Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

SRI PAS IBS PAN

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume I: Foundations

Systems Research Institute
Polish Academy of Sciences

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 9788389475350

Problem of monotonicity for decomposable operations

Paweł Drygaś
Institute of Mathematics, University of Rzeszów,
ul. Rejtana 16A, 35-310 Rzeszów, Poland
e-mail paweldr@univ.rzeszow.pl

Abstract

In this work we consider the problem of monotonicity of decomposable operations on some extensions of fuzzy set. We use binary operations on the lattice $([0,1],\leq)$ to construct these operations. For special cases of these components, i.e. t-norms we obtain intuitionistic t-norms. But in general case the components need not be monotonic. So, we consider the decomposable operation without such assumption.

Keywords: binary operations, t-representable operations, decomposable operations, monotonicity.

1 Introduction

In this paper we study the problem of monotonicity of decomposable operations on some extensions of fuzzy set. A characterization of such problem for operations is interesting not only from a theoretical point of view but also for their applications, since they have proved to be useful in several fields. In Section 2 we put the definitions of fuzzy sets, Atanassov intuitionistic fuzzy sets, interval-valued fuzzy sets and an L-fuzzy sets. Next, we recall relationships between these concepts. Since in L^I (cf. (3)) it is used natural order, we use such notation in the further consideration.

Recent Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2010.*

In Sections 3 we put a description of decomposable operations and next we consider a relationship between the monotonicity of decomposable operations and its components.

2 Intuitionistic and interval-valued fuzzy sets

The goal of this paper is to consider properties of monotonicity of decomposable operations on some extension of fuzzy set theory introduced in [11]. The fuzzy set theory turned out to be a useful tool to describe situations in which the data are imprecise or vague.

Definition 1 (cf. [11])**.** *A fuzzy set A on a universe X is a mapping*

$$A : X \to [0,1].$$

Fuzzy set describe the degree to which a certain point belongs to a set. A is also called a membership function or membership degree.

Intuitionistic fuzzy sets were introduced by Atanassov as an extension of the fuzzy sets in the following way.

Definition 2 (cf. [1], [2])**.** *An Atanassov intuitionistic fuzzy set A on a universe X is a triple*

$$A = \{(x, \mu(x), \nu(x)) : x \in X\}, \tag{1}$$

where $\mu, \nu : X \to [0,1]$ and $\mu(x) + \nu(x) \leq 1$, $x \in X$.

An Atanassov intuitionistic fuzzy set assigns to each element of the universe not only a membership degree $\mu(x)$ but also a nonmembership degree $\nu(x)$, $x \in X$.

An Atanassov intuitionistic fuzzy set A on X can be represented by an L^*-fuzzy set in the sense of Goguen. Namely

Definition 3 (cf. [7])**.** *An L-fuzzy set A on a universe X is a function $A : X \to L$ where L is a lattice.*

In this paper by $(L^*, \leq_{L^*})$ we mean the following complete lattice

$$L^* = \{(x_1, x_2) \in [0,1]^2 : x_1 + x_2 \leq 1\}, \tag{2}$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$.

Another extension of fuzzy sets are interval-valued fuzzy sets introduced independently by Sambuc (1975) and Gorzałczany (1987). In interval-valued fuzzy sets to each element of the universe a closed subinterval of the unit interval is assigned and this is the way of describing the unknown membership degree.

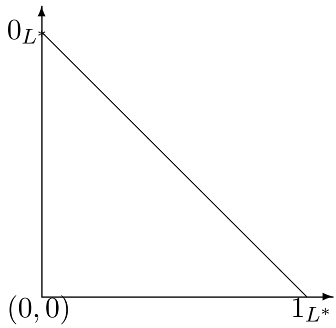

Figure 1: Lattice L^*

Definition 4 ((cf. [10], [8])). *An interval-valued fuzzy set A on X is a mapping*

$$A : X \to L^I,$$

where

$$L^I = \{[x_1, x_2] : x_1, x_2 \in [0,1] : x_1 \le x_2\}, \tag{3}$$

with following order

$$[x_1, x_2] \le_{L^I} [y_1, y_2] \Leftrightarrow x_1 \le y_1 \wedge x_2 \le y_2.$$

$(L^I, \le_L)$ is a complete lattice with operations

$$[x_1, x_2] \wedge [y_1, y_2] = [\min(x_1, y_1), \min(x_2, y_2)],$$

$$[x_1, x_2] \vee [y_1, y_2] = [\max(x_1, y_1), \max(x_2, y_2)].$$

and the boundary elements $1_{L^I} = [1, 1]$ and $0_{L^I} = [0, 0]$.

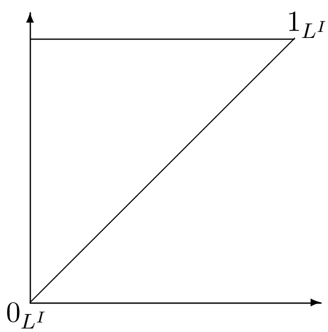

Figure 2: Lattice L^I

Deschrijver and Kerre [3] showed that Atanassov intuitionistic fuzzy sets are equivalent to interval-valued fuzzy sets. The isomorphism assign the Atanassov intuitionistic fuzzy set the interval value fuzzy set as follows: $(x, \mu_A(x), \nu_A(x)) \mapsto [\mu_A(x), 1 - \nu_A(x)]$.

In this article we will develop our investigations for $(L^I, \leq_L)$, since in this case it will be easier to prove the main result.

3 Decomposable operations

Now, we recall the crucial definition for investigations in this paper.

Definition 5 ([5]). *An operation $\mathcal{F} : (L^I)^2 \to L^I$ is called decomposable if there exist operations $F_1, F_2 : [0,1]^2 \to [0,1]$ such that for all $x, y \in L^I$*

$$\mathcal{F}(x, y) = [F_1(x_1, y_1), F_2(x_2, y_2)], \tag{4}$$

where $x = [x_1, x_2]$, $y = [y_1, y_2]$.

At first, before we will serve conditions defining when operation is decomposable we recall properties of monotonicity of binary operations.

Definition 6 (cf. [6]). *A binary operation $\mathcal{F}$ is increasing in $(L^I, \leq)$ if*

$$\underset{x,y,z \in L^I}{\forall} (x \leq y) \Rightarrow \quad \begin{array}{l} (\mathcal{F}(x,z) \leq \mathcal{F}(y,z), \\ \mathcal{F}(z,x) \leq \mathcal{F}(z,y)). \end{array} \tag{5}$$

The following Lemma characterize certain family of decomposable operations

Lemma 1 (cf. [5]). *Increasing operations $F_1, F_2 : [0,1]^2 \to [0,1]$ in* (4) *lead to the decomposable operation $\mathcal{F}$ if and only if $F_1 \leq F_2$.*

Special classes of operations in L^I are triangular norms and triangular conorms which are useful in approximate reasoning, e.g. in medical diagnosis and information retrieval.

Definition 7 (cf. [4], [9]). *A triangular norm $\mathcal{T}$ on L^I is an increasing, commutative, associative operation $\mathcal{T} : (L^I)^2 \to L^I$ with a neutral element 1_{L^I}.*
A triangular conorm $\mathcal{S}$ on L^I is an increasing, commutative, associative operation $\mathcal{S} : (L^I)^2 \to L^I$ with a neutral element 0_{L^I}.

Example 1. *We have the following examples of triangular norms on L^I:*

$$\inf(x, y) = [\min(x_1, y_1), \min(x_2, y_2)],$$
$$\mathcal{T}(x, y) = [\max(0, x_1 + y_1 - 1), \min(x_2, y_2)],$$

and triangular conorm on L^I

$$\sup(x, y) = [\max(x_1, y_1), \max(x_2, y_2)].$$

We use triangular norms from unit interval to construct triangular norms on the lattice L^I. So, we remind the definitions of such operations

Definition 8 ([9])**.** *A triangular norm T is an increasing, commutative, associative operation $T : [0, 1]^2 \to [0, 1]$ with a neutral element 1.*
A triangular conorm S is an increasing, commutative, associative operation S : $[0, 1]^2 \to [0, 1]$ with a neutral element 0.

Remark 1 (cf. [4])**.** *A decomposable triangular norm $\mathcal{T}$ on L^I is also called a t-representable triangular norm. In this case there exist triangular norms T_1 and T_2 on $[0, 1]$ such that for all $x, y \in L^I$*

$$\mathcal{T}(x, y) = [T_1(x_1, y_1), T_2(x_2, y_2)].$$

Remark 2 (cf. [4])**.** *A decomposable triangular conorm $\mathcal{S}$ on L^I is also called t-representable triangular conorm. In this case there exist a triangular conorms S_1 and S_2 on $[0, 1]$ such that for all $x, y \in L^I$*

$$\mathcal{S}(x, y) = [S_1(x_1, y_1), S_2(x_2, y_2)].$$

Example 2. *The operation*

$$\mathcal{T}(x, y) = [\max(x_1 + y_1 - 1, 0), \max(x_2 + y_2 - 1, 0)]$$

is a decomposable triangular norm with the Łukasiewicz triangular norm.

Of course if we consider a decomposable triangular norm then we have the monotonicity of components. In this situation the condition which let us to obtain decomposable operation is given in Lemma 1.

In general case the components need not be monotonic. So, we consider the decomposable operations without such assumption.

Theorem 1. *Operations $F_1, F_2 : [0, 1]^2 \to [0, 1]$ lead to the decomposable operation $\mathcal{F}$ if and only if*

$$\forall_{(x,y)\le(z,t)} F_1(x, y) \le F_2(z, t). \tag{6}$$

Proof. Let $F_1, F_2 : [0,1]^2 \to [0,1]$ fulfill (6), $\mathcal{F}$ be given by (4) and $[x_1, x_2]$, $[y_1, y_2] \in L^I$, i.e. $x_1 \leq x_2$, $y_1 \leq y_2$ then by (6) we have $F_1(x_1, y_1) \leq F_2(x_2, y_2)$. So $\mathcal{F}(x, y) = [F_1(x_1, y_1), F_2(x_2, y_2)] \in L^I$.

Let $x = [x_1, x_2]$, $y = [y_1, y_2] \in L^I$, i.e. $x_1 \leq x_2$, $y_1 \leq y_2$ and $\mathcal{F}$ be given by (4) where $F_1, F_2 : [0,1]^2 \to [0,1]$. If $\mathcal{F}$ is decomposable operation, then $\mathcal{F}(x, y) = [F_1(x_1, y_1), F_2(x_2, y_2)] \in L^I$, i.e. $F_1(x_1, y_1) \leq F_2(x_2, y_2)$ which is equivalent to the condition (6). □

Corollary 1. *If operation F_1 and F_2 are increasing, then conditions* (6) *and $F_1 \leq F_2$ are equivalent. So, we obtain results from Lemma 1.*

Theorem 2. *The decomposable operation $\mathcal{F}$ is increasing if and only if operations $F_1,\ F_2 : [0,1]^2 \to [0,1]$ are increasing.*

Proof. Let $\mathcal{F}$ be increasing and $x_1 \leq z_1$, $y_1 \leq t_1$, $x_2 \leq z_2$, $y_2 \leq t_2$. Let $x = [x_1, x_2]$, $y = [y_1, y_2]$, $z = [z_1, z_2]$, $t = [t_1, t_2]$. Then $x \leq z$, $y \leq t$ and by the monotonicity of $\mathcal{F}$ we have

$$[F_1(x_1, y_1), F_2(x_2, y_2)] = \mathcal{F}(x, y) \leq \mathcal{F}(z, t) = [F_1(z_1, t_1), F_2(z_2, t_2)].$$

Directly from the above we obtain $F_1(x_1, y_1) \leq F_1(z_1, t_1)$ and $F_2(x_2, y_2) \leq F_2(z_2, t_2)$. It means that F_1 and F_2 are increasing.
Conversely, $F_1,\ F_2 : [0,1]^2 \to [0,1]$ are increasing and $x \leq z$, $y \leq t$. So, $x_1 \leq z_1$, $y_1 \leq t_1$, $x_2 \leq z_2$, $y_2 \leq t_2$. Then $F_1(x_1, y_1) \leq F_1(z_1, t_1)$, $F_2(x_2, y_2) \leq F_2(z_2, t_2)$. Thus $\mathcal{F}(x, y) \leq \mathcal{F}(z, t)$ which means that $\mathcal{F}$ is increasing. □

4 Conclusion

In our further considerations we will check the preservation of some properties by decomposable operations if we not assume the monotonicity.

For checking the preservation of the associativity, commutativity, distributivity, idempotency and existence of the neutral element we don't use the properties of monotonicity (it is used only for conditions which guarantee the existence of decomposable operations). So, these properties are preserved by decomposable operation without the assumption of monotonicity. The most interesting problem is to consider preservation of the properties which use an inequality.

In a similar way we investigate whether the properties considered in this article are preserved by pseudo-t-representable operations.

Definition 9 ([4])**.** *A triangular norm $\mathcal{T}$ (triangular conorm $\mathcal{S}$) is called pseudo-t-representable if*

$$\mathcal{T}(x, y) = [T(x_1, y_1), \max(T(x_1, y_2), T(x_2, y_1))] \tag{7}$$

$$(\mathcal{S}(x,y) = [\min(S(x_1,y_2), S(x_2,y_1)), S(x_2,y_2)]) \,. \tag{8}$$

Remark 3. *If in* (7) *and* (8) *we use monotonic operations on* $[0,1]$ *instead triangular norm* T *and triangular conorm* S*, then we obtain the definition of pseudo decomposable operations on* L^I.

In our further considerations we will check what additional assumptions for pseudo-decomposable operations guarantee the preservation of monotonicity and other properties.

Acknowledgement

This work is partially supported by the Ministry of Science and Higher Education Grant Nr N N519 384936.

References

[1] Atanassov K.T. (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96.

[2] Atanassov K.T. (1999) Intuitionistic Fuzzy Sets. Springer-Verlag, Heidelberg.

[3] Deschrijver G., Kerre E.E. (2003) On the relationship beetwen some extensions of fuzzy set theory. Fuzzy Sets Syst. 133, 227–235.

[4] Deschrijver G., Cornelis C., Kerre E.E. (2004) On the Representation of Intuitonistic Fuzzy t-Norms and t-Conorms. IEEE Trans. Fuzzy Syst. 12, 45–61.

[5] Drygaś P., Pękala B. (2008) Properties of decomposable operations on same extension of the fuzzy set theory. In: K.T. Atanassov, O. Hryniewicz, J. Kacprzyk, M. Krawczak, Z. Nahorski, E. Szmidt, S. Zadrożny (eds), Advances in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations, EXIT, Warszawa 2008, 105-118.

[6] Fuchs L. (1963) Partially Ordered Algebraic Systems, Pergamon Press, Oxford.

[7] Goguen A. (1967) L-fuzzy sets. J. Math. Anal. Appl. 18, 145–174.

[8] Gorzałczany M.B. (1987) A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets Syst. 21 (1), 1-17.

[9] Klement E.P., Mesiar R., Pap E. (2000) Triangular norms. Kluwer Acad. Publ., Dordrecht.

[10] Sambuc R. (1975) Functions ϕ-floues. Application á l'aide au diagnostic en pathologie thyroidienne. Ph. D. Thesis, Universitd́e Marseille, France.

[11] Zadeh L.A. (1965) Fuzzy sets. Information and Control, 8, 338–353.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

http://www.ibspan.waw.pl/ifs2010

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475350
ISBN 838947535-9
9 788389 475350