Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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ISBN 9788389475350

Approximations of intuitionistic fuzzy numbers generated from approximations of fuzzy numbers

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Abstract

We give a method to transfer the calculus from the approximations of fuzzy numbers to the approximations of intuitionistic fuzzy numbers. As a consequence, the properties and algorithms of computing are obtained in an easy way.

Keywords: fuzzy number, intuitionistic fuzzy number, approximation.

1 Introduction

As a generalization of the concept of fuzzy set, the notion of intuitionistic fuzzy set was introduced (see [2], [3]). In the present paper we refer to intuitionistic fuzzy numbers (see [15]), which are particular intuitionistic fuzzy sets and extensions of fuzzy numbers as well.

In the last years, several researchers focused on the computing of different approximations of fuzzy numbers, with or without some additional conditions (see e.g. [1], [7], [8], [11], [23], [24], [25]). We approximate fuzzy numbers by real numbers, real intervals, triangular, trapezoidal or parametric fuzzy numbers because in this way it is easy to handle and to have natural interpretations of the results. Uncertainty and incomplete information can be represented by intuitionistic fuzzy numbers too such that the simplification of calculus, including here different kinds of approximations, is relatively important.

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2010.*

The main result of the present paper is quite general: under some non-restrictive conditions, the approximation of an intuitionistic fuzzy number is reduced to the approximation of a fuzzy number. The main benefit is that several results are obtained as immediate consequences, without to use quite technical and complicated methods based on Karush-Kuhn-Tucker theorem or other theoretical results (see [4], [6]). The algorithms of calculus and properties as invariance to translations, scale invariance, additivity and continuity are displaced too.

2 Fuzzy numbers and intuitionistic fuzzy numbers

We consider the following well-known description of a fuzzy number u:

$$u(x) = \begin{cases} 0, & \text{if } x \leq a_1, \\ l_u(x), & \text{if } a_1 \leq x \leq a_2, \\ 1, & \text{if } a_2 \leq x \leq a_3, \\ r_u(x), & \text{if } a_3 \leq x \leq a_4, \\ 0, & \text{if } a_4 \leq x, \end{cases} \tag{1}$$

where $a_1, a_2, a_3, a_4 \in \mathbb{R}, l_u : [a_1, a_2] \to [0, 1]$ is a nondecreasing continuous function, $l_u(a_1) = 0, l_u(a_2) = 1$, called the left side of the fuzzy number u and $r_u : [a_3, a_4] \to [0, 1]$ is a nonincreasing continuous function, $r_u(a_3) = 1, r_u(a_4) = 0$, called the right side of the fuzzy number u. The α-cut, $\alpha \in]0, 1]$, of a fuzzy number u is the crisp set defined as

$$u_\alpha = \{x \in \mathbb{R} : u(x) \geq \alpha\}.$$

The support or 0-cut u_0 of a fuzzy number u is defined as the closure of the set $\{x \in \mathbb{R} : u(x) > 0\}$, that is

$$u_0 = \overline{\{x \in \mathbb{R} : u(x) > 0\}}.$$

Every α-cut, $\alpha \in [0, 1]$, of a fuzzy number u is a closed interval

$$u_\alpha = \left[u^-(\alpha), u^+(\alpha)\right],$$

where

$$u^-(\alpha) = \inf\{x \in \mathbb{R} : u(x) \geq \alpha\},$$
$$u^+(\alpha) = \sup\{x \in \mathbb{R} : u(x) \geq \alpha\}$$

for any $\alpha \in]0, 1]$. If the sides of the fuzzy number u are strictly monotone then one can see easily that u^- and u^+ are inverse functions of l_u and r_u, respectively.

We denote by $F(\mathbb{R})$ the set of all fuzzy numbers. A metric on the set of fuzzy numbers, which is an extension of the Euclidean distance, is defined by

$$d^2(u,v)=\int_0^1\left(u^-(\alpha)-v^-(\alpha)\right)^2 d\alpha+\int_0^1\left(u^+(\alpha)-v^+(\alpha)\right)^2 d\alpha. \quad (2)$$

If $\omega:\mathbb{R}\to[0,1]$ is a fuzzy set such that $1-\omega$, where $(1-\omega)(x)=1-\omega(x)$, for every $x\in\mathbb{R}$, is a fuzzy number and we denote

$$\omega_\alpha=\{x\in\mathbb{R}:\omega(x)\le\alpha\},\alpha\in[0,1[$$
$$\omega_1=\overline{\{x\in\mathbb{R}:\omega(x)<1\}},$$

then

$$\omega_\alpha=(1-\omega)_{1-\alpha},$$

for every $\alpha\in[0,1]$. The set ω_α is a closed interval $[\omega^-(\alpha),\omega^+(\alpha)]$, for every $\alpha\in[0,1]$.

Fuzzy numbers with simple membership functions are preferred in practice. The most used such fuzzy numbers are so-called trapezoidal fuzzy numbers. A trapezoidal fuzzy number T is given by $T_\alpha=[T^-(\alpha),T^+(\alpha)]$,

$$T^-(\alpha)=t_1+(t_2-t_1)\alpha$$
$$T^+(\alpha)=t_4-(t_4-t_3)\alpha,\alpha\in[0,1],$$

where $t_1,t_2,t_3,t_4\in\mathbb{R},t_1\le t_2\le t_3\le t_4$. When $t_2=t_3$ we obtain so-called triangular fuzzy numbers, when $t_1=t_2$ and $t_3=t_4$ we obtain closed intervals and in the case $t_1=t_2=t_3=t_4$ we obtain crisp numbers. We denote $T=(t_1,t_2,t_3,t_4)$ a trapezoidal fuzzy number as above and $F^T(\mathbb{R})$ the set of all trapezoidal fuzzy numbers.

Let $u,v\in F(\mathbb{R}),u_\alpha=[u^-(\alpha),u^+(\alpha)],v_\alpha=[v^-(\alpha),v^+(\alpha)],\alpha\in[0,1]$ and $\lambda\in\mathbb{R}$. We consider the sum $u+v$ and the scalar multiplication $\lambda\cdot u$ by

$$(u+v)_\alpha=u_\alpha+v_\alpha=\left[u^-(\alpha)+v^-(\alpha),u^+(\alpha)+v^+(\alpha)\right] \quad (3)$$

and

$$(\lambda\cdot u)_\alpha=\lambda\cdot u_\alpha=\begin{cases}[\lambda u^-(\alpha),\lambda u^+(\alpha)], & \text{if } \lambda\ge 0,\\ [\lambda u^+(\alpha),\lambda u^-(\alpha)], & \text{if } \lambda<0,\end{cases} \quad (4)$$

respectively, for every $\alpha\in[0,1]$. In the case of the trapezoidal fuzzy numbers $T=(t_1,t_2,t_3,t_4)$ and $S=(s_1,s_2,s_3,s_4)$ we obtain

$$T+S=(t_1+s_1,t_2+s_2,t_3+s_3,t_4+s_4)$$

and

$$\lambda \cdot T = \begin{cases} (\lambda t_1, \lambda t_2, \lambda t_3, \lambda t_4), & \text{if } \lambda \geq 0, \\ (\lambda t_4, \lambda t_3, \lambda t_2, \lambda t_1), & \text{if } \lambda < 0. \end{cases}$$

Let $u \in F(\mathbb{R})$. The expected interval $EI(u)$, expected value $EV(u)$, width $w(u)$, value $Val(u)$, ambiguity $Amb(u)$, left-hand ambiguity $Amb_L(u)$, right-hand ambiguity $Amb_R(u)$ and core $core(u)$ of a fuzzy number u were introduced ([20], [12], [13], [14], [18]) by

$$EI(u) = \left[\int_0^1 u^-(\alpha)\, d\alpha, \int_0^1 u^+(\alpha)\, d\alpha\right] \tag{5}$$

$$EV(u) = \frac{1}{2}\left(\int_0^1 u^-(\alpha)\, d\alpha + \int_0^1 u^+(\alpha)\, d\alpha\right) \tag{6}$$

$$w(u) = \int_0^1 u^+(\alpha)\, d\alpha - \int_0^1 u^-(\alpha)\, d\alpha \tag{7}$$

$$Val(u) = \int_0^1 \alpha u^+(\alpha)\, d\alpha + \int_0^1 \alpha u^-(\alpha)\, d\alpha \tag{8}$$

$$Amb(u) = \int_0^1 \alpha u^+(\alpha)\, d\alpha - \int_0^1 \alpha u^-(\alpha)\, d\alpha \tag{9}$$

$$Amb_L(u) = \int_0^1 \alpha \left(EV(u) - u^-(\alpha)\right) d\alpha \tag{10}$$

$$Amb_U(u) = \int_0^1 \alpha \left(u^+(\alpha) - EV(u)\right) d\alpha \tag{11}$$

$$core(u) = \left[u^-(1), u^+(1)\right]. \tag{12}$$

Definition 1 *([2], [3]) Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in X is an object A given by*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle ; x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ satisfy the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1,$$

for every $x \in X$.

Definition 2 *An intuitionistic fuzzy set $A = \{\langle x, u_A(x), v_A(x)\rangle ; x \in \mathbb{R}\}$ such that u_A and $1 - v_A$ are fuzzy numbers, where*

$$(1 - v_A)(x) = 1 - v_A(x), \forall x \in \mathbb{R},$$

is called an intuitionistic fuzzy number.

We denote by $A = (u_A, v_A)$ an intuitionistic fuzzy number and by $IF(\mathbb{R})$ the set of all intuitionistic fuzzy numbers. It is obvious that any fuzzy number u can be represented as an intuitionistic fuzzy number by $(u, 1-u)$.

With respect to the α-cuts of the fuzzy number $1 - v_A$ the following equalities are immediate:

$$(1-\nu_A)^- (\alpha) = \nu_A^- (1-\alpha) \tag{13}$$

and

$$(1-\nu_A)^+ (\alpha) = \nu_A^+ (1-\alpha), \tag{14}$$

for every $\alpha \in [0,1]$.

We define the addition $A + B \in IF(\mathbb{R})$ of $A = (u_A, v_A), B = (u_B, v_B) \in IF(\mathbb{R})$ by

$$A + B = (u_{A+B}, v_{A+B}),$$

where $u_{A+B} = u_A + u_B$ and v_{A+B} is given by

$$1 - v_{A+B} = (1 - v_A) + (1 - v_B).$$

We define the scalar multiplication $\lambda \cdot A \in IF(\mathbb{R})$ of $A = (u_A, v_A) \in IF(\mathbb{R}), \lambda \in \mathbb{R}$ by

$$\lambda \cdot A = (u_{\lambda \cdot A}, v_{\lambda \cdot A}),$$

where $u_{\lambda \cdot A} = \lambda \cdot u_A$ and $v_{\lambda \cdot A}$ is given by

$$1 - v_{\lambda \cdot A} = \lambda \cdot (1 - v_A).$$

In the intuitionistic fuzzy case the distance d (2) becomes (see [4])

$$\begin{aligned}\widetilde{d}^2(A,B) = &\frac{1}{2}\int_0^1 \left(u_A^-(\alpha) - u_B^-(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(u_A^+(\alpha) - u_B^+(\alpha)\right)^2 d\alpha \\ &+ \frac{1}{2}\int_0^1 \left(v_A^-(\alpha) - v_B^-(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(v_A^+(\alpha) - v_B^+(\alpha)\right)^2 d\alpha.\end{aligned}$$

where $A = (u_A, v_A), B = (u_B, v_B) \in IF(\mathbb{R})$.

It is immediate that

$$\widetilde{d}^2(A,B) = \frac{1}{2} d^2(u_A, u_B) + \frac{1}{2} d^2(1 - v_A, 1 - v_B).$$

Any parameter (real number or real interval) associated with a fuzzy number can be extended in a natural way to an intuitionistic fuzzy number $A = (u_A, v_A)$

as the arithmetic mean of the same parameter applied for u_A and $1 - v_A$. Taking into account (13)-(14) the parameters introduced in (5)-(12) became

$$\widetilde{EI}(A) = \left[\frac{1}{2}\int_0^1 \left(u_A^-(\alpha) + v_A^-(\alpha)\right) d\alpha, \right. \tag{15}$$

$$\left. \frac{1}{2}\int_0^1 \left(u_A^+(\alpha) + v_A^+(\alpha)\right) d\alpha\right]$$

$$\widetilde{EV}(A) = \frac{1}{4}\int_0^1 \left(u_A^-(\alpha) + v_A^-(\alpha) + u_A^+(\alpha) + v_A^+(\alpha)\right) d\alpha \tag{16}$$

$$\widetilde{w}(A) = \frac{1}{2}\int_0^1 \left(u_A^+(\alpha) + v_A^+(\alpha) - u_A^-(\alpha) - v_A^-(\alpha)\right) d\alpha \tag{17}$$

$$\widetilde{Val}(A) = \frac{1}{2}\int_0^1 \alpha u_A^+(\alpha)\, d\alpha + \frac{1}{2}\int_0^1 \alpha u_A^-(\alpha)\, d\alpha \tag{18}$$

$$+ \frac{1}{2}\int_0^1 (1-\alpha)\, v_A^+(\alpha)\, d\alpha + \frac{1}{2}\int_0^1 (1-\alpha)\, v_A^-(\alpha)\, d\alpha$$

$$\widetilde{Amb}(A) = \frac{1}{2}\int_0^1 \alpha u_A^+(\alpha)\, d\alpha - \frac{1}{2}\int_0^1 \alpha u_A^-(\alpha)\, d\alpha \tag{19}$$

$$+ \frac{1}{2}\int_0^1 (1-\alpha)\, v_A^+(\alpha)\, d\alpha - \frac{1}{2}\int_0^1 (1-\alpha)\, v_A^-(\alpha)\, d\alpha$$

$$\widetilde{Amb}_L(A) = \int_0^1 \alpha \left(\widetilde{EV}(A) - \frac{1}{2}u_A^-(\alpha) - \frac{1}{2}(1-v)_A^-(\alpha)\right) d\alpha \tag{20}$$

$$\widetilde{Amb}_U(A) = \int_0^1 \alpha \left(\frac{1}{2}u_A^+(\alpha) + \frac{1}{2}(1-v)_A^+(\alpha) - \widetilde{EV}(A)\right) d\alpha \tag{21}$$

$$\widetilde{core}(A) = \left[\frac{u_A^-(1) + v_A^-(0)}{2}, \frac{u_A^+(1) + v_A^+(0)}{2}\right], \tag{22}$$

where $A = (u_A, v_A) \in IF(\mathbb{R})$.

3 Main result

The benefits of a method to reduce the calculus of approximation (trapezoidal, triangular, parametric, interval, etc.) in intuitionistic fuzzy case to the fuzzy case are obvious. It is sufficient to see the sophisticated calculus in the finding of the nearest trapezoidal fuzzy number of an intuitionistic fuzzy number preserving the expected interval ([6]). The first step in this direction was already proposed in [9]. A more general result is the following

Theorem 1 *Let us consider the function* $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}$ *satisfying the following properties:*

i) For any intuitionistic fuzzy number $A = (u_A, v_A)$ *we have* $M(A) = M(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$*;*

ii) For any fuzzy number $u \in F(\mathbb{R})$ *there exists an unique element* $s(u) \in M(u)$ *such that*

$$d(u, s(u)) = \min_{w \in M(u)} d(u, w).$$

Then, for any intuitionistic fuzzy number $A = (u_A, v_A)$*, there exists an unique element* $S(A) \in M(A)$ *such that*

$$\widetilde{d}(A, S(A)) = \min_{w \in M(A)} \widetilde{d}(A, w).$$

In addition, we have $S(A) = s(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$*.*

Proof. Let us choose arbitrary $A = (u_A, v_A) \in IF(\mathbb{R})$ and $w \in M(A)$. We have

$$\begin{aligned}
&\widetilde{d}^2(A, w) \\
&= \frac{1}{2} d^2(u_A, w) + \frac{1}{2} d^2(1 - v_A, w) \\
&= \frac{1}{2} \int_0^1 \left(u_A^-(\alpha) - w^-(\alpha)\right)^2 d\alpha + \frac{1}{2} \int_0^1 \left(u_A^+(\alpha) - w^+(\alpha)\right)^2 d\alpha \\
&+ \frac{1}{2} \int_0^1 \left((1 - v_A)^-(\alpha) - w^-(\alpha)\right)^2 d\alpha + \frac{1}{2} \int_0^1 \left((1 - v_A)^+(\alpha) - w^+(\alpha)\right)^2 d\alpha \\
&= d^2(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A), w) + \frac{1}{4} \int_0^1 \left(u_A^-(\alpha) - (1 - v_A)^-(\alpha)\right)^2 d\alpha \\
&+ \frac{1}{4} \int_0^1 \left(u_A^+(\alpha) - (1 - v_A)^+(\alpha)\right)^2 d\alpha.
\end{aligned}$$

Since the expression

$$\frac{1}{4} \int_0^1 \left(u_A^-(\alpha) - (1 - v_A)^-(\alpha)\right)^2 d\alpha + \frac{1}{4} \int_0^1 \left(u_A^+(\alpha) - (1 - v_A)^+(\alpha)\right)^2 d\alpha$$

is constant, it follows that $\widetilde{d}^2(A, w)$ is minimum if and only if $d^2(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A), w)$ is minimum. Taking into account the hypothesis of the theorem we easily obtain the desired conclusion. ■

4 Applications

4.1 Approximation of intuitionistic fuzzy numbers

4.1.1 Nearest fuzzy number to an intuitionistic fuzzy number

Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}$, $M(A) = F(\mathbb{R})$ for all $A \in IF(\mathbb{R})$. It is immediate that the hypothesis in Theorem 1 are satisfied and since in this case $s(u) = u$, for all $u \in F(\mathbb{R})$, we obtain:

Theorem 2 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then*

$$S(A) = \frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)$$

is the nearest fuzzy number to A with respect to the distance $\widetilde{d}$ and $S(A)$ is unique with this property.

If $A = (u_A, v_A)$ is a trapezoidal intuitionistic fuzzy number, that is $u_A = (t_1, t_2, t_3, t_4)$ and $1 - v_A = (s_1, s_2, s_3, s_4)$ then

$$S(A) = \left(\frac{t_1 + s_1}{2}, \frac{t_2 + s_2}{2}, \frac{t_3 + s_3}{2}, \frac{t_4 + s_4}{2} \right)$$

is the nearest trapezoidal fuzzy number of A.

4.1.2 Trapezoidal approximation of an intuitionistic fuzzy number

Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}$, $M(A) = F^T(\mathbb{R})$ for all $A \in IF(\mathbb{R})$. For $u \in F(\mathbb{R})$ let us denote $s(u) = t(u)$ the trapezoidal approximation of u with respect to the distance d. From Theorem 1 we obtain:

Theorem 3 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then*

$$S(A) = T(A) = t(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$$

is the nearest trapezoidal fuzzy number to A with respect to the distance $\widetilde{d}$ and $T(A)$ is unique with this property.

The nearest trapezoidal fuzzy number to an intuitionistic fuzzy number was already computed in [9].

4.1.3 Parametric approximation of an intuitionistic fuzzy number

Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}$, $M(A) = F^{s_L,s_R}(\mathbb{R})$ for all $A \in IF(\mathbb{R})$, where $F^{s_L,s_R}(\mathbb{R})$ is the set of (s_L, s_R) parametric fuzzy numbers (see [22]). For $u \in F(\mathbb{R})$ let us denote $s(u) = p_{s_L,s_R}(u)$ the parametric (s_L, s_R) approximation of u with respect to the distance d (see [22], [8]). From Theorem 1 we obtain:

Theorem 4 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then $S(A) = P_{s_L,s_R}(A) = p_{s_L,s_R}(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$ is the nearest parametric fuzzy number to A with respect to the distance $\widetilde{d}$ and $P_{s_L,s_R}(A)$ is unique with this property.*

The nearest parametric fuzzy number to a given fuzzy number was computed in [8]. Taking into account Theorem 4 we can obtain the result for intuitionistic fuzzy numbers.

4.1.4 Trapezoidal approximation of an intuitionistic fuzzy number preserving the expected interval

Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}, M(A) = \{w \in F(\mathbb{R}) : \widetilde{EI}(A) = EI(w)\}$. The equality $M(A) = M(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$ is immediate from (15). Now, for $u \in F(\mathbb{R})$ let us denote by $t_e(u)$ the nearest fuzzy number to u (with respect to metric d), preserving the expected interval of u. From Theorem 1 we obtain:

Theorem 5 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then*

$$S(A) = T_e(A) = t_e(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$$

is the nearest trapezoidal fuzzy number to A (with respect to the distance $\widetilde{d}$), preserving the expected interval of A, and $T_e(A)$ is unique with this property.

The nearest trapezoidal fuzzy number of an intuitionistic fuzzy number preserving the expected was already computed in [6]. The method is based on Karush-Kuhn-Tucker theorem and it is very complicated. We obtain the same result from Theorem 5 and Theorem 7 in [7].

4.1.5 Trapezoidal approximation of an intuitionistic fuzzy number preserving value and ambiguity

It is immediate that (see (19) and (18))

$$\widetilde{Amb}(A) = Amb(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)) \tag{23}$$

and

$$\widetilde{Val}(A) = Val(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)), \tag{24}$$

for every $A = (u_A, v_A) \in IF(\mathbb{R})$. Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}, M(A) = \{w \in F(\mathbb{R}) : \widetilde{Val}(A) = Val(w)$ and $\widetilde{Amb}(A) = Amb(w)\}$. We have $M(A) = M(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$. If we denote by $t_{a,v}(u)$ the nearest trapezoidal approximation of $u \in F(\mathbb{R})$ (with respect to d), preserving the value and ambiguity of u, then from Theorem 1 we obtain:

Theorem 6 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then*

$$S(A) = T_{a,v}(A) = t_{a,v}(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$$

is the nearest trapezoidal fuzzy number to A (with respect to the distance $\widetilde{d}$), preserving the value and ambiguity of A, and $T_{a,v}(A)$ is unique with this property.

The nearest trapezoidal fuzzy number to a given fuzzy number (with respect to the distance d), preserving the value and ambiguity was computed in [11]. Taking into account Theorem 6 we immediately obtain.

Theorem 7 *Let $A = (u_A, v_A)$ be an intuitionistic fuzzy number,*

$$\begin{aligned}
m^- &= \int_0^1 u_A^-(\alpha)\, d\alpha, m^+ = \int_0^1 u_A^+(\alpha)\, d\alpha, \\
n^- &= \int_0^1 v_A^-(\alpha)\, d\alpha, n^+ = \int_0^1 v_A^+(\alpha)\, d\alpha, \\
M^- &= \int_0^1 \alpha u_A^-(\alpha)\, d\alpha, M^+ = \int_0^1 \alpha u_A^+(\alpha)\, d\alpha, \\
N^- &= \int_0^1 \alpha v_A^-(\alpha)\, d\alpha, N^+ = \int_0^1 \alpha v_A^+(\alpha)\, d\alpha
\end{aligned}$$

and

$$T_{a,v}(A) = (t_1(A), t_2(A), t_3(A), t_4(A)) = (t_1, t_2, t_3, t_4)$$

the nearest (with respect to the metric $\widetilde{d}$) trapezoidal fuzzy number to intuitionistic fuzzy number A, which preserves its value and ambiguity.

(i) *If*

$$-m^- + m^+ + n^- - n^+ + 3M^- - 3N^- - 3M^+ + 3N^+ \leq 0$$

then

$$t_1 = 2m^- - n^- - 3M^- + 3N^- \tag{25}$$

$$t_2 = -m^- + 2n^- + 3M^- - 3N^- \tag{26}$$

$$t_3 = -m^+ + 2n^+ + 3M^+ - 3N^+ \tag{27}$$

$$t_4 = 2m^+ - n^+ - 3M^+ + 3N^+. \tag{28}$$

(ii) *If*

$$-m^- - m^+ + 2n^- + 3M^- + M^+ - 3N^- - N^+ > 0$$

then

$$t_1 = 3n^- - 2n^+ + 3M^- - 2M^+ - 3N^- + 2N^+ \tag{29}$$

$$t_2 = t_3 = t_4 = n^+ + M^+ - N^+. \tag{30}$$

(iii) *If*

$$-m^- - m^+ + 2n^+ + M^- + 3M^+ - N^- - 3N^+ < 0$$

then

$$t_1 = t_2 = t_3 = n^- + M^- - N^- \tag{31}$$

$$t_4 = -2n^- + 3n^+ - 2M^- + 3M^+ + 2N^- - 3N^+. \tag{32}$$

(iv) *If*

$$-m^- + m^+ + n^- - n^+ + 3M^- - 3M^+ - 3N^- + 3N^+ > 0$$
$$-m^- - m^+ + 2n^- + 3M^- + M^+ - 3N^- - N^+ \leq 0$$

and

$$-m^- - m^+ + 2n^+ + M^- + 3M^+ - N^- - 3N^+ \geq 0$$

then

$$t_1 = m^- + m^+ + n^- - 2n^+ - 3M^+ + 3N^+ \tag{33}$$

$$t_2 = t_3 = -\frac{1}{2}m^- - \frac{1}{2}m^+ + n^- + n^+ + \frac{3}{2}M^- + \frac{3}{2}M^+ - \frac{3}{2}N^- - \frac{3}{2}N^+ \tag{34}$$

$$t_4 = m^- + m^+ - 2n^- + n^+ - 3M^- + 3N^- \tag{35}$$

4.1.6 Trapezoidal approximation of an intuitionistic fuzzy number preserving core

It is immediate that (see (22))

$$\widetilde{core}\,(A) = core(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$$

for every $A = (u_A, v_A) \in IF(\mathbb{R})$. Let us consider the function $M : IF(\mathbb{R}) \to 2^{F(\mathbb{R})}, M(A) = \{w \in F(\mathbb{R}) : \widetilde{core}(A) = core(w)\}$. We have $M(A) = M(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$. If we denote by $t_c(u)$ the nearest trapezoidal fuzzy number to $u \in F(\mathbb{R})$ (with respect to metric d), preserving the core of u, then from Theorem 1 we obtain:

Theorem 8 *If $A = (u_A, v_A)$ is an intuitionistic fuzzy number, then*

$$S(A) = T_c(A) = t_c(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$$

is the nearest (with respect to the distance $\widetilde{d}$) trapezoidal fuzzy number to A, preserving the core of A and $T_c(A)$ is unique with this property.

The nearest trapezoidal fuzzy number to a given fuzzy number preserving the core was computed in [1]. Taking into account Theorem 8 we immediately obtain

Theorem 9 *The nearest (with respect to the metric $\widetilde{d}$) trapezoidal fuzzy number to an intuitionistic fuzzy number $A = (u_A, v_A)$, which preserves the core of A, $T_c\,(A) = (t_1, t_2, t_3, t_4)$, is given by*

$$\begin{aligned}
t_1 &= \frac{3}{2}\int_0^1 u_A^-\,(\alpha)\,d\alpha - \frac{3}{2}\int_0^1 \alpha u_A^-\,(\alpha)\,d\alpha \\
&\quad + \frac{3}{2}\int_0^1 \alpha v_A^-\,(\alpha)\,d\alpha - \frac{1}{4}\left(u_A^-\,(1) + v_A^-\,(0)\right), \\
t_2 &= \frac{1}{2}\left(u_A^-\,(1) + v_A^-\,(0)\right), \\
t_3 &= \frac{1}{2}\left(u_A^+\,(1) + v_A^+\,(0)\right), \\
t_4 &= -\frac{3}{2}\int_0^1 \alpha u_A^+\,(\alpha)\,d\alpha + \frac{3}{2}\int_0^1 \alpha v_A^+\,(\alpha)\,d\alpha \\
&\quad + \frac{3}{2}\int_0^1 u_A^+\,(\alpha)\,d\alpha - \frac{1}{4}\left(u_A^+\,(1) + v_A^+\,(0)\right).
\end{aligned}$$

4.2 Transfer of algorithms

The idea to use some parameters associated with fuzzy numbers in the expression of conditions and trapezoidal approximations was proposed in [17] and continued in [19], [8], [11]. Let us assume every parameter P which appears in the algorithm in the fuzzy case satisfies $P(A) = P(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A))$, where $A = (u_A, v_A)$. Taking into account Theorem 1 we obtain a similar algorithm in the intuitionistic fuzzy case.

The algorithms for calculus of the nearest trapezoidal fuzzy number to an intuitionistic fuzzy number and of the nearest trapezoidal fuzzy number to an intuitionistic fuzzy number preserving the expected interval were presented in [5]. As a new result, let us consider the algorithm for calculus of the nearest trapezoidal fuzzy number preserving value and ambiguity of a given intuitionistic fuzzy number.

Because

$$\widetilde{w}(A) = w\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right),$$

$$\widetilde{Amb}_L(A) = Amb_L\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right)$$

and

$$\widetilde{Amb}_U(A) = Amb_U\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right)$$

for every $A = (u_A, v_A) \in IF(\mathbb{R})$, we obtain the following similar algorithm with the fuzzy case (see [11]) to obtain the nearest trapezoidal fuzzy number $T_{a,v}(A) = (t_1, t_2, t_3, t_4)$ to a given intuitionistic fuzzy number A, which preserves value and ambiguity, that is

$$\widetilde{Val}(T_{a,v}(A)) = \widetilde{Val}(A)$$

and

$$\widetilde{Amb}(T_{a,v}(A)) = \widetilde{Amb}(A).$$

Algorithm 1 *Step 1: If $\widetilde{w}(A) \leq 3\widetilde{Amb}(A)$ then apply (25)-(28) to compute the approximation, else*

Step 2: If $4\widetilde{Amb}_L(A) < \widetilde{Amb}(A)$ then apply (29)-(30) to compute the approximation, else

Step 3: If $4\widetilde{Amb}_U(A) < \widetilde{Amb}(A)$ then apply (31)-(32) to compute the approximation, else

Step 4: apply (33)-(35).

4.3 Transfer of properties

With respect to the properties of mappings $s : F(\mathbb{R}) \to F(\mathbb{R})$ and $S : IF(\mathbb{R}) \to F(\mathbb{R})$ in Theorem 1 we can formulate the following result:

Theorem 10 (i) *If s is additive then S is additive too;*
(ii) *If s is invariant to translations then S is invariant to translations too;*
(iii) *If s is scale invariant then S is scale invariant too;*
(iv) *If s has the Lipschitz constant c then S has the same Lipschitz constant c;*
(v) *If s is continuous then S is continuous too.*

Proof. (i) Let $A, B \in IF(\mathbb{R}), A = (u_A, v_A)$ and $B = (u_B, v_B)$. According with the definition of addition we have $A + B = (u_{A+B}, v_{A+B})$, where $u_{A+B} = u_A + u_B, 1 - v_{A+B} = (1 - v_A) + (1 - v_B)$. Taking into account Theorem 1 and the hypothesis we get

$$\begin{aligned}
&S(A+B) \\
&= s\left(\frac{1}{2}\cdot(u_A+u_B)+\frac{1}{2}\cdot((1-v_A)+(1-v_B))\right) \\
&= s\left(\frac{1}{2}\cdot u_A+\frac{1}{2}\cdot(1-\nu_A)\right)+s\left(\frac{1}{2}\cdot u_B+\frac{1}{2}\cdot(1-\nu_B)\right) \\
&= S(A)+S\left(B\right).
\end{aligned}$$

(ii) It is immediate from (i).

(iii) Let $A = (u_A, v_A) \in IF(\mathbb{R})$ and $\lambda \in \mathbb{R}$. The definition of the scalar multiplication and Theorem 1 imply

$$\begin{aligned}
&S(\lambda\cdot A) \\
&= S((\lambda\cdot u_A, 1-\lambda\cdot(1-v_A))) \\
&= s\left(\frac{1}{2}\cdot\lambda\cdot u_A+\frac{1}{2}\cdot\lambda\cdot(1-v_A)\right) \\
&= s\left(\lambda\cdot(\frac{1}{2}\cdot u_A+\frac{1}{2}\cdot(1-v_A))\right) \\
&= \lambda\cdot s\left(\frac{1}{2}\cdot u_A+\frac{1}{2}\cdot(1-v_A)\right) \\
&= \lambda\cdot S(A).
\end{aligned}$$

(iv) Let $A, B \in IF(\mathbb{R}), A = (u_A, v_A)$ and $B = (u_B, v_B)$. According with Theorem 1 we get

$$S(A) = s\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right)$$

and

$$S(B) = s\left(\frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right).$$

If s is Lipschitz with the constant c then

$$d\left(s\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right), s\left(\frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right)\right)$$
$$\leq cd\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A), \frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right).$$

On the other hand,

$$\begin{aligned}
&d^2\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A), \frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right)\\
=&\int_0^1\left(\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right)^-(\alpha) - \left(\frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right)^-(\alpha)\right)^2 d\alpha\\
+&\int_0^1\left(\left(\frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)\right)^+(\alpha) - \left(\frac{1}{2} \cdot u_B + \frac{1}{2} \cdot (1 - v_B)\right)^+(\alpha)\right)^2 d\alpha\\
=&\int_0^1 \left(\frac{1}{2}u_A^-(\alpha) + \frac{1}{2}(1 - v_A)^-(\alpha) - \frac{1}{2}u_B^-(\alpha) - \frac{1}{2}(1 - v_B)^-(\alpha)\right)^2 d\alpha\\
+&\int_0^1 \left(\frac{1}{2}u_A^+(\alpha) + \frac{1}{2}(1 - v_A)^+(\alpha) - \frac{1}{2}u_B^+(\alpha) - \frac{1}{2}(1 - v_B)^+(\alpha)\right)^2 d\alpha\\
=&\frac{1}{4}\int_0^1 \left(u_A^-(\alpha) - u_B^-(\alpha) + (1 - v_A)^-(\alpha) - (1 - \nu_B)^-(\alpha)\right)^2 d\alpha\\
+&\frac{1}{4}\int_0^1 \left(u_A^+(\alpha) - u_B^+(\alpha) + (1 - v_A)^+(\alpha) - (1 - v_B)^+(\alpha)\right)^2 d\alpha\\
\leq& \frac{1}{2}\int_0^1 \left(u_A^-(\alpha) - u_B^-(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left((1 - v_A)^-(\alpha) - (1 - \nu_B)^-(\alpha)\right)^2 d\alpha\\
+& \frac{1}{2}\int_0^1 \left(u_A^+(\alpha) - u_B^+(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left((1 - v_A)^+(\alpha) - (1 - v_B)^+(\alpha)\right)^2 d\alpha\\
=& \frac{1}{2}d^2(u_A, u_B) + \frac{1}{2}d^2(1 - v_A, 1 - v_B)\\
=& \widetilde{d^2}(A, B)
\end{aligned}$$

We get

$$\widetilde{d}\,(S(A), S(B)) \leq c\widetilde{d}(A, B).$$

(v) We have $S = s \circ f$, where $f : IF(\mathbb{R}) \rightarrow F(\mathbb{R})$ is defined by $f\,(A) = f((u_A, v_A)) = \frac{1}{2} \cdot u_A + \frac{1}{2} \cdot (1 - v_A)$. Since s and f are continuous, we obtain the continuity of S. ■

Lists of criteria which a crisp approximation (or defuzzification) and a trapezoidal approximation operator on fuzzy numbers should or just possess were proposed in [21] and [16]. They include additivity, invariance to translations, scale invariance and continuity.

Corollary 1 *The operator $T_e : IF(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $T_e\,(A)$ is the nearest (with respect to distance $\widetilde{d}$) trapezoidal fuzzy number to intuitionistic fuzzy number A, preserving the expected interval of A, is invariant to translations, scale invariant and continuous.*

Proof. It is immediate from Theorem 10 taking into account the operator $t_e : F(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $t_e\,(u)$ is the nearest (with respect to distance d) trapezoidal fuzzy number to fuzzy number u, preserving the expected interval of u, is invariant to translations, scale invariant and continuous (see [7], [10]). ■

Corollary 2 *The operator $T_{a,v} : IF(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $T_{a,v}\,(A)$ is the nearest (with respect to distance $\widetilde{d}$) trapezoidal fuzzy number to intuitionistic fuzzy number A, preserving the ambiguity and value of A, is invariant to translations, scale invariant and continuous.*

Proof. It is immediate from Theorem 10 taking into account the operator $t_{a,v} : F(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $t_{a,v}\,(u)$ is the nearest (with respect to distance d) trapezoidal fuzzy number to fuzzy number u, preserving the value and ambiguity of u, is invariant to translations, scale invariant and continuous (see [11]). ■

Corollary 3 *The operator $T_c : IF(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $T_c\,(A)$ is the nearest (with respect to distance $\widetilde{d}$) trapezoidal fuzzy number to intuitionistic fuzzy number A, preserving the core of A, is invariant to translations and scale invariant.*

Proof. It is immediate from Theorem 10 taking into account the operator $t_c : F(\mathbb{R}) \rightarrow F(\mathbb{R})$, where $t_c\,(u)$ is the nearest (with respect to distance d) trapezoidal fuzzy number to fuzzy number u preserving the core of u, is invariant to translations and scale invariant (see [1]). ■

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475350
ISBN 838947535-9
9 788389 475350