Polska Akademia Nauk • Instytut Badań Systemowych

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Książka jubileuszowa
z okazji
70-lecia urodzin

PROFESORA KAZIMIERZA MAŃCZAKA

pod redakcją
Jakuba Gutenbauma

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ROOT LOCUS AND FREQUENCY DESIGN OF INDUSTRIAL PID CONTROLLERS - A TUTORIAL

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Abstract: *The tutorial deals with the problem of tuning industrial PID controller given overshoot and settling time. Plants typical for process control and DC servos are considered. Controller settings are computed by means of root locus and frequency methods. Analytical expressions are derived for some simple models employed by self-tuning controllers. A number of numerical examples for illustration of design cases is presented.*

Keywords: *PID controller, root-locus, frequency method, process control.*

1. Introduction

Working knowledge of industrial controller design, what effectively means quick calculation of PID settings given basic specifications, still seems rather insufficient both among teachers, students and engineers. Teaching programs focus essentially on control theory. Textbooks often hide practical aspects among various abstract issues of limited usefulness. Besides, in Poland and other East European countries familiarity with root locus method, which allows for quick design given plant transfer function, is also rather inadequate.

The purpose of this tutorial is to fill these gaps in part by presenting solutions to basic design problems involving industrial PID controller of IEC 1131 standard (Lewis 1995). Root locus and frequency methods are employed (e.g. Phillips and Harbor 1991, Franklin et al. 1995). We consider only typical process control plants and DC servos, i.e. the models with time constants, delays and integrators (e.g. Findeisen 1969, Unbehauen and Rao 1987). Two basic specifications, overshoot and settling time, are given. Selection of specific PID transfer function, with four or three parameters, depends on noise content in the process variable. Analytical solutions for a few simple models identified by some commercial self-tuning controllers are derived (Åström et al. 1993). Fifteen brief numerical examples illustrating various design cases are provided to support the tutorial.

2. Preliminaries

PID controller. The PID transfer function specified in the IEC 1131 standard (Lewis 1995) and implemented now in most of industrial controllers and larger PLCs has the form

$$\text{PID:}\quad k_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{\frac{T_d}{D}s+1}\right). \tag{1}$$

We assume that the settings T_i, T_d, D satisfy the following restriction

$$(T_i + T_d/D)^2 \geq 4T_iT_d(1+1/D)\,, \tag{2}$$

which assures that the PID has two real zeros. So one can write

$$\text{PID:}\qquad k\frac{s+z_1}{s}\cdot\frac{s+z_2}{s+p} \tag{3}$$

$$k = k_p(D+1),\ z_1+z_2 = \frac{T_i+T_d/D}{T_iT_d(1+1/D)},\ z_1z_2 = \frac{1}{T_iT_d(1+1/D)},\ p = \frac{D}{T_d}.$$

The PID of the type (3) has long been used in Honeywell controllers (Bibbero 1977). The case with double zero is of particular interest, so

$$\text{PID:}\qquad k\frac{(s+z)^2}{s(s+vz)},\qquad k = k_p(D+1) \tag{4}$$

$$z = \frac{1}{T_i}\frac{d+1/D}{2(1+1/D)},\ d = \frac{1}{D(2D+1-2\sqrt{D(D+1)})},\ v = \frac{2(D+1)}{1+1/(dD)},\ T_d = \frac{T_i}{d}.$$

For $D = 0.5, 1, 2, 5, 8$ we get $v = 2.3, 3.4, 5.4, 11.5, 17.5$, respectively ($D = 5, 8$ are default values in Siemens and Honeywell controllers). Small values of D are necessary when process variable is not filtered well enough.

If the process variable is noise-free, large D can be set and (1) reduces to the textbook form $k_p(1+1/(T_is)+T_ds)$. Assuming $T_d \leq T_i/4$ we can write

$$\text{PID:}\qquad k\frac{(s+z_1)(s+z_2)}{s} \tag{5}$$

$$k = k_p T_d, \qquad z_1 + z_2 = 1/T_d, \qquad z_1 z_2 = 1/(T_i T_d).$$

If $T_d = T_i/4$, as in the well-known Ziegler-Nichols tunings (e.g. Findeisen 1969, Åström et al. 1993), then

$$\text{PID:} \quad k\frac{(s+z)^2}{s}, \qquad k = k_p \frac{T_i}{4}, \qquad z = \frac{2}{T_i}. \tag{6}$$

In this tutorial, depending on design specifications and process noise, we consider each of the PID representations (3) to (6). Return to the original settings k_p, T_i, T_d, D is straightforward.

Typical industrial plants. The designs presented here apply only to the plants of common industrial practice. In the process control area the plants are usually modeled by

$$\frac{1}{Ts+1}e^{-\tau s}, \quad \frac{1}{(T_1 s+1)(T_2 s+1)}e^{-\tau s}, \quad \frac{1}{(Ts+1)^n}, \quad \frac{1}{s(Ts+1)}e^{-\tau s}, \quad e^{-\tau s} \tag{7}$$

(plant gain included into PID). Time constants and delays are identified from time or frequency responses. While writing a plant transfer function $G_p(s)$ or frequency characteristics $|G_p(j\omega)|$, $\angle G_p(j\omega)$ we implicitly assume, loosely speaking, that $G_p(s)$ is of the same class as the models (7). Oscillatory, unstable, non-minimum phase and other nonstandard models are not considered here.

In addition to (7) we also deal with two following transfer functions

$$\frac{1}{s^2}, \qquad \frac{1}{s(Ts+1)} \tag{8}$$

which represent a DC servomotor equipped with current or voltage driver, respectively.

Design problems. General problem involves tuning a feedback loop with PID controller and the plants as above to get transients roughly similar to those of standard 2nd order system

$$G_{II}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}. \tag{9}$$

The poles are $s_{1,2} = R \pm jI$, with $R = -\xi\omega_n$, $I = \omega_n\sqrt{1-\xi^2}$. Overshoot $p_{\%}$ and settling time t_s are given by

$$p_{\%} = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \cdot 100, \qquad t_s = \frac{4}{|R|} = \frac{4}{\xi\omega_n}. \tag{10}$$

For $\xi = 1, 1/\sqrt{2}, 0.5$ we have $p_{\%} = 0, 4.3, 16.3$, respectively. Specifications imposed on the loop may involve different number of data. Here we consider two basic problems, having given:

1) overshoot $p_{\%}$ only
2) overshoot $p_{\%}$ and settling time t_s.

If t_s is not specified, then $t_{s,PID} \overset{\Delta}{=} t_{s,P}$ is reasonable choice as far as industrial applications are concerned (the same settling time for PID as for P).

3. Root locus designs

The plant must be given in the form of a proper transfer function $G_p(s)$, so delay $e^{-\tau s}$ is replaced by Padé approximation. Parameters of G_p are identified from step responses by means of graphical methods or least-squares approximation (e.g. Mańczak 1971, Unbehauen and Rao 1987, Ljung 1987, Niederliński et al. 1998). We review six standard problems given type of the controller and specifications. Numerical examples are provided.

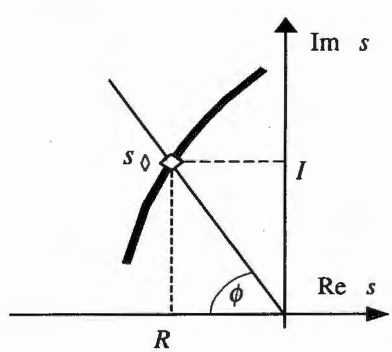

Fig.1. Illustration of root-locus design.

P $+ p_{\%}$. Open-loop transfer function is $G_{open}(s) = k\,G_p(s)$, so the root-locus is determined by the equation $1 + k\,G_p(s) = 0$. Let r be the table of closed-loop poles resulting from Matlab instructions

$$\begin{aligned} & k = kmin{:}dk{:}kmax \\ & r = rlocus(num, den, k) \end{aligned} \tag{11}$$

with numerator *num* and denominator *den* from G_p. In r we select a particular column $r(:,i)$ such that its plot is closest to imaginary axis (Fig.1). The angle

$$\phi = \mathrm{atan}\frac{\sqrt{1-\xi^2}}{\xi} \tag{12}$$

determines straight line corresponding to the overshoot $p_{\%}$. The crossing $s_{\Diamond}$ of this line with $r(:,i)$ is obtained for the gain k which yields overshoot similar to $p_{\%}$ (exactly $p_{\%}$ if G_p is of 2nd order as in (9)). $s_{\Diamond}$ and k are found using Matlab instructions

$$\begin{aligned} & Re = real(r(:,i)); \quad Im = imag(r(:,i)) \\ & [k, \quad r(:,i), \quad atan(Im./(-Re))*180/pi]. \end{aligned} \tag{13}$$

The *atan*() above represents the angle $180° - \angle G_p(s)$ for s along $r(:,i)$. In the three column table generated by (13) we choose a row whose third element equals ϕ. The first element in the row represents the value of k and the second the crossing $s_{\Diamond} = R + jI$. Settling time t_s is estimated as $4/|R|$ (see (10)).

Example. Given the plant $1/(s+1)^3$ and specification $p_{\%} = 16.3$ we get $\phi = 60°$, $k = 1$, $s_{\Diamond} = 0.5(-1 + j\sqrt{3})$, $t_s = 8$, $p_{\%,actual} = 13.9$ (from Matlab *step*() response). The actual overshoot characterizes standard feedback system, i.e. the one with controller and plant in the forward path and with unity feedback.

We repeat that in the problems considered here the closed-loop transfer function G_{closed} is different than the 2nd order one in (9), so one cannot expect $p_{\%,actual}$ to be exactly the same as the specified $p_{\%}$. Therefore while defining the general design objective in Sec.2 we have used the words "roughly similar". If one really needs $p_{\%,actual} = p_{\%}$, subsequent fine-tuning must be performed. A step-by-step algorithm for precise tuning of the PID controller is described in Świder and Trybus (1998).

PI + $p_{\%}$. PI transfer function $k_p(1+\frac{1}{T_i s})$ is written as $k\frac{s+z}{s}$ with $k = k_p$, $z = 1/T_i$. Let p_1 denote a stable pole of G_p which is closest to the origin (inverse of the largest time constant). Zero z is chosen by *pole-zero cancellation*, i.e.

$$z \stackrel{\Delta}{=} p_1 \tag{14}$$

Now $G_{open}(s) = k\,G'_p(s)$, where

$$G'_p(s) = \frac{1}{s}[(s+p_1)\cdot G_p(s)], \tag{15}$$

what means that the component $(s+p_1)$ in the plant denominator is replaced by s. For G'_p we determine k as above.

Ex. Data as before. Results: $z = p_1 = 1$, $k = 0.375$, $s_\Diamond = -0.25 + j0.433$, $p_{\%,actual} = 15.25\%$, $t_s = 16$ (twice as much as for P).

PD + $p_{\%}, t_s$. PD part of the PID in (1) is equivalent to lead compensator $k\frac{s+z}{s+p}$ since

$$\text{PD:}\quad k_p(1+\frac{T_d s}{\frac{T_d}{D}s+1}) = k\frac{s+z}{s+p} \tag{16a}$$

with

$$k = k_p(D+1), \qquad z = \frac{1}{T_d(1+1/D)}, \qquad p = \frac{D}{T_d}. \tag{16b}$$

Open-loop transfer function thus becomes

$$G_{open}(s) = k\frac{s+z}{s+p}\cdot G_p(s) = k\,G(s), \qquad G(s) = \frac{s+z}{s+p}\cdot G_p(s). \tag{16c}$$

The crossing $s_\Diamond = R + jI$ is specified by the data $p_{\%}, t_s$ (Sec.2). Since there are three unknowns, k, p, z, one of them must be assumed *a priori*.

Typically one chooses zero z as the inverse of *a quarter of settling time* (Phillips and Harbor 1991), so

$$z = \frac{4}{t_s}. \tag{17}$$

The root-locus equation $1 + k\,G(s) = 0$ is split into the angle and gain conditions

$$\angle G(s) = \pm 180°, \qquad k = -\frac{1}{G(s)}. \tag{18a,b}$$

Using $G(s) = \frac{s+z}{s+p} G_p(s)$ in (18a), taking $s = s_\Diamond$ and employing standard geometry in the complex plane we can find

$$p = z + I \cdot \tan\alpha, \tag{19}$$

where $\alpha = \pm 180° - \angle G_p(s_\Diamond)$ (α can be made acute by ±360°). Note that $\alpha \triangleq \angle PD(s_\Diamond)$, i.e. α is the angle of the transfer function (16a) computed at $s_\Diamond$. Having z and p one gets

$$k = -\frac{s+p}{s+z} \cdot \frac{1}{G_p(s)}\bigg|_{s_\Diamond} \tag{20}$$

from (18b) (Matlab may leave small imaginary part in k due to roundoff).

Ex. Data as before, $t_s = 6$ (less than for P). Results: $s_\Diamond = -0.67 + j1.15$, $z = 0.67$, $p = 1.69$, $k = 2.32$, $p_{\%,actual} = 14.6\%$.

<u>PID $k\frac{(s+z)^2}{s} + p_\%, t_s$</u>. We repeat that such PID can be used if the process variable is well filtered. For

$$G_{open}(s) = k\frac{(s+z)^2}{s} \cdot G_p(s) \tag{21}$$

the angle condition (18a) yields

$$z = |R| + I \cdot \mathrm{ctg}\,\alpha, \qquad \alpha = \pm 90° - \tfrac{1}{2}\angle \frac{G_p(s)}{s}\bigg|_{s_\Diamond}. \tag{22}$$

Therefore

$$k = -\frac{s}{(s+z)^2} \cdot \frac{1}{G_p(s)}\bigg|_{s_0} . \tag{23}$$

Ex. Data as before, $t_s = 6$. Results: $z = 0.853$, $k = 1.69$, $p_{\%,actual} = 14.5\%$ ($k_p = 2.89$, $T_i = 2.34$, $T_d = 0.58$).

PID $k\frac{s+z_1}{s} \cdot \frac{s+z_2}{s+p} + p_{\%}, t_s$. Now the process variable does not have to be so well filtered. However, the price of it is somewhat longer settling time t_s. As for PI, zero $z_1 = p_1$ cancels out the pole p_1, so

$$G_{open}(s) = k\frac{s+z_2}{s+p} \cdot G_p'(s) \tag{24}$$

with G_p' given in (15). Thus we have returned to lead compensator design, another words, to PD controller.

Ex. Data as before, $t_s = 8$ (as $t_{s,P}$ for P). Results: $z_1 = 1$, $z_2 = 0.5$, $p = 2$, $k = 2$, $p_{\%,actual} = 3.6\%$.

PID $k\frac{(s+z)^2}{s(s+\nu z)} + p_{\%}, t_s$, ν given. This represents the case when the divisor D in (1) is specified. The angle condition (18a) yields the equation

$$2\,\mathrm{atan}\frac{\tan\phi}{z'-1} - \mathrm{atan}\frac{\tan\phi}{\nu z'-1} + \angle G_p(s_0) + \phi = 0 \tag{25}$$

where $z' = z \cdot t_s/4$. (25) must be solved for z' using Matlab (or iteratively). Then

$$k = -\frac{s(s+\nu z)}{(s+z)^2} \cdot \frac{1}{G_p(s)}\bigg|_{s_0} . \tag{26}$$

Ex. Data as before ($t_s = 8$), $D = 1$ so $\nu = 3.4$. Results: $z = 0.805$, $k = 2.84$, $p_{\%,actual} = 11.85\%$.

In the last two cases we have taken $t_{s,PID} \overset{\Delta}{=} t_{s,P}$. As indicated in Sec.2, this is a reasonable choice when the PID control is required but settling time is not specified.

4. Some analytical solutions

Here we present solutions "by hand" to the plants (7), (8) typical for industrial practice. This requires restriction of Padé delay approximation to 1[st] order. Resulting expressions for PID settings can be used for automatic tuning. First two transfer functions of (7) have already been applied for this purpose (Åström et al. 1993).

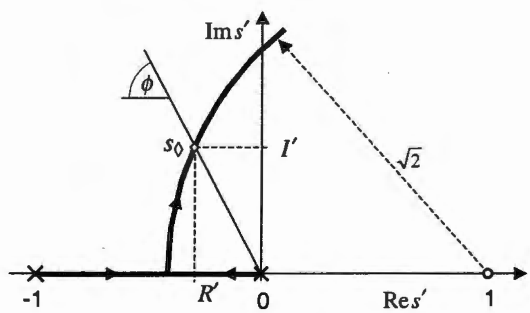

Fig.2. Root-locus plot for $\frac{1}{(T_1 s+1)(T_2 s+1)} e^{-\tau s}$ + PID $k \frac{(s+z_1)(s+z_2)}{s} + p_{\%}$.

<u>$\frac{1}{(T_1 s+1)(T_2 s+1)} e^{-\tau s}$ + PID $k \frac{(s+z_1)(s+z_2)}{s} + p_{\%}$</u>. Such model is identified by Honeywell UDC controllers during self-tuning. Pole-zero cancellation can be employed, so $z_1 = 1/T_1$, $z_2 = 1/T_2$. With 1[st] order Padé we get

$$G_{open}(s) = \frac{k}{T_1 T_2} \cdot \frac{e^{-\tau s}}{s} \cong k' \frac{-s'+1}{s'(s'+1)} \tag{27}$$

where $k' = k\tau/(2T_1T_2)$, $s' = s\tau/2$. Root-locus plot is a circle with the center at $(-1, j0)$ and the radius $\sqrt{2}$ (Fig.2). The crossing $s'_0 = R' + jI'$ results from solving

$$\begin{cases} (R'-1)^2 + I'^2 = 2 \\ I' = -\tan\phi \cdot R'. \end{cases} \tag{28}$$

Having $s'_\Diamond$ we calculate

$$k' = \left. -\frac{s'(s'+1)}{-s'+1} \right|_{s_\Diamond} . \tag{29}$$

Finally $k = 2k'T_1T_2/\tau$, $t_s = 2\tau/|R'|$.

Ex. Data: $T_1 = T_2 = \tau = 1$, $p_\% = 16.3\%$. Results: $z_1 = z_2 = 1$, $s'_\Diamond = -0.31 + j0.54$, $k' = 0.38$, $k = 0.76$, $t_s = 6.5$, $p_{\%,actual} = 26.3\%$ for 3rd order Padé delay approximation (1st for design, 3rd for simulation).

$\underline{\frac{1}{(Ts+1)^n} + \text{PID } k\frac{(s+z)^2}{s} + p_\%}$. Such model is identified by Siemens SIPART controllers (Linzenkirchner 1980). After cancellation by setting $z = 1/T$ we get

$$G_{open}(s) = \frac{k}{T^2} \cdot \frac{1}{s(Ts+1)^{n-2}} = k' \frac{1}{s'(s'+1)^{n-2}} \tag{30}$$

where $k' = k/T$, $s' = sT$. If $n = 2$, then $G_{closed}(s) = \frac{1}{sT^2/k+1}$. For $n \geq 3$ the breakpoint s'_b of the root-locus equals $-1/(n-1)$ and the corresponding gain is

$$k' = \frac{1}{n-2}\left(\frac{n-2}{n-1}\right)^{n-1} \qquad \text{for} \quad p_\% = 0. \tag{31}$$

Settling time can be estimated as

$$t_s = 4(n-1)T \tag{32}$$

(roughly). If $p_\% > 0$ is required then Matlab instructions (11), (13) must be employed.

Ex. Data: $T = 1$, $p_\% = 0$, $n = 3, 4, 5$. Results: $z = 1$, $k = 0.25, 0.148, 0.105$, $t_s = 8, 12, 16$, respectively. $p_{\%,actual} = 0$ in all cases; $t_{s,actual}$ exceeds (32) somewhat.

$\underline{\frac{1}{s(Ts+1)}e^{-\tau s} + \text{PID } k\frac{(s+z_1)(s+z_2)}{s} + p_\%}$. With $z_1 = 1/T$ and 1st order Padé we get

$$G_{open}(s) = \frac{k}{T} \cdot \frac{s+z_2}{s^2} e^{-\tau s} \cong k' \frac{s+z_2'}{s^2} \cdot \frac{-s'+1}{s'+1} \qquad (33)$$

where $k' = k\tau/(2T)$, $s' = s\tau/2$, $z_2' = z\tau/2$. To get small t_s we must choose z_2 large enough. The largest z_2', for which the system has a triple pole and the root-locus looks as in Fig.3, equals 0.08. Then $s_b' = -0.26$ and $k' = 0.22$. Finally $k = 2k'T/\tau$, $z_2 = 2z_2'/\tau$.

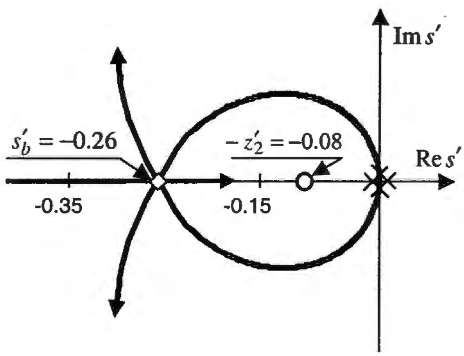

Fig.3. Root-locus plot for $\frac{1}{s(Ts+1)}e^{-\tau s}$ + PID $k\frac{(s+z_1)(s+z_2)}{s}$ + $p_{\%}$.

Ex. Data: $T = \tau = 1$, $p_{\%} = 0$. Results: $z_1 = 1$, $z_2 = 0.16$, $k = 0.44$, $p_{\%,actual} = 17.5\%$. The overshoot can be removed by set-point filter $\frac{1}{s/z_2+1}$. Settling time t_s is in the range $(25\ldots30)\tau$.

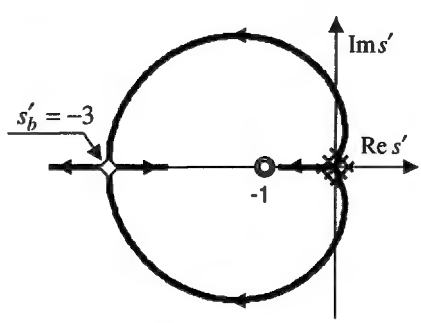

Fig.4. Root-locus plot for $\frac{1}{s^2}$ + PID $k\frac{(s+z)^2}{s}$ + $p_{\%} = 0$, t_s .

$\frac{1}{s^2}$ + PID $k\frac{(s+z)^2}{s} + p_{\%} = 0,\ t_s$. This is the problem of tuning a current driven DC servo (Sec.2). Here

$$G_{open}(s) = k\frac{(s+z)^2}{s^3} = k'\frac{(s'+1)^2}{s'^3} \tag{34}$$

with $s' = s/z$, $k' = k/z$. Root-locus plot is shown in Fig.4. The breakpoint $s'_b = -3$ results from $k' = 27/4$ ($p_{\%} = 0$) (Irzeński, Trybus 1992). The third pole is then $s'_3 = -3/4$, what allows one to estimate settling time as $t'_s \cong 3/|s'_3| = 4$. Final settings are given by

$$z = \frac{4}{t_s}, \qquad k = \frac{27}{4}z. \tag{35}$$

Due to $(s+z)^2$ in the numerator of G_{closed} the step response exhibits 17.9% overshoot. This is removed by $\frac{1}{s/z_2+1}$ set-point filter.

Others. Designs for the other plants in (7), (8) are carried out as follows:

- $\frac{1}{Ts+1}e^{-\tau s} + \mathrm{PI} + p_{\%}$. PI controller is suitable for such plant. T is cancelled by $z = 1/T$ (in PI $k\frac{s+z}{s}$). Then $G_{open}(s) = \frac{k}{T}\frac{e^{-\tau s}}{s}$ as in (27) (see also Fig.2).
- $e^{-\tau s} + \mathrm{I} + p_{\%}$. I controller $\frac{k}{s}$ suits pure delay plant. Since $G_{open}(s) = k\frac{e^{-\tau s}}{s}$ the problem is the same as above.
- $\frac{1}{s(Ts+1)}$ + PID $k\frac{(s+z_1)(s+z_2)}{s} + p_{\%} = 0, t_s$. This represents a voltage driven DC servo. We take $z_1 = 1/T$ and get $G_{open}(s) = \frac{k}{T}\frac{s+z_2}{s^2}$. Root-locus is a circle with breakpoint at $-2z_2$. Hence $z_2 = 2/t_s$ and $k = 4Tz_2$.

5. Frequency designs

Here the plant is given by the magnitude $|G_p(j\omega)|$ and phase $\angle G_p(j\omega)$ frequency characteristics whose samples are shown in Fig.5. We stress that the transfer function representation is *not needed*. The characteristics in frequency range necessary for the design can be obtained by slight extension of relay control experiment used for self-tuning (Åström at al. 1993 and Åström, Wittenmark 1995). After bringing the relay controlled system to ultimate oscillation the relay hysteresis must be gradually increased to keep oscillation frequency decreasing. Recorded input and output are then processed by FFT. Such experiment resembles excitation by "chirp" signal recommended for open-loop identification (Franklin et al. 1996).

$\underline{\mathrm{P} + p_{\%}}$. Phase margin PM of the open-loop system is defined by

$$G_{open}(j\omega_1) = k\,G_p(j\omega_1) \stackrel{\Delta}{=} 1 \cdot e^{j(-180^\circ + PM)}. \tag{36}$$

Since the closed-loop system is expected to behave similarly as the 2[nd] order system, the PM must be equal to

$$PM = 90^\circ - \operatorname{atan} \frac{\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}{2\xi} \tag{37a}$$

(Phillips, Harbor 1991). For $\xi < 0.7$ (oscillatory responses), (37a) can be approximated by

$$PM \cong 100 \cdot \xi \qquad (\xi < 0.7). \tag{37b}$$

Since ξ follows from $p_{\%}$, the phase margin PM computed from (37a) or (37b) becomes now direct data for the design.

The frequency ω_1 in (36) is read out from the $\angle G_p(j\omega)$ characteristic employing the phase condition

$$\angle G_{open}(j\omega_1) = \angle G_p(j\omega_1) = -180^\circ + PM \tag{38}$$

(see Fig.5). k follows from the magnitude condition $|G_{open}(j\omega_1)| = 1$

$$k = \frac{1}{M}$$

where $M = |G_p(j\omega_1)|$ is read out from $|G_p(j\omega)|$. Settling time is evaluated as

$$t_s \cong \frac{8}{\omega_1 \tan PM} \tag{39}$$

(Phillips, Harbor 1991).

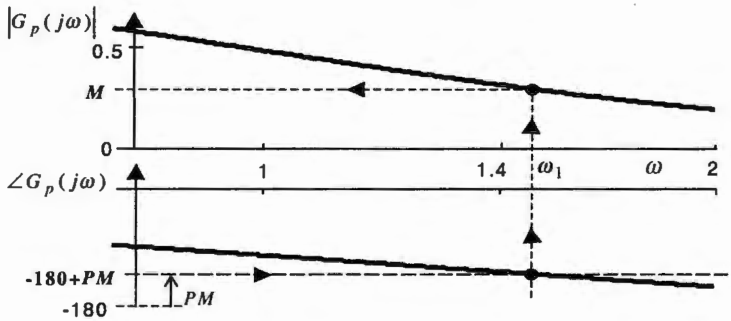

Fig.5. Frequency characteristics of a sample plant $\left(\frac{e^{-0.2s}}{(s+1)^2}\right)$.

Ex. Data: $|G_p(j\omega)|$, $\angle G_p(j\omega)$ in Fig.5, $p_{\%} = 16.3$. Results: $\xi = 0.5$, $PM = 50°$, $\omega_1 = 1.5$, $M = 0.31$, $t_s = 4.5$. The characteristics represent the plant $e^{-0.2s}/(s+1)^2$. For 3rd order Padé approximation of the delay one gets $p_{\%} = 34.6$.

<u>PI + $p_{\%}$</u>. Zero of $k\frac{s+z}{s}$ is chosen as Phillips and Harbor (1991)

$$z = (0.1 \ldots 1.0)\omega_1 , \tag{40}$$

so the corresponding angle $\angle$PI at ω_1 equals $-5\ldots-45°$. To keep $\angle G_{open}(j\omega_1)$ at $-180° + PM$, the frequency ω_1 must be determined for the phase margin PM enlarged by $|\angle \mathrm{PI}|$, i.e.

$$G_p(j\omega_1) = -180° + PM + |\angle \mathrm{PI}| . \tag{41}$$

Naturally, such ω_1 is lower than the one in (38). Having ω_1, we choose z according to (40) and calculate k from the magnitude condition

$$k = \frac{\omega_1}{\sqrt{\omega_1^2 + z^2}} \cdot \frac{1}{M}. \tag{42}$$

Ex. Data as before, 0.3 in (40) (almost half of decade). Results: $\angle \text{PI} = -16.7°$, $\omega_1 = 1.19$, $z = 0.357$, $M = 0.41$, $k = 2.31$, $t_s = 5.6$, $p_{\%,actual} = 13.6\%$.

We remark that the lower factor 0.1 in (40) yields short raise time (10% to 90%) but transients settle down quite slowly. Choice of the upper 1.0 gives the transients similar to 2nd order and may be compared to pole-zero cancellation.

PD + $p_{\%}$, t_s. Recall from (16a) that PD controller is equivalent to lead compensator. Zero of $k\frac{s+z}{s+p}$ is chosen according to (17) ($z = 4/t_s$). Having *PM* and t_s we *calculate* ω_1 from (39), so

$$\omega_1 = \frac{8}{t_s tanPM}. \tag{43}$$

Now p and k follow from the phase and magnitude conditions, i.e.

$$p = \frac{z + \omega_1 tan\theta}{1 - \frac{z}{\omega_1} tan\theta}, \qquad k = \frac{1}{M}\sqrt{\frac{\omega_1^2 + p^2}{\omega_1^2 + z^2}}, \tag{44a}$$

where

$$\theta = -180° + PM - \angle G_p(j\omega_1) \tag{44b}$$

represents the lead compensator phase at ω_1 ($\theta = \angle \text{Lead}(j\omega_1)$).

Ex. Data as before, $t_s = 2.5$ (less than $t_{s,p} = 4.5$). Results: $z = 1.6$, $\omega_1 = 2.67$, $M = 0.12$, $\angle G_p = -170°$, $\theta = 40°$, $p = 7.54$, $k = 21$, $t_s = 8.6$, $p_{\%,actual} = 14.0$.

PID $k\frac{(s+z)^2}{s}$ + $p_{\%}$, t_s. Here again ω_1 is computed from (43). Phase and magnitude conditions yield

$$z = \frac{\omega_1}{tan\theta + \sqrt{tan^2\theta + 1}}, \qquad k = \frac{1 + \sin\theta}{2\omega_1 M}. \tag{45}$$

Now $\theta = -180° + PM - \angle G_p(j\omega_1)$ represents $\angle \mathrm{PID}(j\omega_1)$.

Ex. Data as before. Results: $\omega_1 = 2.67$, $\theta = 40°$, $z = 1.25$, $k = 2.5$, $p_{\%,actual} = 18.1$.

$p_{\%}$ *only*. We take $t_s \stackrel{\Delta}{=} t_{s,P}$ (compare Secs.2,3). This means that $\angle G_p(j\omega_1) = -180° + PM$, so $\theta = 0°$. Hence $z = \omega_1$, $k = 1/(2\omega_1 M)$ or

$$k_p = \frac{1}{M}, \qquad T_i = \frac{2}{\omega_1}, \qquad T_d = \frac{T_i}{4}. \tag{46}$$

For our *example*: $k_p = 3.2$, $T_i = 1.3$, $T_d = 0.33$ ($\omega_1 = 1.5$, $M = 0.31$).

<u>PID $k\frac{s+z_1}{s}\cdot\frac{s+z_2}{s+p}$ + $p_{\%}$, t_s</u>. z_1 is chosen according to (40), i.e. as for PI controller. The plant "seen" by the remaining lead compensator $k\frac{s+z_2}{s+p}$ has the transfer function

$$G_p''(s) = \frac{s+z_1}{s} G_p(s), \tag{47}$$

so the original plant characteristics must be modified accordingly: $\left|G_p''(j\omega)\right| = \left|G_p(j\omega)\right| \cdot \sqrt{\omega^2 + z_1^2}\,/\omega$, $\angle G_p''(j\omega) = \angle G_p(j\omega) + \mathrm{atan}\frac{\omega}{z_1} - 90°$.

Now we design the lead $k\frac{s+z_2}{s+p}$ for G_p''.

Ex. Data as before, $t_s = 4.5 (= t_{s,P})$. Results: $z_1 = 0.357$ (see PI), $\omega_1 = 1.5$, $\left|G_p''(j\omega_1)\right| = M = 0.32$, $\angle G_p''(j\omega_1) = -143°$, $\theta = 13°$, $z_2 = 0.89$, $p =$ $= 1.65$, $k = 4.04$, $p_{\%,actual} = 8.2$.

<u>PID $k\frac{(s+z)^2}{s(s+vz)}$ + $p_{\%}$, t_s, v</u> given. The condition $\angle \mathrm{PID}(j\omega_1) = \theta$ yields the following equation for z

$$2\,\mathrm{atan}\frac{\omega_1}{z} - \mathrm{atan}\frac{\omega_1}{vz} - 90° = \theta \tag{48}$$

Now

$$k = \frac{1}{\omega_1 M} \cdot \frac{\omega_1^2 + z^2}{\sqrt{\omega_1^2 + (\nu z)^2}} \tag{49}$$

If $t_s = t_{s,P}$, then $\theta = 0°$.

Ex. Data as before ($t_s = t_{s,P}$). Results: $z = 0.95$, $k = 5.5$, $p_{\%,actual} = 17.5\%$.

6. Conclusions

Quick selection of PID settings for typical industrial plants given basic specifications, such as overshoot and settling time, is still a skill not so commonly encountered among control engineers and students. Familiarity of Polish control community with root locus method also seems inadequate. Therefore presentation of typical design problems solved quickly by means of root locus or frequency methods has been basic objective of this tutorial. Standard process control plants and DC servos, whose models include time constants, delays and integrators, have been considered. A few design cases related to commercial self-tuning controllers have been solved analytically. 1st order Padé approximation of the delay has turned out sufficient for the design, confirming robustness of the PID control (3rd order have been used for simulations). Transfer function of the plant, which is necessary for root locus design, is usually obtained from step response identification. Frequency characteristics are generated by fast Fourier transform (FFT) employing periodic excitation with varying frequency ("chirp"-type).

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