_n) = L_{mx}.$$

As can be easily noted, the bar diagram is a (discrete) approximation of the actual image of income distribution, and as the value of Δd decreases (meaning that N increases) it approaches the continuous image of the income distribution, i.e.

$$\lim_{\Delta d \to 0} \frac{L(d + \Delta d) - L(d)}{\Delta d} = L_{mx} \cdot l(d).$$

Knowing the intensity λ_0 of the product use by an individual customer (buyer), we can determine the magnitude of demand from the formula

$$L(d_{lim}) = L_{mx}\lambda_0 q(d_{lim}) = \lambda_{mx} q(d_{lim}),$$

or, if we substitute the value of d_{lim}, we get:

$$\Lambda(C) = \lambda_{mx} \cdot q(\frac{C}{\gamma \cdot T}) = \lambda_{mx} \cdot q(C) = L_{mx} \cdot \lambda_0 \cdot q(C).$$

Let us note that the value of the function $q(C)$ can be interpreted as the value of probability that a randomly chosen customer disposes of income d, sufficient for purchasing the product. Hence, we can write down that the expected (mean) intensity of demand for the product from a potential customer equals $\lambda = \lambda_0 \cdot q(C)$ and $\Lambda = L_{mx} \cdot \lambda$. It must be added, at this point, that such a manner of noting the respective relations makes it possible to clearly separate the dependence of demand upon price C and – the situation considered in greater detail later on – the radius R of the zone of sales, for $L_{mx} = \pi g R^2$, where g is the area density of customers.

Thus, in order to determine the concrete formula, defining the dependence of demand (λ) upon the price (C) of the product, one must know the income structure (q) of the potential customers or their income distribution (l), as well as technical parameters of the product, which, in our simplified example, are reduced to the period of use (durability) of the product (T). It is also necessary to know the “intensity of wish” or “impatience” regarding the purchasing of the product, expressed through the value of γ.

The value of λ_0 is usually adopted as equal the inverse of T.

We shall show further on various examples of dependence of demand λ upon the price C of the product, according to the specific features of the incomes of the local society. These features are described with the various shapes of the function of income density distribution. Let us consider some examples, then.

1. The uniform distribution

Assume that the income density function has the following form: $l(d) = l_0 = \text{const.}$, for $0 < d < d_{mx}$, $l(d) = 0$ for all other values of d, see Fig. 1.4.

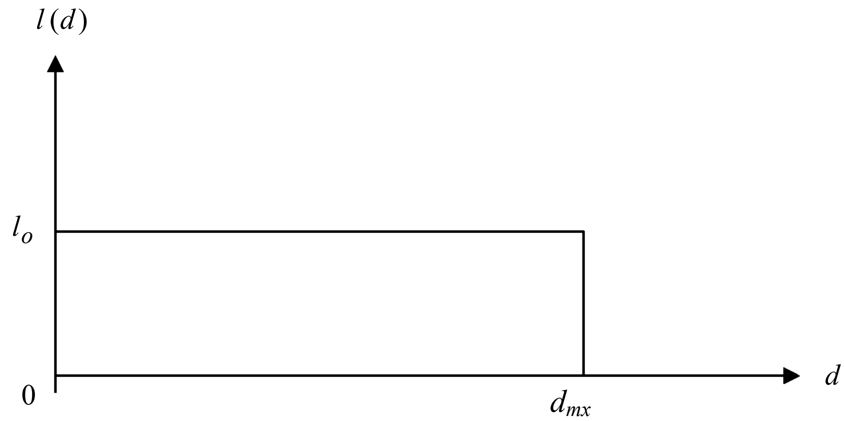

Figure 1.4. The uniform distribution

More useful for our purposes is the function of income structure, defined as

$$q(d) = \frac{1}{A}\int_{d}^{d_{\max}} l(x)dx\,,$$

which corresponds to the fraction of the local population, having annual income not lower than d, and where

$$A = \int_{0}^{d_{mx}} l(x)dx.$$

As we substitute $l(x) = l_0$, we get

$$q(d) = \frac{1}{l_0}\frac{1}{d_{mx}}\int_{d}^{d_{mx}} l_0 dx = 1 - \frac{d}{d_{mx}}.$$

The shape of the corresponding income structure function is shown in Fig. 1.5.

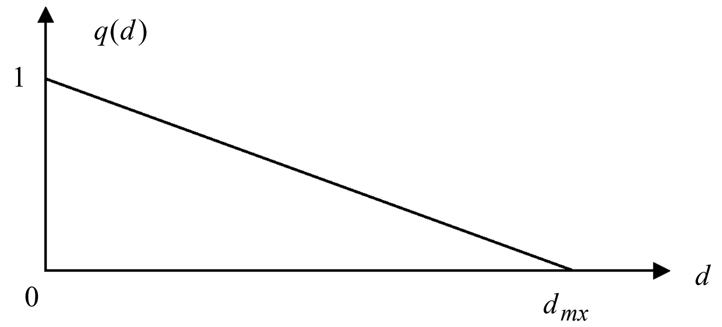

Figure 1.5. Shape of the income structure function, corresponding to uniform income distribution

Let us recall that we introduced the magnitude d_{lim}, the limit income, below which a potential customer will not be able to afford buying the product having durability T and price C, $d_{lim} = C/(\gamma T)$.

Hence, if there are L_{mx} of potential customers within the zone of influence of the sales outlet, then the expression $q(d_{lim})L_{mx}$ determines the number of buyers of the product. Each such product, after (average) time T will have to be replaced by a new one (perhaps featuring better parameters. As already discussed, this replacement process defines the intensity of purchase, i.e. $\lambda_0 = 1/T$. Consequently, the intensity of sales of the product shall be equal

$\Lambda = \lambda_0 q(d_{lim})L_{mx}$,

which, for the case of uniform distribution of incomes means

$\Lambda = \lambda_0(1-(d_{lim}/d_{mx}))L_{mx} = \lambda_0(1-C/(\gamma T d_{mx}))L_{mx}$.

Now, if we use notation $C_{mx} = \gamma T d_{mx}$ to denote the maximum price of the product, for which the number of persons, who can afford it, falls to zero, then we obtain

$$\Lambda = \lambda_{mx}(1\text{-}C/C_{mx}) \text{ or } \Lambda = \lambda_{mx} - aC$$

where $\lambda_{mx} = \lambda_0 L_{mx}$ and $a = \lambda_{mx}/C_{mx}$.

The product we consider serves to satisfy a certain need of the potential (and actual) customers. We can imagine that there also exists a product of higher quality, and, correspondingly, higher price, which satisfies the same kind of need. In such a broader context we are obliged to introduce, side by side with the value of d_{lim}, i.e. the income, above which a potential customer is able to afford buying the product, also the value d_M, indicating the limit income, above which a potential customer can afford the better product and so will not buy the cheaper one.

This kind of phenomenon is most easily observed on the market of home appliances, TV equipment or cars. For such a situation we can write down

$$\Lambda = \lambda_{mx}[q(d)\text{-}q(d_M)] = \lambda_{mx}(d_M\text{-}d)/d_{mx} = \lambda_{mx}(C_M\text{-}C)/C_{mx},$$

where C_M is the price of the higher quality product, serving to satisfy the same kind of need.

2. The triangle distribution

Let us now consider the case, when the income density function has the shape shown in Fig. 1.6. For this case have:

$$q(d) = \frac{1}{A}\int_{d}^{d_{mx}} l(x)dx = \frac{1}{A}\frac{1}{2} l_{mx} d_{mx} (1 - \frac{d}{d_{mx}})^2 ,$$

where $A = \int_0^{\infty} l(x)dx = \frac{1}{2} l_{mx} d_{mx}$.

If we now substitute $d_{lim} = C/\gamma T$, then we obtain

$$q(C) = (1\text{-}C/(\gamma T d_{mx}))^2 = (1\text{-}C/C_{mx})^2,$$

where, as before, $C_{mx} = \gamma T d_{mx}$.

Next, we obtain therefrom

$$\Lambda = \lambda_{mx}(1\text{-}C/C_{mx})^2 = \lambda_{mx}(1\text{-}aC)^2$$

where $a = 1/C_{mx} = 1/(\gamma T d_{mx})$.

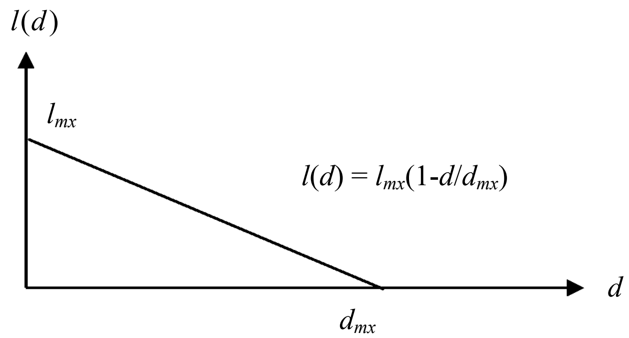

Figure 1.6. The triangle distribution

If, like in the preceding case, we assume that there exists a product of higher quality, satisfying the very same need, sold for a higher price, C_M, then the demand function shall be expressed as

$$\Lambda = \lambda_{mx}[q(d)\text{-}q(d_M)] =$$

$$\lambda_{mx}(C/C_M)[2\text{-}(C/C_M)(1+(C_M/C_{mx}))](1\text{-}(C_M/C_{mx})).$$

3. The power function distribution

Assume now that the income density function has the form

$$l(d) = l_{mx} \cdot \frac{d^2}{(d_0 + d)^2}.$$

The shape of the respective function is shown in Fig. 1.7. We select the value of d_0 so as to ensure the fulfilment of

$$l(d_0) = \frac{1}{4} l_{mx}.$$

For the power distribution the function $q(d)$ takes on the form

$$q(d) = \frac{1}{A}\int_d^{\infty} l_{mx} d_0^2 \frac{1}{(d_0 + x)^2} dx = \frac{1}{A}\frac{l_{mx} d_0^2}{d_0 + d} = \frac{d_0}{d_0 + d}.$$

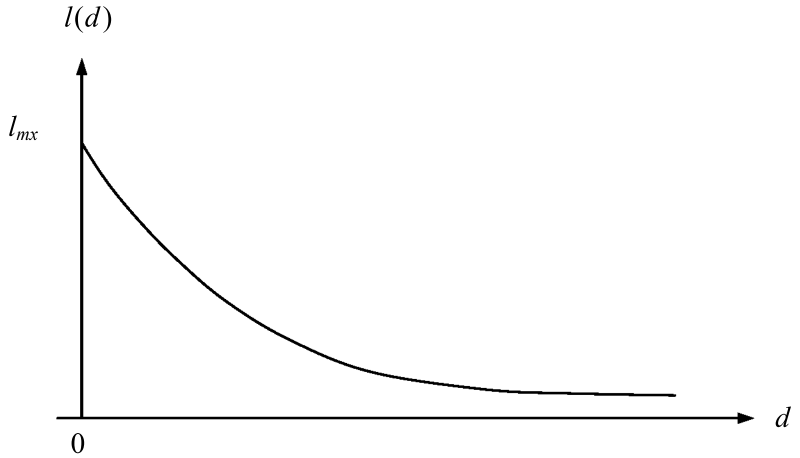

Figure 1.7. The shape of the power distribution

By appropriate substitution, we get

$$q(C) = \frac{d_0}{d_0 + \frac{C}{\gamma T}} = \frac{C_0}{C_0 + C}, \text{ where } C_0 = d_0\gamma T.$$

Consequently, we obtain

$$\Lambda = \lambda_0 L_{mx} C_0/(C_0+C) = \lambda_{mx} C_0/(C_0+C) = \lambda_{mx}(1+C/C_0)^{-1}.$$

If the product considered is being sold on the market, on which there exists another product, satisfying the same kind of need, but of higher quality and higher price, C_M, then, of course, there is no perspective for selling the lower quality product for a price $C > C_M$. In the case of such two products the demand function takes on the following form:

$$\Lambda = \lambda_{mx} \cdot \left[q(C) - q(C_M) \right] = \lambda_{mx} \cdot \frac{C_0}{C_0 + C} \cdot \frac{C_M - C}{C_0 + C_M}.$$

4. Non-monotone power distribution

Let us now consider the function of income density distribution of the following general form:

$$l(x) = \frac{2x}{(1+x^2)^2}, \text{ where } x = d/d_0.$$

Its shape is shown in Fig. 1.8.

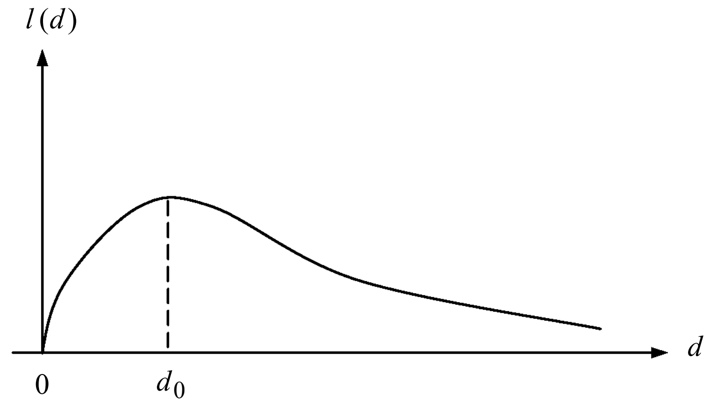

Figure 1.8. The shape of a non-monotone power function considered

For this shape of the income density function, the function $q(d)$ takes the form

$$q(d) = \int_{\frac{d}{d_0}}^{\infty} l(x)dx = \frac{1}{1+\left(\frac{d}{d_0}\right)^2}.$$

or, if we substitute $d = C/\gamma T$, we get

$$q(C) = \frac{1}{1+(\frac{C}{C_0})^2}, \text{ where } C_0 = \gamma T d_0,$$

and, consequently:

$$\Lambda = \lambda_{mx} \cdot \frac{1}{1 + \left(\dfrac{C}{C_0}\right)^2}.$$

If the sales of our product are limited by the existence of competitive products of higher quality class, then, after we have established the lowest price C_M of this higher quality product, we can determine demand with the following formula:

$$\Lambda = \lambda_{mx}\left[q(C) - q(C_M)\right] = \lambda_{mx} \frac{\left(\dfrac{C_M}{C_0}\right)^2 - \left(\dfrac{C}{C_0}\right)^2}{\left[1 + \left(\dfrac{C_M}{C_0}\right)^2\right] \cdot \left[1 + \left(\dfrac{C}{C_0}\right)^2\right]}.$$

5. Trimmed power distribution

The previously analysed distribution had in a way an academic character, since it admitted existence of infinite incomes, which is, of course, not a real case. Hence, we shall now consider a trimmed power distribution, whose shape is shown in Fig. 1.9, i.e. $l(d) = 2d/(1+d^2)^2$ for $0 \le d \le d_{mx}$, and $l(d) = 0$ for other values of d.

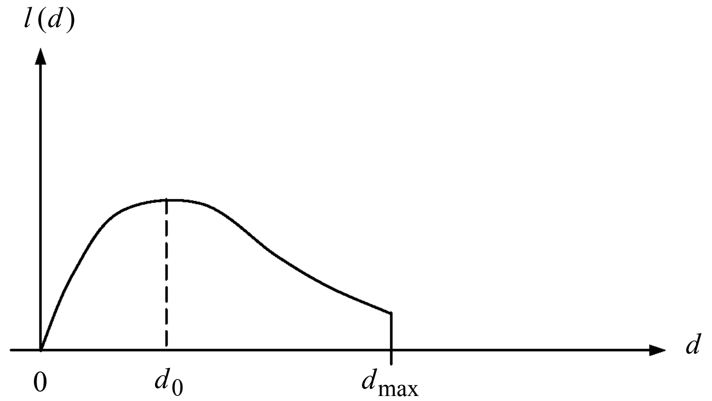

Figure 1.9. The shape of a trimmed power distribution

Now, the income structure function shall take on the form

$$q(d) = \int_{\frac{d}{d_{mx}}}^{1} \frac{2x}{(1+x^2)^2}\,dx = \frac{1}{1+\left(\dfrac{d}{d_{mx}}\right)^2} - \frac{1}{2}$$

As we normalise this function, we get

$$q(d) = 2\frac{1-\left(\dfrac{d}{d_{mx}}\right)^2}{1+\left(\dfrac{d}{d_{mx}}\right)^2}$$, where $q(0) = 1$, $q(d_{mx}) = 0$.

If we now substitute $d = C/\gamma T$, we obtain

$$q(C) = 2\cdot\frac{1-(\dfrac{C}{C_{mx}})^2}{1+(\dfrac{C}{C_{mx}})^2}$$, where, as before, $C_{mx} = \gamma T d_{mx}$.

And so, ultimately we obtain

$$\Lambda = \lambda_0.q(d).L_{mx} = \lambda_{mx}.\frac{1-(\dfrac{C}{C_{mx}})^2}{1+(\dfrac{C}{C_{mx}})^2}.$$

6. The polynomial distribution

In the wealthier societies, the essential mass of the potential customers has incomes concentrated around the average, with relatively less of those with significantly lower and higher incomes.

The simplest approximation of such a structure of incomes is provided by the distribution of income density described with the second order polynomial of the form $l(d) = (d_{mx} - d)d$.

The shape of this function is shown in Fig. 1.10.

The range of applicability of the formula for density, whose shape is shown in Fig. 1.10, is limited to the interval $0<d<d_{mx}$. The respective distribution corresponds in a certain manner to the bar diagram, shown in Fig. 1.3, with the appropriately defined limits on the bars.

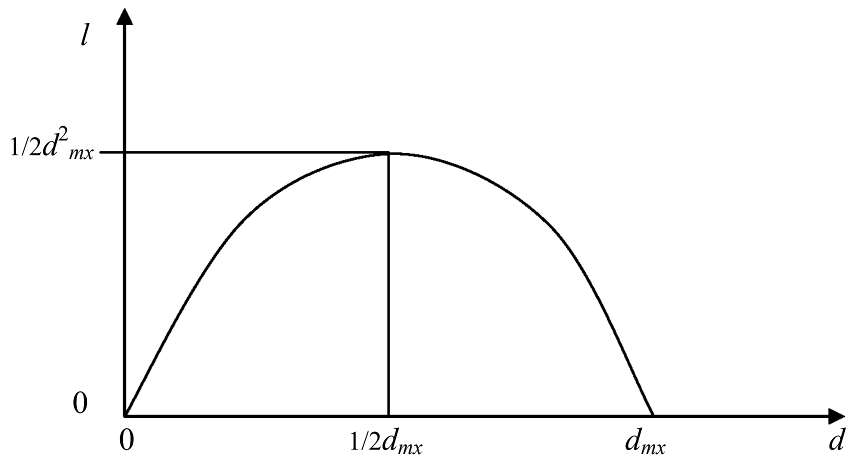

Figure 1.10. An example of the polynomial (quadratic) density function

For this distribution the number of persons with incomes higher than d shall be defined by the formula

$$\int_{d}^{d_{mx}} l(x)dx = \frac{1}{6}d_{mx}^{3} - \frac{1}{3}(\frac{3}{2}d_{mx} - d)\cdot d^{2} .$$

As before, in order to define the function $q(d)$, we have to perform appropriate normalisation, from which we obtain:

$$q(d_{gr}) = \frac{\int_{d_{gr}}^{d_{mx}} l(x)dx}{\int_{0}^{d_{mx}} l(x)dx} = 1 - \left(3 - 2\frac{d}{d_{mx}}\right)\cdot\left(\frac{d}{d_{mx}}\right)^{2},$$

so that $q(d_{mx}) = 0$, $q(0) = 1$.

If we denote, as before, $\lambda_{mx} = \lambda_0 L_{mx}$, and we assume that purchasing of the product can only be afforded by the customers, whose income exceeds d_{lim}, then the dependence of demand upon the price of the product shall take on the following form:

$$\Lambda = \lambda_{mx}\left[1-\left(3-2\frac{d_{gr}}{d_{mx}}\right)\cdot\left(\frac{d_{gr}}{d_{mx}}\right)^2\right],$$

where $d_{lim} = d_{lim}(C)$ is the function of the product price C: $d_{lim} = C/(\gamma T)$.

The shape of the respective function is shown in Fig. 1.11.

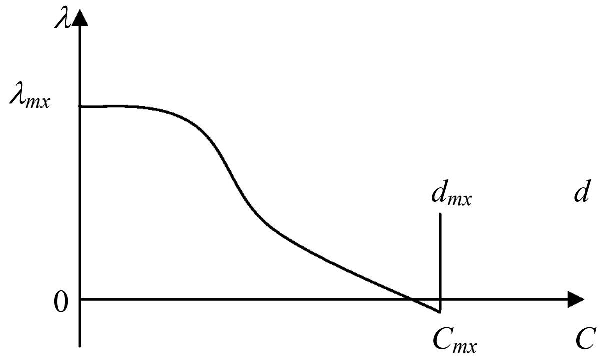

Figure 1.11. Illustration of the demand function for the polynomial (quadratic) income distribution

At this point we shall terminate the consideration of different forms of the income density functions, $l(d)$, and the associated with them expected stationary intensity of purchasing (demand), for the product characterised by C and T, and the value of γ, which is proper for a given product and the wealth of the local community.

Some remarks on dependence of demand upon the influence of "fashion"

We have not been considering until now the influence exerted by the common opinion and variable fashion on the decision of purchase of a given product by a customer, against the back ground of other, competitive products.

We have been assuming, therefore, that the customer is motivated uniquely by an own economic interest, that is – if s/he has the possibility of choosing among buying product *A* or *B*, which serve to satisfy the same need, for the price, respectively, C_A or C_B, then s/he will choose the one, for which the quotient C_A/T_A or C_B/T_B is lower, T_A and T_B being the respective time periods of use (durability) of the two products.

In a particular case, when the use of a product involves energy consumption, the role of the value of T can be played by the period of use that we can obtain for the two products with the same cost of purchased energy. Thus, for instance, if we were considering a passenger car, the values of T_A and T_B could be the numbers of hours of driving on fuel purchased for the same amount of money.

It turns out, however, that we sometimes act otherwise. Namely, we frequently give in to the opinion of our societal environment (or the environment we would like to belong to) and purchase not the product that fulfils the here outlined criterion of an own interest, but the one that it is well seen to (that one should) have. In this context it is common that company CEOs or owners consider it appropriate to own, or drive, a luxury car, and at that – of a definite brand or brands. This is perceived as necessary from the point of view of prestige of the company and the position. At the same time, in terms of functionality, a much cheaper car would quite suffice. By driving a more expensive car, despite the obvious inconsistency with the simple economic rationality, we actually may economically ultimately gain by maintaining the intangible prestige (which may, e.g., help in acquiring better credit conditions in a bank).

In particular, stupefying successes are sometimes achieved in exploiting these intangible advantages by the various kinds of tricksters.

Another example of rejecting the criterion of own interest is provided by the influence of fashion, so variable along time. A clear influence of fashion on decisions, concerning purchasing of products, can be observed on the example of women's garments. And so, for instance, a piece of clothes that is more expensive and of lower quality shall be sought and bought only because it is of the currently fashionable colour.

Thus, while the opinion of the environment is relatively constant over time, or changes quite slowly in its general features, fashions change much more frequently (annually or seasonally). Besides, while the opinion of the environment concerns usually price characteristics of products, fashion refers to features inessential from the point of view of product use (like colour). On the other hand, both kinds of influence motivate to purchase products that are much more expensive than it would result simply from the production cost (which is especially true for the impact of fashion).

Let us try to identify the mechanism of influence of the non-economic factors on the magnitude of demand, Λ. We shall do this on the example of two products, A and B, satisfying the very same need of the customers, but sold for distinctly different prices, $C_A > C_B$. Let us take as example the sports footwear produced by a known brand A (Adidas, Nike, Puma, …) and a much less known company, B.

Assume, further, that the difference of prices may be, after all, justified by the difference in durability of the footwear, T_A and T_B. It may occur that the values $d^A{}_{lim} = C_A/(\gamma T_A)$ and $d^B{}_{lim} = C_B/(\gamma T_B)$, which define the lower limits of the incomes of potential customers, when they are still capable of affording the footwear of, respectively, company *A* and *B*, become (approximately) equal, so that $d^{A}_{lim} = d^{B}_{lim} = d_{lim}$.

The two products, even though physically different, but satisfying the same kind of need, become competitive on a given market and consequently might be considered "equivalent" or even "identical" from the point of view of competition.

It may, otherwise, happen that the durability of the two products is the same, i.e. $T_A = T_B = T$. We then deduce $d^{A}_{lim} > d^{B}_{lim}$. In this situation these two are different products from the point of view of competition, even though they still satisfy the same need of the potential customers. Naturally, one of them – the more expensive one – should "normally" be wiped out of the market. This, however, shall not (necessarily) happen, when, for instance, an essential role is played by the opinion of the snobbish community of the wealthier customers, who prefer to purchase the more expensive product.

It may namely be that a customer, using the more expensive product (even though it is less advantageous from the point of view of own economic interest), considers him/herself as somebody "better" than those, who use the cheaper product. This might even take on the aspect of contempt for the users of the cheap stuff.

In such a situation, a part of demand for the footwear shall be directed towards the more expensive products *A*, excepting, of course, the part of demand, which originates from the customers, who cannot afford buying for the price C_A, i.e. their income *d* is such that $d < d^{A}_{lim} = C_A/(\gamma T)$.

Hence, the initially considered group of potential purchasers of sports footwear shall get split into, first, the sub-group that satisfies the income condition $d > d^{A}_{lim} = C_A/(\gamma T)$, and, second, those, whose income *d* fulfils the condition $d^{A}_{lim} > d > d^{B}_{lim}$.

So, we deal here with a new phenomenon. Namely, demand for product *B*, better in purely economic terms, shall constitute only a part of the total demand for the sports footwear, and if we admit that this part shall originate from the customers, whose incomes are determined by the latter double inequality, then this part shall be proportional to the difference $q(d^B{}_{lim}) - q(d^A{}_{lim})$, and shall be expressed for the simplest, linear model of demand as

$$\Lambda_B = \lambda_0 \cdot \left[q(d_{gr}^B) - q(d_{gr}^A) \right] \cdot L_{mx},$$

where λ_0 is, in this case, the average number of pairs of sports footwear, purchased by a customer during unit time period (a year), usually the inverse of the time period of use (durability), T, and L_{mx} is the total number of customers, purchasing sports footwear on the given market.

If we admit the linear model with constant income density, i.e. $q(d) = 1 - d/d_{mx}$, then we get as demand for product *A*:

$$\Lambda_B = \lambda_0 L_{mx} \left[(1 - d_{\lim}^B / d_{mx}) - (1 - d_{\lim}^A / d_{mx}) \right] = \lambda_0 L_{mx} \left[(C_A - C_B) / \gamma T \right] / d_{mx}$$

$$= \lambda_{mx} (d_{\lim}^B - d_{\lim}^A) / d_{mx} = \lambda_{mx} (C_A - C_B) / C_{mx} = a(C_A - C_B),$$

where $a = \lambda_{mx}/C_{mx}$ and $d_{mx} = C_{mx}/\gamma T$, with the prices fulfilling the inequalities $0 < C_B < C_A < C_{mx}$.

Likewise, demand for product *B* shall be expressed as

$$\Lambda_A = \lambda_0 L_{mx} q(d^A{}_{lim}) = \lambda_0 L_{mx}(1 - d^A{}_{lim}/d_{mx}) = \lambda_{mx}(1 - C_A/C_{mx}) = a(C_{mx} - C_A)$$

while the total demand for sports footwear shall be equal

$$\Lambda_A + \Lambda_B = \lambda_{mx} q(d^A{}_{lim}) + \lambda_{mx}[q(d^B{}_{lim}) - q(d^A{}_{lim})] = \lambda_{mx}(1 - C_B/C_{mx}).$$

Let us note, at this point, that the non-rational decision of the purchaser (buying product *A*) from the point of view of own economic interest, could also be justified by the fact that purchasing of this product gives rise to an additional benefit M, supposedly decreasing the cost of purchasing down to C_A-M, so that the inequality $(C_A\text{-}M)/T_A > C_B/T_B$ is satisfied.

Alas, we do not know how to determine the value of M, and even – how to precisely define the respective quantity.

The situation on the market may, however, get even more complicated, if we account, additionally, for the influence of fashion. This may concern the influence of current fashion on the colours of sports footwear, or some details of outlook, little important from the point of view of functionality of the shoes.

If the fashion regarding such small details changes from year to year, then the period of use, now T', shrinks to just one year. Consequently, values $d^B_{lim} = C_B/(\gamma T')$ and $d^A_{lim} = C_A/(\gamma T')$ shall dramatically increase (as we assume that the period of use, or durability, for this kind of good is much longer than one year), and so the values of Λ_A and Λ_B shall also drop. Ultimately, the group motivated by "prestige" shall get further split into two sub-groups: those, who shall buy every year the (newly designed) more expensive footwear for the price C_A, and the ones, who will not afford such annual change of footwear and will buy the more expensive product, but use for more than a year. The income limits, defining the resulting three groups are expressed by the values

$$d^B_{lim} = C_B/(\gamma T_B),\ d^A_{lim} = C_A/(\gamma T_A), \text{ and } d^{A'}_{lim} = C_A/(\gamma T'_A),$$

provided, of course, that appropriate sequence of these values is preserved.

It may also happen that the group of buyers, who purchase the cheaper footwear for the price C_B, shall get split into two subgroups, defined by the following income limits

$$d^B_{lim} = C_B/(\gamma T_B),\ d^{B'}_{lim} = C_B/(\gamma T'_B), \text{ and } d^A_{lim} = C_A/(\gamma T_A).$$

It can be easily noted that, depending upon the relations of intervening parameter values, *C* and *T*, (some of) the groups, characterised by the above limits, may actually coincide.

A Reader has certainly noticed by now that, altogether, determination of demand for the "prestige" goods, which can, additionally, be subject to fashion changes, is a very complex issue.

In the second part of the book we shall limit the development of the theory of competition to the case, when demand for products depends uniquely upon the economic interest of the purchaser (the case of "homo oeconomicus"). It can be hoped that this kind of attitude shall dominate even more than today, as people become aware that development of fashion largely serves the interest of the producers, trying to incite an artificial demand for their products and to increase it. This, again, can be best observed on the instance of the great "Fashion Houses", dictating fashion changes in cooperation (or collusion) with garment producers.

2. Cost and profitability of production activity

Cost of manufacturing a product

Every modern technological installation for producing anything requires a definite period of time and definite investment outlays, I, in order to start functioning. These outlays are covered, as a rule, from a bank credit, which must be repaid in a definite time period T_0. This repayment, including interest and service fees, weighs on the future proceeds from sales by the "instalment" value equal I/T_0.

Besides, after construction has been finished, additional time and outlays (U) are needed for establishing the appropriate stock of materials even before production starts (these outlays are recuperated when production activity of a given kind or facility is terminated). Hence, we deal with a significant delay of appearance of the ready product with respect to the instant, when costs started to be borne, this delay resulting from summation of durations of all the necessary preparatory activities. This delay may in fact be quite significant, e.g. in agriculture – not less than one year, in shipbuilding – several years, and in forestry – even a couple of decades! The outlays U are usually financed by means of a recurring, or turnover credit, with a predefined interest rate ρ_0. This cost, therefore, weighs on future proceeds from sales with the (unit) value of $\rho_0 U$.

After production has started, technological devices, buildings, and the entire value of fixed assets is subject to wear and tear, requiring appropriate repairs, inspections etc., which entail respective costs. In addition, all the technological devices, buildings and the like have definite periods of durability, after which they must be replaced by the new ones, irrespective of the repairs. Thus, the entire productive capital has a definite (average) durability, T, which can be determined by analysing the periods of durability of the particular elements of the fixed capital. If we denote by B the costs of repairs, inspections etc. of all the elements of the fixed productive capital in the period T, then the quotient $(I+B)/T$ (often referred to as the generalised amortisation cost) shall have to be deducted from the future revenue from product sale.

The installed and functioning technology may be made use on production process with varying intensity μ (number or volume of product per unit time), not bigger, though, than the one defined in the design, μ_{mx}. Depending upon the intensity of production activity the demand shall change for materials and subassemblies, energy and fuels, as well as human labour, usually in proportion to the intensity μ. The coefficients of proportionality, η, are constituted by the respective norms and rates of use and consumption, and effectiveness. This makes possible calculation of the “direct cost” of producing a single (unit) product, b, and the resulting “variable cost” of production, $b\mu$.

Let us analyse in greater detail the cost of producing a single item (unit volume) of a product.

Producing a complex good requires performing a sequence of definite operations on consecutive stages of the production process. Each operation is carried out at some work station, equipped with appropriate technical means (machines, devices, tools), used by an operator or (more and more frequently) a control automaton.

The technical means, with which a work station is equipped, depend upon the adopted way of carrying out the operations. There is usually a choice of possible technical realisations of individual operations, and so also of the equipment used.

The technical means, taken together with the method used to carry out a given operation, shall be referred to as technology of realisation of a given operation, whether of productive or service character.

Designing a working station to carry out a given operation requires selection of an adequate method and technical means, produced by concrete companies. This, in turn, makes it necessary to have a possibly objective assessment of the various technologies of realisation of a given operation, so that the production line designed could have the best qualities for the producer.

In conditions of a relatively free market, the most important criterion of evaluation of the production technologies is unit cost of production. The lower this cost, for a predefined level of product quality obtained, the cheaper our product can be, and the more effective our struggle on the market, i.e. the bigger our chances to drive out the competitors.

Before we start analysing the way to determine the unit cost of production, we shall devote some attention to two basic notions, characterising production technology, understood as an ordered set of technologies of realisation of individual operations, resulting in turning out the product in question. These two notions are:

- the normative (designed, maximum) effectiveness (output capacity) μ_{mx}, and
- the time of producing a given product, τ, i.e. time from the start of the execution of the first operation until the end of the last one within a production technology.

Let us note at this point that it is generally not true that the value of τ is the inverse of μ, namely, it is not necessarily so that $\tau = 1/\mu$. This is, namely, true only for the most primitive of production technologies, e.g., when a single person performs consecutively all the production operations, without a break, during one shift.

We shall now pass on to determination of costs of turning out a product, resulting from the choice of a production technology, not accounting for the costs of maintenance of administration, the board etc. In general terms, the (operational) costs of production (of performing a sequence of production operations) are composed of the following elements:

1. the cost of materials and energy consumed, necessary for manufacturing a unit of product, defined by the formula $b_0 = \Sigma_i \eta_i C_i$, where, as indicated before, η_i are the norms of use (intensities of consumption), determined for a given technology, of the material or means i, per unit of product considered, and C_i are the prices of materials and means used, also per unit of product; b_0 is often called direct unit cost of production, and μb_0 is referred to as variable production cost;
2. the cost of capital "frozen" in the production (and sales) stock, often called cost of turnover credit; the value of such credit that has to be drawn from a bank is determined by the expression $\tau\mu b_0$, with the value of μb_0 determining the financial stream, necessary for the continued realisation of the production process; if we denote by ρ the interest rate

on the turnover credit, then the expression $\rho\tau\mu b_0$ defines the cost of this credit, which is equivalent to the cost of financing the stock "in production"; let us emphasise that a production plant requires for continued activity relatively quite significant credit line, since this manner of financing must cover all kinds of stock – materials, subassemblies, as well as ready products; these costs do largely not depend upon the purely technical matters, but also on the general organisation of the plant, as well as on the negotiated terms of credit repayment and the dates of payment for the materials etc. purchased for production, not to mention the terms of payment for the products that the company sells;

3. the cost of "frozen" investment capital I, necessary for creation of productive apparatus, including construction of appropriate premises, purchasing and installation of machines and equipment, etc., as well as the cost of servicing the repayment of credit, taken for these purposes; for simplicity, let us assume that the value of credit servicing rate is constant, equal $\rho_0 I$, where ρ_0 is the interest rate on investment credit, with, usually, $\rho > \rho_0$; similarly, we shall assume that the credit repayment instalments are also constant and equal I/T_0, where T_0 is, as before the period of repayment of the investment credit; consequently, the cost of investment shall be equal $\rho_0 I + I/T_0$ over the period of T_0;

   let us indicate, though, that the above expression might be interpreted otherwise; namely, when a company disposes of own funds for the development of the production infrastructure, there is no need to recur to credit taking at the bank and the investment of the value of I is done from own means; in such a case the magnitude ρI can be interpreted as the cost resulting from the forgone profits that could be gained if the investment were kept in the bank as a deposit; the second component of the investment cost would also have an analogous form as before, with the difference that the denominator would be the value of T, interpreted as durability of the production infrastructure, and the value of the quotient is known as amortisation cost or rate; hence, the overall structure of the formula for investment cost would not change; in addition, if it so happened that $T = T_0$, then investment cost in both cases would be the same, the difference resulting from the difference of interest rates on credits and deposits;

4. the cost of running the productive infrastructure, B, which includes total costs of repairs, modernisations, conservation etc. of the fixed assets over their entire period of exploitation T, which must be carried out in view of the wear and tear processes; all the components of the infrastructure have definite time periods of durability, after which they are scrapped, as no fulfilling any more the technical, economic or safety conditions; more generally, the entire invested productive capital (fixed assets) has a definite period of durability, T, which can be determined through analysis of individual elements of the productive assets; in addition, running cost must encompass the costs of lighting, air conditioning, as well as some kinds of taxes, associated with productive infrastructure (like those related to the volume of buildings or the area covered by them), insurance and protection costs, etc.; altogether, running costs can be defined as B/T; regarding the value of B, it is common to assume it as a proportion of the total investment cost, I, with the coefficient of proportionality η_0, i.e. in the form
$$B = \eta_0 I.$$

All of the last three kinds of costs are called constant costs, since, in distinction from the variable cost, they do not depend upon the intensity of production, μ.

Summing up, production costs, in monetary units per time unit, shall be expressed as

$$I \cdot \left(\rho_0 + \frac{1}{T_0} + \frac{\eta_0}{T} \right) + b_0 \cdot (1 + \rho \cdot \tau) \cdot \mu$$

while the cost of producing a unit (a single item) of the product shall be equal

$$\kappa = \frac{I \cdot \left(\rho_0 + \frac{1}{T_0} + \frac{\eta_0}{T} \right)}{\mu} + b_0 \cdot (1 + \rho \cdot \tau).$$

The above expression can be useful in terms of assessment of utility of a given technology both for the technology designer and for the investor – the future user of the technology.

Let us note that the above measure of utility of production technology depends only partly on the technical characteristics of the technology in question, such as the designed capacity μ_{mx}, τ, η_I and T. It, namely, depends also on the financial conditions (ρ, ρ_0, T_0), as well as the economic ones: prices of production materials (C_i), investment costs (I), running costs (η_0). That is why different technologies may turn out best under various economic and financial conditions, i.e. entail the lowest cost of production. Whenever we mention in this book, further on, the comparable (or the same) conditions of market competition, this will mean that the competitors have access to the same choice of production technologies and act in the same economic and financial circumstances.

It is perhaps worth mentioning, for clarity, that if a typical technology offers too low capacity μ_{mx}, then we can always install two identical production lines, and thus ensure double capacity. This, in general, though, ought not change the value of κ, the criterion function for the assessment of production technology.

So, if we want to achieve a higher capacity, we should make use of another technological design, the one that is characterised by a lower value of κ. New designs appear incessantly on the market, as research and development push technological knowledge forward.

If we introduce notation $Q = I \cdot \left(\rho_0 + \frac{1}{T_0} + \frac{\eta_0}{T} \right)$ and remark that usually the value of the expression $\rho_0 \tau$ is much below 1, then we can represent the unit cost κ of turning out a product with the formula having the structure of

$\kappa = b + Q/\mu$, where $b = b_0(1+\rho\tau)$.

At the same time, as one learns from the history of industrial activity, technological progress leads to continuing decrease of the value of b, and the steady increase (alas) of the value of Q, as we try to increase μ, but with always decreasing κ. In economics, the fact that κ decreases, while μ is increased, is referred to as the "scale effect" or the "economies of scale". If we may admit a reasonable assumption that the value of b remains (approximately) constant over some period of time, and does not depend upon μ, the same cannot be said of $Q(\mu)$, which is an obvious function of μ, increasing as μ increases. Note that if Q were increasing proportionally to μ, then production costs would be constant (given our assumption of constant b. Hence, the existence of the "economies of scale" is associated with the behaviour of the quotient $Q(\mu)/\mu$ over a relatively large interval of values of μ.

In the majority of introductory texts to similar domains of economics it is assumed that the unit costs of production is constant and does not depend upon the scale of production (intensity). Assume this is true.

Then, assume we dispose of a technological production setting composed of machines, tools, buildings and premises, such that if we employ people, we could produce a definite good with intensity μ_0. Let us also assume that this intensity is relatively low. We shall refer to this setting as to the technological line.

Suppose that maintenance of this technological line costs us Q_0 per unit time, with the direct cost of producing the good equal k_0. Hence, the cost of producing this good shall be equal k_0+Q_0/μ_0. At this point we would take the handbook assumption mentioned, namely that this cost of production is constant and does not depend upon the intensity of production μ.

For a definite level of development of technology we can agree that the value of k_0 does not depend on μ and is defined by the established technical standards and norms of effectiveness and efficiency. In accordance with previous considerations, the constancy of production costs is equivalent to assuming that $Q(\mu)/\mu$ is constant, in particular – irrespective of the value of μ. In our case this constant value shall result from the quotient Q_0/μ_0.

Let us note that if in fact $Q(\mu)/\mu = Q_0/\mu_0$ = const., then for all values $\mu = n\mu_0$, where n is a natural number, we could establish a production setting consisting of n parallel technological lines, each with intensity μ_0. The cost of maintaining these lines would be equal nQ_0.

Consequently, for each n (and so also $\mu = n\mu_0$) the cost of turning out the product would be equal

$$k_0 + \frac{Q(\mu)}{\mu} = k_0 + \frac{Q(n \cdot \mu_0)}{n \cdot \mu_0} = k_0 + \frac{n \cdot Q_0}{n \cdot \mu_0} = k_0 + \frac{Q_0}{\mu_0} = const.$$

In such a situation the cost borne for producing a good would be independent of the scale of production, μ, and so no "economies of scale" would exist. In close association with this, there would be no process of concentration of production – to the contrary, by distributing production facilities in space we would be able to economise on costs of transport to the customers.

At the same time, we actually observe, at least to an extent, that the process of concentration exists, as well as do the "economies of scale", even though not without limitations. Hence, the assumption made in the reasoning here presented was incorrect.

Further, it can be also easily noted that it would be even more unreasonable to assume that the quotient $Q(\mu_0)/\mu$ is an increasing function of μ. This would, namely, mean that only the most primitive production lines, characterised by the unit value μ_0, would be most economic, so that no concentration of production would be reasonable, and any price competition would become insignificant, given the assumption we made at the outset that all the market players have equal access to the same set of production technologies.

Hence, in order to stay in agreement with the observed general principles, applying – though, as mentioned, within definite limits – throughout modern civilisation, we shall assume that the quotient $Q(\mu)/\mu$ is a decreasing function of μ. Thus, we admit that the "economies of scale" or the "scale effect" exist in reality, that concentration – with all pertinent reservations – is economically justified, and that price competition is important for the development and decline of companies (along with the quality and technology competition).

Let us note that the limits to the decreasing character of the quotient $Q(\mu)/\mu$ result from a variety of factors. One of them is associated with environmental considerations, and the notion of "carrying capacity" (how much of a definite activity can be carried out on a definite area without causing irreversible negative effects).

In the further course of this book we shall often return to the problem of finding the optimum production intensity (scale), μ^*, that shall be equal respective demand, using the substitution $Q(\mu)/\mu = Q/\mu$. The respective calculations might start from introduction of the value of Q based on a "catalogue" of the available production technologies, featuring the capacities μ that are possibly close to the expected value μ^*; then, after having calculated μ^*, we ought to check, whether this calculated value is covered by the range of reasonable values of μ, selected from the "catalogue" of the production technologies with constant costs Q. If the technology selected does not guarantee the rational production with intensity μ^*, then we have to repeat the calculations, choosing another technology. Hence, use of the substitution mentioned is advised not only in the interval of the possible changes of the value of μ for a given production line.

Let us also note that the value of the quotient $Q(\mu)/\mu$ depends upon time, as well. In this context, the inequality

$$\frac{Q(\mu,t)}{\mu} > \frac{Q(\mu,t+\Delta t)}{\mu}$$

takes place, due to the incessant technological progress, associated with the appearing inventions in the domain of production technology. It must be emphasised that if this inequality were not taking place, the advance of civilisation would not be possible.

In what follows we shall be considering the manufacturing activity, in which economies of scale exist. This effect, in quantitative terms referring to the unit cost of production, is expressed through the following expression for κ:

$$\kappa = b + \frac{Q}{\mu}.$$

The issue of the "economies of scale" (in production, as well as in service) is closely associated with the diagram of dependence of production costs upon the scale of production, known in economics, shown here in Fig. 1.12. This figure shows the decreasing unit cost of production, for a definite technology of production, with $Q(\mu_{mx}) = Q$, for $0<\mu<\mu_{mx}$.[3]

[3] More detailed analysis of the ways to determine (find) the best (optimum, dominant – in the case of multicriteria choices) technologies can be found in Piasecki (1968, 1986, 2000a).

In particular cases, when no tools are needed to turn out a product (or their cost is so low as to be neglected), then we can assume $Q = 0$. Then, cost of production, $\kappa = b$, does not depend upon μ. This applies, in particular, to the hand-made products, which are made to order.

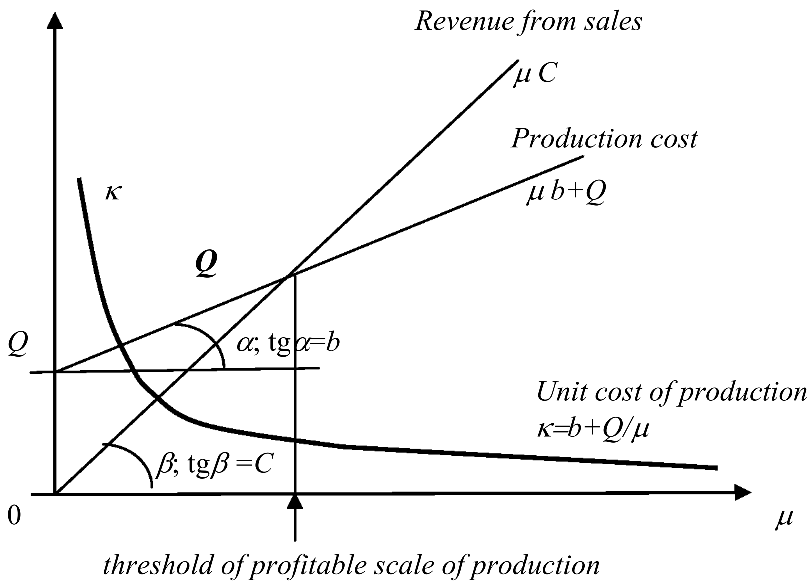

Figure 1.12. Schematic representation of production costs as a function of production scale (intensity)

There are also quite different situations, like in, for instance, service, where constant cost, Q, dominates, while the value of b can be neglected in comparison with the ratio Q/μ within quite a broad interval of values of μ, like in the systems of automatic information and responding (e.g. time table information services), or in the human-operated responding and information services, when the staff is paid flat monthly salaries.

It is quite rare, but still imaginable, to encounter a situation, when the unit cost increases with production intensity μ. Let us, namely, imagine a known artist, who needs six hours to elaborate a "product", and gets paid for it a certain sum. If we wanted this person to produce two items a day, it is quite conceivable that the sum asked for each of them could go up ("unit effort" having gone up, along with the expected profit of the intermediary). If so, the cost would go even higher when three items a day were to be produced. Although this is in a way an academic situation, it actually occurs when we recur to (and pay for) overtime to get the required production.

Yet, the issue of establishing production cost gets much more complex, when we produce many different products.

Let us assume, for simplicity, that a company manufactures two kinds of products. Assume also that the constant costs of maintaining the technological lines for manufacturing both products are equal D (per time unit). These costs must be recovered from the proceeds from sales of the two products. When establishing the sales prices of these products we must know the cost of their production. It is common to determine these costs in the following manner:

$$\kappa_1 = Q_1/\mu_1 + b_1; \quad \kappa_2 = Q_2/\mu_2 + b_2; \quad \text{where } Q_1 + Q_2 = D.$$

If some general (bookkeeping) rules do not determine unambiguously the way of splitting D into Q_1 and Q_2, then we could, at least in theory, assume an arbitrary division of D among two products.

This leads to a possibility of an ("artificial") maximum lowering of the price of one of the products (e.g. no. 1). This, however, would have to entail simultaneous increase of the price of the second product, and hence to the drop of sales of the latter.

We should explain at this point, what is the usual manner of dividing the constant cost of maintaining the technological lines of an enterprise among products manufactured by this enterprise. If we now introduce notation d_1 for the cost of direct labour used (a part of b_1) and, analogously, d_2, for, respectively, products no. 1 and 2, then the usually adopted division of costs is given by

$$Q_1 = D\frac{d_1}{d_1 + d_2}\text{; and } Q_2 = D\frac{d_2}{d_1 + d_2},$$

and then the cost of producing the two kinds of products shall be equal

$$\kappa_1 = \frac{d_1}{d_1 + d_2}\frac{D}{\mu_1} + b_1\text{, and } \kappa_2 = \frac{d_2}{d_1 + d_2}\frac{D}{\mu_2} + b_2.$$

This principle of establishing the split of constant costs results from the fact that the value added tax (VAT) is being calculated in a similar manner. Sometimes, in place of the values of d_1 and d_2 the values of b_1 and b_2 are directly used. This may occur when, for instance, costs of energy consumed for production are significant.

There are, of course, cases, very rare, though, when the production lines of different products are entirely separate and then the values of Q_i for these products, I = 1, 2, 3,…, can be established unambiguously.

An example of the difficulties mentioned can be provided by a "bakery", producing various kinds of bakery products. In stable economic conditions one can determine statistically the structure (proportions) of market demand for these various kinds of products. Consequently, we can define a single (composite) product: "bread".

Yet, in order to determine the cost of producing a unit of "bread", we must know the costs of production of all the actual bakery products, composing it.

The direct costs of turning out individual products can be uniquely determined given knowledge of specific parameters of intensity of use of flour, sugar, yeasts, etc., and of their prices.

It is much more difficult to determine the values of Q_i for individual products *I* in the situation, when we only know the total sum of these costs, *Q*, encompassing costs of using ovens, mixers and other equipment, including buildings and their amortisation, as well as wage or, more generally, labour remuneration fund, transport of "bread" to the retailers, and so on.

Some of these do not have to be split, of course, among the individual products, if we just want to establish the cost of "bread", but if we want to have a more general procedure for establishing this cost, then such splitting may turn out indispensable. In what follows, we shall formulate such a procedure of establishing the cost of production of a composite product.

On an extension of the notion of "product"

The usually adopted notion of product does not refer, naturally, to a single item produced, but is associated with a definite, distinct type of good. This "type" is identified through a characteristic code, defined by the producer, visible on the product, and, besides, each item may, in fact, bear an individual serial number.

In our considerations, concerning competition, we shall need a somewhat different comprehension of the notion of "product". Namely, two (or more) goods, manufactured by different companies, shall be considered *competitively equivalent* (*homogeneous*), if they satisfy the same kind of need of the purchaser (their function is the same) and do not differ as to production costs (under the same technology) nor transport costs. From the point of view of the processes of competition, products can be considered on a market as homogeneous even though they might differ as to their aspect and be marked by different codes by their (different) producers.

The notion of a product can further be extended by encompassing with this notion also the *composite products*. This extension – partly already illustrated by the previous example – shall be explained on another instance.

Assume that a producer is supplying customers in the neighbourhood with various kinds of products. Let these be four kinds of products, indexed j, $j = A, B, C$ and D. The intensity of the need for the product, generated by the i^{th} customer in a certain period of time, shall be denoted λ_{ij}, $i = 1, 2, 3, \ldots, j = A, B, C, D$.

For pragmatic calculation purposes, we can introduce the coefficients of the structure of needs, $\gamma_{ij} = \lambda_{ij}/\lambda_{iA}$ (i.e. $\gamma_{iA} = 1$). This notation can be extended over the relations for the entire population of customers considered, $\lambda_j = \Sigma_i\lambda_{ij}$ and $\gamma_j = \lambda_j/\lambda_A$. Therefrom, of course, $\lambda_j = \gamma_j\lambda_A$.

Now, every kind of product that customers buy, is characterised by the unit price C_j and the loading (or storage) volume δ_j (δ_{ij} for particular customers). Further, we can define the values of $C^h{}_j = \rho C_j$, where ρ is the interest on the short-term (renewable, turnover) credit. If an i^{th} customer is supplied with goods of the volume of $\boldsymbol{G}_i = \{G_{ij}\}$, then the necessary capacity of the transport means shall be equal to $\Sigma_j G_{ij}\delta_j$.

If the structure of the transported "package" is established, then we have

$$\Sigma_j G_{ij}\delta_{ij} = \Sigma_j G_{iA}\gamma_{ij}\delta_j = G_{iA}\Sigma_j\gamma_{ij}\delta_j = G_{iA}\delta^i.$$

The above introduced quantity δ^I is the loading (stock) volume of the unit of a specific kind of good A_i. This unit, proper for the customer i, is, in fact, a composition of four kinds of products, consisting of one nominal unit of product A ($\lambda_{iA} = 1$); $\lambda_{iA}\gamma_{iB}$ of nominal units of product B; $\lambda_{iA}\gamma_{iC}$ of nominal units of product C and $\lambda_{iA}\gamma_{iD}$ of nominal units of product D. This particular "mix" of products, constituting the composite product A_i, differs from the "mixes", proper for other customers, $A_{i'}$, $i' \neq i$, corresponding to other composite "products".

Unit price of "product" A_i equals

$$C^i = 1 \cdot C_A + \gamma_{iB}C_B + \gamma_{iC}C_C + \gamma_{iD}C_D$$

while the intensity of need equals $\lambda^i = \lambda_{iA}$, and $C^h{}_i = \rho C^i$.

Further on, we shall use for our purposes the intensity of needs (potential purchases) generated by a "statistical customer", λ_A, defined as

$$\lambda_A = \frac{1}{L}\sum_{i=1}^{L}\lambda_{iA}\ ,$$

where L is the number of customers within a given area.

Of course, the composite "product", denoted A_i, can now be treated like any other product, with respective parameters and characteristics, including cost, price and transport volume.

A good example of the composite product is provided by fuel for car combustion engines. Different fuels are needed for various vehicles. Given the differentiated "population" of vehicles on the road within a given area, stable, statistically determined demand for individual kinds of fuel are established for the thus defined market. This, in turn, makes it possible to define a composite product "fuel", produced by the refinery. This "product" is being distributed over the market through the network of pumping stations.

If the network of stations belongs to the same company as the refinery, then we deal with one company, producing and selling fuel. On a higher level, we could treat all the refineries and distribution networks as one enterprise, equivalent to the fuel branch.

In the following parts of the book considerations concerning competition shall refer to a definite product and a company, producing it, all the involved quantities having respective interpretation.

On the profitability of economic (business) activity

It is quite common to conduct business on the basis of current (turnover) and investment credits. If so, profitability of production is the fundamental criterion of granting the credits by the bank.

In order to avoid misunderstandings, we shall define the notion of profitability, called also "return" in definite contexts. Thus, we shall measure profitability of an economic activity with the ratio ε of profit (over a certain period of time), Z, to cost K of the activity in the same period. Since profit is the resultant of revenues, P, and the same costs, K, we get

$$\varepsilon = Z/K = (P-K)/K = P/K-1 = \frac{\Lambda C}{\mu \cdot \left(\frac{Q}{\mu} + b\right)} - 1 = \frac{C}{\frac{Q}{\mu} + b} - 1 \text{ for } \Lambda = \mu.$$

If the activity is to be profitable (bring positive net returns), then value of ε must be positive, or the ratio of revenues (P) to costs (K) must exceed unity. The limit of profitability is defined by the equality of revenues from sales with the associated costs:

$$C \cdot \mu = Q + \mu \cdot b.$$

We can derive from this equality the limit (threshold) value of production intensity, μ_{thr}, for some definite price, or the threshold price, C_{thr}, for a definite production intensity, i.e.

$$\mu_{thr} = Q/(C-b) \text{ or } C_{thr} = b + Q/\mu.$$

At this point we must enter into details, concerning determination of the value of b in connection with the necessity of extending the nation of τ.

We shall now denote with the symbol τ the period of time that elapses since the instant of purchase of energy, materials and parts needed to manufacture the product, until the moment of sale of these "ingredients" in the form of the product and obtaining respective payment (and not, as before, until manufacturing is terminated). If we use symbol ρ to denote the interest rate on turnover credit, then we must increase the direct, material cost of production, b_0, by the cost of this credit, yielding $b = b_0+\rho\tau b_0 = b_0(1+\rho\tau)$.

Then, production cost K shall be given by the expression

$$K = Q + b_0(1+\rho\tau) = Q + b\mu.$$

This extension of the way of calculating some quantities was instrumental for attracting attention to the role of banks. Let us, namely, consider the costs of servicing two kinds of bank credits. The repayment of the first of them, the investment credit, entails a part of the constant cost, Q, while the second, turnover credit, contributes to the direct cost.

The repayment and servicing costs, related to credits, depend primarily upon the interest rate ρ, which is established by the banks on the basis of the referential interest rate, set by the respective national bank. In a vast proportion of cases the actual interest rates on credits granted the companies, whether for investment or current production purposes, are in fact negotiated. The outcome of such negotiations weighs heavily on the economic performance of the enterprise, by influencing the values of both b and Q. The interest rate on turnover credit may have decisive impact on conditions of short-term competitiveness of some enterprises, while the interest on investment credits may not only motivate or discourage companies with regard to investment making, but also exert influence on the value of the threshold of minimum profitable production intensity.

Let us add that the referential interest rate, set by the respective (central) national bank influences in quite a similar manner the dynamics of the entire national economy, and, in case of large open national economies, may also exert influence significantly exceeding the territory of the particular country in question.

3. Classification of adopted models of economic activity

This classification shall be based on the essential properties, defined by the forms of functions of demand and unit costs.

Thus, the following functions, describing demand, are considered in the present study:

$$I. \quad \Lambda = a \cdot (C_{mx} - C) = \lambda_{mx} \cdot \left(1 - \frac{C}{C_{mx}}\right)$$

$$II. \quad \Lambda = \lambda_{mx} \cdot \left(1 - \frac{C}{C_{mx}}\right)^2$$

$$III. \quad \Lambda = \lambda_{mx} \frac{1}{1 + \dfrac{C}{C_0}} = \lambda_{mx} \cdot \frac{C_0}{C_0 + C}$$

$$IV. \quad \Lambda = \lambda_{mx} \frac{1}{1 + \left(\dfrac{C}{C_0}\right)}$$

$$V. \quad \Lambda = \lambda_{mx} \cdot \frac{1 - (\dfrac{C}{C_{mx}})^2}{1 + (\dfrac{C}{C_0})^2}$$

$$VI. \quad \Lambda = \lambda_{mx} \cdot \left[1 - \left(3 - \frac{C}{C_{mx}}\right) \cdot \left(\frac{C}{C_{mx}}\right)^2\right]$$

Notwithstanding the above, the following models of unit costs are considered:

1. $\kappa = b$
2. $\kappa = \mu \cdot b$
3. $\kappa = \dfrac{Q}{\mu}$
4. $\kappa = \dfrac{Q}{\mu} + b$
5. $\kappa = \dfrac{Q}{\mu} + b + K_T(R)$

where $K_T(R)$ is the cost of transporting the product to the customers, who reside in the distance up to R from the sales facility.

Besides, individual models of profit, accounting for actual delivery of products, depend upon the distribution of customers over a given area, and may significantly differ, depending upon the form of the function of density of customer distribution.

There are three cases that we distinguish in this context, namely:

- pointwise distribution (distances and delivery costs are neglected),
- uniform distribution, with density g = const., over the entire area considered,
- cone-like distribution, with density decreasing as distance from the centre (of the circle) increases, i.e. $g(r)$ decreases as r, radius, being the distance from the centre, equivalent to the location of the sales facility, increases.

The here outlined division of the problem domain of market competition is certainly incomplete, and was introduced by the authors for the general orientation in this domain, since the plausible assumptions and the sensible questions are, indeed, so many, that a clear choice must be made.

This book presents a complete exposition of a coherent and far-reaching theory of market competition. It is based on simple precepts, does not require deep knowledge of either economics or mathematics, and is therefore aimed primarily at undergraduate students and all those trying to put in order their vision of how the essential market mechanisms might work. Volume II, now in preparation, shall bring the theory to further problems and results.

The logic of the presentation is straightforward; it associates the microeconomic elements to arrive at both more general conclusions and at concrete formulae defining the way the market mechanisms work under definite assumed conditions.

Some may consider this exposition too simplistic. In fact, it is deliberately kept very simple, for heuristic purposes, as well as in order to make the conclusions more clear. Adding a lot of details that make theory more realistic – these details, indeed, changing from country to country, and from sector to sector – is mainly left to the Reader, who is supposed to be able to design the more accurate image on the basis of the foundations, provided in the book.

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