Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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Systems Research Institute
Polish Academy of Sciences

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Conditional distributivity of increasing binary operations

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Abstract

The problem of distributivity is of a great interest both for particular as well as fundamental reasons. This relates for instance to the theory of binary operations like triangular norms, conorms and their generalizations, i.e. uninorms and nullnorms. This paper is precisely devoted to solve conditional distributivity of monotonic operations with neutral element. We simplify here some assumptions and results about conditional distributivity and distributivity of uninorms and triangular operations (norms and conorms) in four cases. In particular, the assumptions of associativity and commutativity are not necessary in consideration of these functional equations.

Keywords: fuzzy connectives, uninorm, conditional distributivity equation, idempotent operation.

1 Introduction

The problem of distributivity has been posed many years ago and it has many diverse aspects (see [1, 3, 4]). Currently, there are many new directions of examinations related to distributivity of different generalizations of logical connectives used in fuzzy set theory, e.g. triangular norms and conorms ([2, 6]); aggregation

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functions, quasi-arithmetic means [7]; fuzzy implications ([5], [20], [21]); uninorms and nullnorms ([10, 15, 16, 22]). Recently authors deal with solutions of the distributivity inequalities and domination property (cf. e.g. [11, 23, 24]).

This paper is mainly devoted to pairs of distributive weak algebraic operations in fuzzy set theory. Section 2 discusses basic concepts of families of binary operations on the unit interval $[0,1]$ such as triangular norms, triangular conorms, uninorms and their generalizations. Characterizations of such binary operations are interesting not only from a theoretical point of view but they are also useful for example in utility theory [13], fuzzy logic framework [12] or neural networks [14]. In sections 3 the left and right distributivity equations are recalled. The basic facts about the (conditional) distributivity equations of binary operations are presented. The last sections (4, 5) deal with the solutions of conditional distributivity and distributivity for pairs of operations from the described families.

2 Increasing binary operation with unit

We start with basic definitions and facts.

Definition 1 ([10]). *Let $e \in [0,1]$. By N_e we denote the family of all operations $F : [0,1]^2 \to [0,1]$ which are increasing with respect to both variables and have neutral element $e \in [0,1]$ i.e.*

$$\forall_{x\in[0,1]}\ F(e,x) = F(x,e) = x.$$

Definition 2 (cf. [26]). *An operation $F \in N_e$ is called uninorm if it is associative and commutative.*

We use the following notation $D_e = [0,e) \times (e,1] \cup (e,1] \times [0,e)$ for $e \in (0,1)$.

Theorem 1 ([10]). *Let $e \in [0,1]$. $F \in N_e$ iff*

$$F = \begin{cases} A & in\ [0,e]^2 \\ B & in\ [e,1]^2 \\ C & in\ D_e \end{cases},$$

where $A : [0,e]^2 \to [0,e]$, $B : [e,1]^2 \to [e,1]$ are increasing with neutral element e and $C : D_e \to [0,1]$ is also increasing and fulfils

$$\min(x,y) \le C(x,y) \le \max(x,y)\ \ for\ \ (x,y) \in D_e.$$

Corollary 1. *Operations A and B from Theorem 1 fulfil* $0 \le A \le \min$, $\max \le B \le 1$.

Definition 3 ([10])**.** *Let $e \in [0,1]$. By $N_e^{\max}$ ($N_e^{\min}$) we denote the family of all operations $F \in N_e$ fulfilling the additional condition:*

$$\forall_{x\in(e,1]}\, F(0,x) = F(x,0) = x,\ (\forall_{x\in[0,e)}\, F(1,x) = F(x,1) = x). \tag{1}$$

Remark 1. *Any operation $F \in N_e$ has zero elements $s = 0$ in $[0,e]$ and $s = 1$ in $[e,1]$. If $e > 0$ then any operation $F \in N_e^{\min}$ has zero element $s = 0$. If $e < 1$ then any operation $F \in N_e^{\max}$ has zero element $s = 1$. Moreover $N_0^{\min} = N_0$, $N_1^{\min} = N_1$, $N_1^{\max} = N_1$, $N_0^{\max} = N_0$.*

Theorem 2 ([10])**.**

$$F \in N_e^{\min} \Leftrightarrow F = \begin{cases} A & in\ [0,e]^2 \\ B & in\ [e,1]^2 \\ \min & in\ D_e \end{cases},\ F \in N_e^{\max} \Leftrightarrow F = \begin{cases} A & in\ [0,e]^2 \\ B & in\ [e,1]^2 \\ \max & in\ D_e \end{cases},$$

where $A : [0,e]^2 \to [0,e]$, $B : [e,1]^2 \to [e,1]$ are increasing with neutral element e.

Theorem 3 (cf.[25], Theorem 4.5 and Theorem 4.7)**.** *Let $e \in [0,1]$. Operations*

$$U_e^{\min} = \begin{cases} \max & in\ [e,1]^2 \\ \min & otherwise \end{cases},\ U_e^{\max} = \begin{cases} \min & in\ [0,e]^2 \\ \max & otherwise \end{cases} \tag{2}$$

are unique idempotent uninorms in $N_e^{\min}$ and $N_e^{\max}$, respectively.

Lemma 1 (cf.[8], Theorem 2)**.** $F = \min$ *is the unique idempotent operation from the family N_1.* $F = \max$ *is the unique idempotent operation from the family N_0.*

3 Functional equations of distributivity

We consider here the functional equations of distributivity and conditional distributivity of two binary operations. Let us remind some of the most important facts relating to this topic.

Definition 4. *(cf. [1], p. 318) Let $F, G : [0,1]^2 \to [0,1]$. Operation F is distributive over G, if they fulfil the left and right distributivity conditions:*

$$\forall_{x,y,z\in[0,1]}\ F(x, G(y,z)) = G(F(x,y), F(x,z)), \tag{3}$$

$$\forall_{x,y,z\in[0,1]}\ F(G(y,z),x) = G(F(y,x),F(z,x)) \tag{4}$$

(i.e. F is left distributive and right distributive over G).

Operation F is left and right conditionally distributive over G, if they fulfil (3) *and* (4) *with appropriate additional condition.*

Lemma 2 (cf. [9], Lemma 3)**.** *If operation $F : [0,1]^2 \to [0,1]$ with neutral element $e \in (0,1)$ is left or right distributive (conditionally distributive) over operation $G : [0,1]^2 \to [0,1]$ fulfilling $G(e,e) = e$, then G is idempotent.*

Lemma 3 ([17])**.** *Every increasing operation $F : [0,1]^2 \to [0,1]$ is distributive (conditionally distributive) over* $\max$ *and* $\min$.

4 Conditional distributivity between $F \in N_e$ and $G \in \{N_0, N_1\}$

At first our consideration will concern the conditional distributivity of $F \in N_e^{\max}$ and $F \in N_e^{\min}$ over $G \in N_0$ and/or $G \in N_1$.

Definition 5 (cf. [22])**.** *Let $F \in N_e$ and $G \in N_0$. We say that operation F is conditionally distributive over G, if*

$$\forall_{x,y,z\in[0,1]}\ F(x,G(y,z)) = G(F(x,y),F(x,z)),\ \textit{whenever}\ G(y,z) < 1. \tag{5}$$

Lemma 4. *Let $F \in N_e$ with neutral element $e \in (0,1)$ and $G \in N_0$. If F is left or right conditionally distributive over G fulfilling $G(e,e) = e$, then $G = \max$.*

Proof. Directly from Lemma 2 and Lemma 1 we obtain that $G = \max$. □

Theorem 4. *Let $e \in (0,1)$. Operation $F \in N_e^{\max}$ is left or right conditionally distributive over $G \in N_0$ if and only if $G = \max$.*

Proof. Let operation $F \in N_e^{\max}$ be left conditionally distributive over $G \in N_0$. First we show that $G(e,e) = e$. Taking $y = z = 0$ in equation (5), by (1) we get $x = F(x,0) = F(x,G(0,0)) = G(F(x,0),F(x,0)) = G(x,x)$ for all $x > e$. Now, by increasingness of G we have $e = G(0,e) \le G(e,e)$, because $e > 0$. Moreover, $G(e,e) \le G(e,x) \le G(x,x) = x$ for $x \in (e,1]$. Thus $G(e,e)$ is a lower bound of $(e,1]$, i.e. $G(e,e) \le e = inf(e,1]$.
Hence $G(e,e) = e$. Then by Lemma 4 we obtain $G = \max$.

Conversely, according to Lemma 3 condition (5) holds immediately. In a similar way we can prove the case of the right conditional distributivity equation. □

Theorem 5. *Let $e \in (0,1)$. Operation $F \in N_e^{\min}$ is left or right conditionally distributive over a continuous operation $G \in N_0$ if and only if $G = \max$.*

Proof. Let operation $F \in N_e^{\min}$ be left conditionally distributive over a continuous operation $G \in N_0$. We already know that $G(e,e) \geq G(e,0) = e$. Let us suppose that $G(e,e) > e$. Since G is continuous, there exist $z < e$ such that $G(z,z) = e < 1$. Applying $y = z$ in (5) we have $x = F(x,e) = F(x, G(z,z)) = G(F(x,z), F(x,z))$ for all $x \in [0,1]$. Now taking $x > e > z$ we get $x = G(\min(x,z), \min(x,z)) = G(z,z) = e$, which is a contradiction. Thus $G(e,e) = e$ and by Lemma 4 we obtain $G = \max$.

Conversely, according to Lemma 3 condition (5) holds immediately. Proof of the right conditional distributivity equation is analogous. □

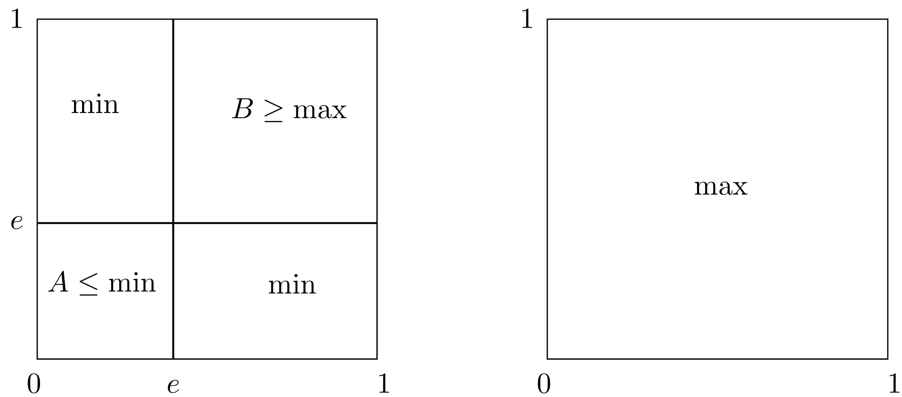

Figure 1: Structure of conditionally distributive operations from Theorem 5.

Example 1. *Operation $F \in N_{\frac{1}{4}}^{\min}$ (which is not associative and commutative) given by the formula*

$$F(x,y) = \begin{cases} xy, & if \ \ (x,y) \in [0, \frac{1}{4}) \times [0, \frac{1}{8}] \\ \max, & if \ \ (x,y) \in [\frac{1}{4}, 1] \\ \min & otherwise \end{cases}$$

is conditionally distributive over maximum operation.

Theorem 6. *Let $e \in (0,1)$. An idempotent operation $F \in N_e$ is left or right conditionally distributive over a continuous operation $G \in N_0$ if and only if $G = \max$.*

Proof. Let idempotent operation $F \in N_e$ be left conditionally distributive over a continuous operation $G \in N_0$. As G is continuous, there exist $z \leq e$ such that $G(z,z) = e < 1$. Taking $x = y = z$ in (5) because of idempotency of F we get $z = F(z,e) = F(z,G(z,z)) = G(F(z,z),F(z,z)) = G(z,z) = e$. Thus $G(e,e) = e$. Then directly from Lemma 4 we obtain $G = \max$.

Conversely, according to Lemma 3 condition (5) holds immediately. In a similar way we can prove the case of the right conditional distributivity equation. □

If we consider the conditional distributivity equations for operations $F \in N_e$ and $G \in N_1$ we obtain results similar as the above.

Definition 6 ([22]). *Let $F \in N_e$ and $G \in N_1$. We say that operation F is conditionally distributive over G, if*

$$\forall_{x,y,z\in[0,1]}\ F(x,G(y,z)) = G(F(x,y),F(x,z)), whenever\ G(y,z) > 0. \quad (6)$$

Lemma 5. *Let $F \in N_e$ with neutral element $e \in (0,1)$ and $G \in N_1$. Then if F is left or right conditionally distributive over G fulfilling $G(e,e) = e$, then $G = \min$.*

Theorem 7. *Let $e \in (0,1)$. Operation $F \in N_e^{\min}$ is left or right conditionally distributive over $G \in N_1$ if and only if $G = \min$.*

Theorem 8. *Let $e \in (0,1)$. Operation $F \in N_e^{\max}$ is left or right conditionally distributive over a continuous operation $G \in N_1$ if and only if $G = \min$.*

Theorem 9. *Let $e \in (0,1)$. An idempotent operation $F \in N_e$ is left or right conditionally distributive over a continuous operation $G \in N_1$ if and only if $G = \min$.*

5 Distributivity between $F \in \{N_0, N_1\}$ and $G \in N_e$

In this part we present the solutions of distributivity equations for operations $F \in N_0$ ($F \in N_1$) and $G \in N_e$.

Lemma 6. *Let $e \in (0,1)$. If $F \in N_0$ is left or right distributive over $G \in N_e$, then $G = U_e^{\max}$ (cf. (2)).*

Proof. Let $F \in N_0$ and $G \in N_e$, $e \in (0,1)$. Taking $y = z = 0$ in (3) we have $x = F(x,0) = F(x,G(0,0)) = G(F(x,0),F(x,0)) = G(x,x)$ for $x \in [0,1]$, which proves that G is idempotent.

Now, suppose that $G(0,z) = 0$ for some $e < z < 1$. Then for $x = e$ and $y = 0$ in (3) we get $e = F(e,0) = F(e, G(0,z)) = G(F(e,0), F(e,z)) = G(e, F(e,z)) = F(e,z)$. However, $F \in N_0$ and for $e > 0$ we have $F(e,z) \geq F(0,z) = z > e$, which leads to a contradiction. Therefore, $G(0,z) = z$ for all $z > e$ and then $G \in N_e^{\max}$. Now, on account of idempotency of operation G we obtain that $G = U_e^{\max}$. Similarly we can consider condition (4). $\square$

Lemma 7. *Let $e \in (0,1)$. If $F \in N_0$ is left or right distributive over $G \in N_e$, then $F(x,y) = \max(x,y)$ for x, y fulfilling the condition* $\min(x,y) \leq e \leq \max(x,y)$.

Proof. Let us take x and y satisfying $\min(x,y) \leq e \leq \max(x,y) \leq F(x,y)$. Then using assumptions, Remark 1 for operation G and condition (3) we get

$$\begin{aligned} \max(x,y) &= F(\max(x,y), 0) = F(\max(x,y), G(\min(x,y), 0)) \\ &= G(F(\max(x,y), \min(x,y)), F(\max(x,y), 0)) = G(F(x,y), \max(x,y)) \\ &\geq G(F(x,y), e) = F(x,e). \end{aligned}$$

Thus we obtain $F(x,y) = \max(x,y)$ for all (x,y) satisfying $\min(x,y) \leq e \leq \max(x,y)$. $\square$

Now we have the following result

Theorem 10. *Let $e \in (0,1)$. $F \in N_0$ is left or right distributive over $G \in N_e$ if and only if $F(x,y) = \max(x,y)$ for $\min(x,y) \leq e \leq \max(x,y)$ and $G = U_e^{\max}$.*

Proof. Let $F \in N_0$ be left distributive over $G \in N_e$. Then directly from Lemmas 6 and 7 we get $F(x,y) = \max(x,y)$, for $\min(x,y) \leq e \leq \max(x,y)$ and $G = U_e^{\max}$.
Conversely, let $G = U_e^{\max}$ and $F \in N_0$ satisfy $F(x,y) = \max(x,y)$ if $\min(x,y) \leq e \leq \max(x,y)$. Now, since $G(x,x) = x$ for $x \in [0,1]$ and $F(x,y) = F(x,z)$, then (3) is fulfilled.
If $F(x,y) \neq F(x,z)$, suppose first that $y < z$. In this case we consider three cases:
• if $y < e < z$ we have $F(x, G(y,z)) = F(x, \max(y,z)) = F(x,z) = \max(F(x,y), F(x,z)) = G(F(x,y), F(x,z))$,
• if $y < z \leq e$ and $x < e$ we have $F(x, G(y,z)) = F(x, \min(y,z)) = F(x,y) = \min(F(x,y), F(x,z)) = G(F(x,y), F(x,z))$,
• if $y < z \leq e$ and $x \geq e$ we have $F(x, G(y,z)) = F(x, \min(y,z)) = F(x,y) = x = G(x,x) = G(F(x,y), F(x,z))$.
In all cases condition (3) is satisfied. In the case where $z \leq y$ and for condition (4) the proof is similar. $\square$

We can write this theorem in the following equivalent way

Theorem 11. *Let $e \in (0,1)$. $F \in N_0$ is left or right distributive over $G \in N_e$ if and only if F is an ordinal sum of the form*
$F = (< 0, e, F_1 >, < e, 1, F_2 >)$ *and* $G = U_e^{\max}$.

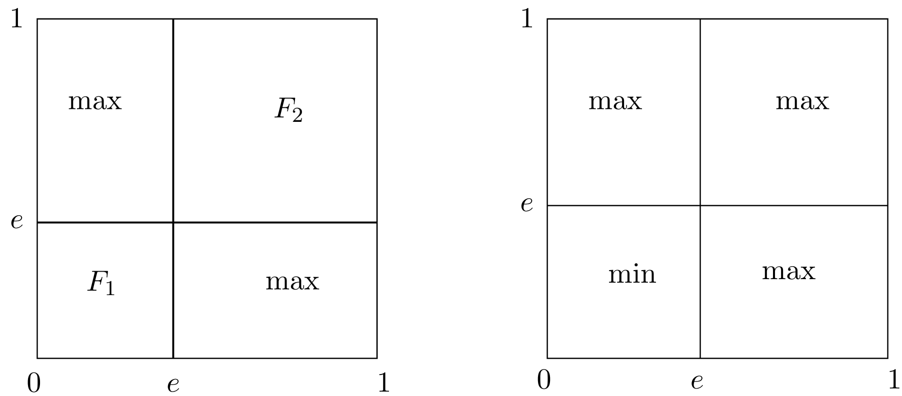

Figure 2: Distributive operations from Theorem 11.

Next we deal with the distributivity between $F \in N_1$ and operation $G \in N_e$. Similarly as in above results one has

Lemma 8. *Let $e \in (0,1)$. If $F \in N_1$ is left or right distributive over $G \in N_e$, then $G = U_e^{\min}$ (cf. (2)).*

Lemma 9. *Let $e \in (0,1)$. If $F \in N_1$ is left or right distributive over $G \in N_e$, then $F(x,y) = \min(x,y)$, for x, y fulfilling the condition*
$\min(x,y) \leq e \leq \max(x,y)$.

Theorem 12. *Let $e \in (0,1)$. $F \in N_1$ is left or right distributive over $G \in N_e$ if and only if $F(x,y) = \min(x,y)$ for $\min(x,y) \leq e \leq \max(x,y)$ and $G = U_e^{\min}$.*

Equivalently

Theorem 13. *Let $e \in (0,1)$. $F \in N_1$ is left or right distributive over $G \in N_e$ if and only if F is an ordinal sum of the form*
$F = (< 0, e, F_1 >, < e, 1, F_2 >)$ *and* $G = U_e^{\min}$.

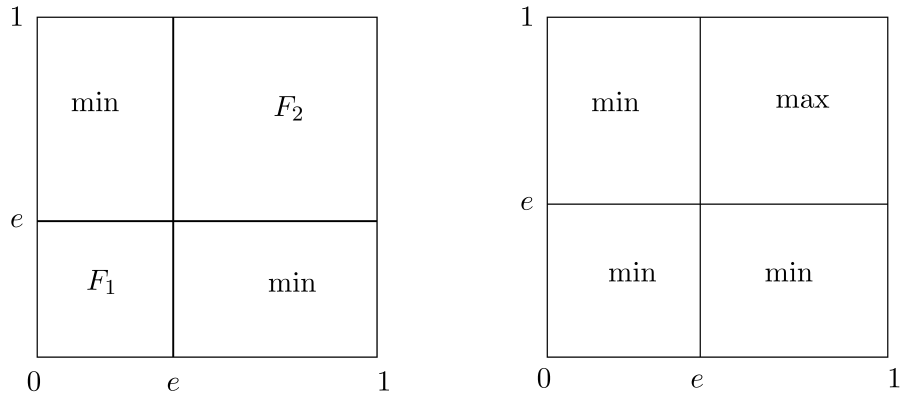

Figure 3: Distributive operations from Theorem 13.

6 Conclusion

In this presentation we gave the partial characterization of conditional distributivity and distributivity equations for binary operations
$F \in N_e^{\min} \cup N_e^{\max}$, $G \in N_1 \cup N_0$ and conversely.
The considerable part of the results connected with distributivity in case $F, G \in \mathcal{F}_e$, where $e \neq 0, 1$ was already published (see [11], [18], [19]), next paper was submitted.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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