POLISH ACADEMY OF SCIENCES
SYSTEMS RESEARCH INSTITUTE

THE INTERNATIONAL ECONOMIC COOPERATION

THEORETICAL FOUNDATIONS

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PREFACE

The main difference between the work here presented and the other studies related to the same, generally speaking, domain, consists in the fact that considerations contained in this book indicate the possibility of resolving questions concerning the choice of the subject and establishment of profitability of international trade and cooperation in conditions when:

* prices on the internal market do not correspond to social costs,

* there is lack of conviction as to correctness of exchange rates,

* prices in international trade are subject to manipulations, resulting from definite interests of some countries, or they simply cannot follow the development of world production system.

As can easily be noticed these are just the conditions in which currently the international trade and cooperation system is being shaped. These particular conditions result, for instance, from governmental subsidies oriented at individual commodities or groups of commodities (e.g. food products), from existing custom tax barriers and from an extremely quick pace of technological progress in the techniques of production.

INTRODUCTION

The problem of international exchange was presented for the first time in precise mathematical terms by Wassily Leontief in his paper entitled "Factor Proportions and the Structure of American Trade", published in *Review of Economics and Statistics* (1956, vol. 38, no. 4).

The first mathematical approach to the problem presented in Poland, was of international industrial cooperation formulated in the Doctoral dissertation of Andrzej Ameljańczyk (Military Technical Academy, 1975), supervised by this author.

Earlier, a similar formulation of the problem of international trade exchange had been forwarded in the Doctoral dissertation of J.Kotyński (Main School of Planimg and Statistics, Warsaw, 1968).

If we distinquish the specific problem of international economic cooperation within the broader domain of international trade exchange then the first monograph devoted entirely to economic cooperation is the book in Polish by S.Piasecki, J.Hołuniec and A.Ameljańczyk, entitled "International economic cooperation - Modelling and Optimization" (PWN, Warsaw-Łódź, 1982).

The assumption of complementarity of goods, characteristic for the problem of cooperation, was first introduced by D.Graham in 1923 in his paper "The Theory of International Values Examined" (Quarterly Journal of Economics, vol. 38, no.1).

The present publication contains the original results of studies conducted during the years 1982-1985, being a continuation of work started a dozen years before.

Models of international cooperation considered there (see Chapters 1 to 3) were much simpler than in the ones presented here. Still, they are, alas, only theoretical models, which cannot be practically applied in economic activity.

Notwithstanding this situation, the models give certain possibilities with respect to applications. I am convinced that further in-depth studies in and broadening of the theory presented here will make out of it in the future a perfect instrument for economic practice. I think that conclusions resulting from it may contribute to quicker reequilibration of the international economic system, which has been put so much off the equilibrium by the existing debts.

Against the background of existing numerous publications dealing with international trade and cooperation, as well as international specialization, the theory here presented does not require acceptance of the commonly up to date adopted assumption concerning economic equilibria within the cooperating countries, and, furthermore, this theory has much greater practical potential than the previous theories, in which it has been necessary to assume existence of economic equilibrium prices for comparing profitability of trade.

Since the theory presented in this book is independent of existence of prices, it can also be used in determination of the price structure of goods included in the trade, profitable for the partners in such an international trade deal. Thus, the structure determined ("terms of trade") guarantees stimulation of international cooperation and improvement of international specialization.

On the other hand, the theory can also be used in deciding whether the structure of prices actually existing in the international market is enhancing or, to the contrary, hindering, the development of trade, whether it does not lead to an unsound development of some of the partners at the expense of the other ones. It is not difficult to realize that the theory presented, and especially its results, concern one of the essential economic problems of present time.

The theory has, indeed, its weak points as well. A number of technical simplifying assumptions put aside (their number shall be decreasing as the theory develops), there is one fundamental assumption. It says that every participant of cooperation relation (of international trade) tries to produce the maximum of necessary goods of a given structure, entering the group considered.

When these ones are consumption goods, we are dealing with the situation, when every partner (every national economy) participating in international exchange, is geared towards maximization of the living standard of own population, given a consumption structure characteristic for this population.

When, however, these are not consumption goods, but, e.g. semi-products, then this corresponds to the situation in which every participant-producer tries to maximize own production, this production determining the structure of demand for semi-products encompassed by cooperation. From this point of view the theory presented may get applied beyond the domain of international cooperation.

Technical simplifications adopted in the book result from the wish of possibly clear and understandable presentation of the theory. Thus, wanting to show graphically the mechanism of cooperation and to illustrate the results of the theory, the present author emphasizes in the book bilateral cooperation encompassing only two kinds or groups of commodities. Analysis of the thus simplified problem is contained in first seven chapters of the book.

The eighth chapter is in a way a generalization of considerations presented in the previous chapters so as to account for the case of multilateral cooperation, involving multiple goods. This chapter may constitute a separate whole - a summary of the contents of the book.

Note, then, that we can express the product $b_{12}b_{21}$ in the following manner:

$$b_{12}\cdot\frac{1}{\frac{1}{b_{21}}} = \frac{b_{12}}{q_{21}}, \quad \text{where} \quad q_{21} = \frac{1}{b_{21}}$$

with q_{21} denoting the "energy effectiveness" of coal, meaning the amount of energy (in kilowatt hours) produced out of one unit (ton) of coal (when the amount of coal used up in producing one unit of energy is b_{21}).

The quotient b_{12}/q_{21} can be interpreted therefore, as the amount of energy used up in producing one ton of coal divided by the quantity of energy obtained from this ton of coal.

It is therefore obvious that for all the domains of economy this quotient must be less - or even much less - than 1. It is only namely then that a pair of coupled economic activities can bring a positive economic effect.

In our example this pair produces coal or energy, or - coal and energy. Thus, if the use of electric power per unit of coal produced was greater than the amount of electricity which can be obtained from this unit of coal then such a system would have to be supplied from outside with energy since the coal produced would not compensate for the energy used up.

Summing up we can assume that in the real economic systems the product $b_{12}b_{21}$ is always less than one, so that the quantity $1-b_{12}b_{21}$ is strictly positive.

4. A MODEL OF AN OPEN ECONOMY WITH UNLIMITED EXCHANGE CAPACITIES

We shall consider now the previous case assuming, however, that there exists a possibility of buying from an external market both products - "1" and "2", with no limitations (besides the necessity of balancing exports and imports), with their prices being, respectively, C_1 and C_2. The statement that there are no limitations on purchases of commodities "1" and "2", with prices C_1 and C_2, is equivalent to the assumption that buying of "1" and "2" for our purposes is possible, that it does not exceed the capacities of our partners in the market (meaning that these quantities are much smaller than total trade volumes in the market), and that it would not lead to price increase in the market - would not disequilibrate the supply/demand system in a significant way. This assumption is also equivalent to accepting that in case we were selling goods we have produced in surplus quantities the market is capable of absorbing this surplus without a drop in the price of the goods we would be selling.

Since the accurate values of the goods considered are of lesser importance, while we are primarily interested in their relation, the "terms of trade", then we put:

$$\omega_2 = \frac{C_2}{C_1}$$

If we now denote by μ_i the magnitude of exports (when $\mu_i>0$) or imports (when $\mu_i<0$) in a year, then the requirement of financial balancing of foreign trade is expressed as the equality

$$\mu_1 C_1 + \mu_2 C_2 = 0 ,$$

where

$$\mu_1 = \varepsilon_1 - \beta_1 = \alpha_1 - \alpha_2 b_{21} - \beta_1$$

$$\mu_2 = \varepsilon_2 - \beta_2 = \alpha_2 - \alpha_1 b_{12} - \beta_2$$

with condition

$$\frac{\beta_1}{\beta_2} = \gamma_2$$

On the other hand the requirement of maximum use of production capacities is expressed by the equality

$$\frac{\alpha_1}{A_1} + \frac{\alpha_2}{A_2} = 1$$

Since, from the first equality, we have

$$\mu_1 = -\mu_2\omega_2$$

then, after substituting this expression and the equality

$$\beta_2 = \beta_1\gamma_2$$

in the formulae defining the magnitude of exports (or imports) of goods we get

$$-\mu_2\omega_2 = \alpha_1 - \alpha_2 b_{21} - \beta_1$$
$$\mu_2 = \alpha_2 - \alpha_1 b_{12} - \beta_1\gamma_2$$

or, having solved this with respect to μ_2:

$$\alpha_1(1-\omega_2 b_{12}) - \alpha_2(b_{21}-\omega_2) = \beta_1(1+\omega_2\gamma_2)$$

The, solving this latter equation together with equation

$$\alpha_1 A_2 + \alpha_2 A_1 = A_1 A_2$$

we get

$$\alpha_1 = A_1 \cdot \frac{1 + \frac{\beta_1}{A_2} \cdot \frac{1 + \omega_2\gamma_2}{b_{21} - \omega_2}}{1 + \frac{A_1}{A_2} \cdot \frac{1 - \omega_2 b_{12}}{b_{21} - \omega_2}} = \frac{A_2(b_{21}-\omega_2) + \beta_1(1+\omega_2\gamma_2)}{1 + \frac{A_2}{A_1}b_{21} - \omega_2(b_{12} + \frac{A_2}{A_1})}$$

$$\alpha_2 = A_2 \cdot \frac{1 + \frac{\beta_1}{A_2} \cdot \frac{1 + \omega_2 \gamma_2}{1 - \omega_2 b_{12}}}{1 + \frac{A_2}{A_1} \cdot \frac{b_{21} - \omega_2}{1 - \omega_2 b_{12}}} = \frac{A_1(1-\omega_2 b_{12}) - \beta_1(1+\omega_2\gamma_2)}{1 + \frac{A_2}{A_1} b_{21} - \omega_2(b_{12} + \frac{A_2}{A_1})} \cdot \frac{A_2}{A_1}$$

Graphical solution to this problem is shown in Figure 3.

This solution determines the magnitudes of production of goods "1" and "2", i.e. α_1, α_2, such that their "consumption" β_1, β_2 have a given structure γ_2, that imports (purchases) be financially fully compensated by the exports, and the productive potential be fully utilized.

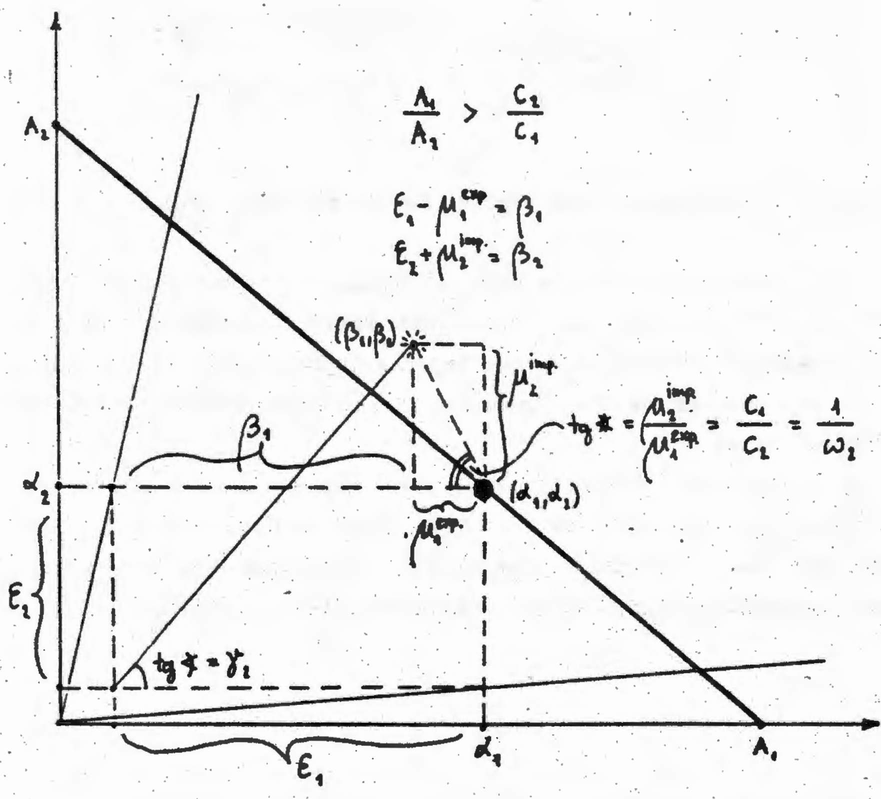
We would like, though, to have the value β_1 of consumption for satisfaction of demand possibly high, just as we postulated in the previous case. In order to determine this value let us consider the problem assuming that the inequality given below holds:

$$\frac{A_1}{A_2} \geq \omega_2$$

This assumption is in fact inessential, for it only requires such a numbering of the two goods considered that the inequality

$$\frac{A_1}{A_2} \geq \frac{C_2}{C_1}$$

holds. Adoption of such an indexing of commodities facilitates further considerations, and also makes it simpler to graphically illustrate the method of solving the problem of maximization of quantity β_1.



$\frac{A_1}{A_2} > \frac{C_2}{C_1}$
$\varepsilon_1 - \mu_1^{exp} = \beta_1$
$\varepsilon_2 + \mu_2^{imp} = \beta_2$
(β_1, β_2)
μ_2^{imp}
$tg\,\sphericalangle = \frac{\mu_2^{imp}}{\mu_1^{exp}} = \frac{C_1}{C_2} = \frac{1}{\omega_2}$
β_1
A_2
α_2
(α_1, α_2)
μ_1^{exp}
ε_2
$tg\,\sphericalangle = \gamma_2$
ε_1
α_1
A_1

Figure 3.

Let us turn attention at Figure 3, in which indexing was adopted satisfying the above inequality. Analysing possible locations of the point α_1, α_2 on the straight line limiting production, which passes through points $(0, A_2)$ and $(A_1, 0)$, we can conclude that β_1 will grow when α_1 grows and α_2 decreases.

Since relations involved are linear, it would be the best from the point of view of maximization of β_1 to take values $\alpha_1 = A_1$ and $\alpha_2 = 0$ or a maximum value $\alpha_1 < A_1$, the latter together with

$$\alpha_2 = \alpha_1 b_{12}$$

a. Consider the first case, i.e. when

$$\alpha_1 = A_1, \ \alpha_2 = 0$$

For this case, in order to ensure the given magnitude of production of commodity "1", we need to import, for production purposes, $A_1 b_{12}$ of commodity "2", irrespective of imports meant to satisfy internal consumption demand, $\beta_2 = \beta_1 \gamma_2$.

Look at Figure 4 illustrating the solution to our problem for the first case:

The magnitude of exports of the commodity "1" equals

$A_1 b_{12} \omega_2$ - when meant for financial balancing of purchases (imports) of commodity "2" for purposes of producing commodity "1" in quantity $\alpha_1 = A_1$, and

$\beta_2 \omega_2$ - meant for balancing purchases (imports) of commodity "2" for satisfaction of consumption demand, $\beta_2 = \beta_1 \gamma_2$.

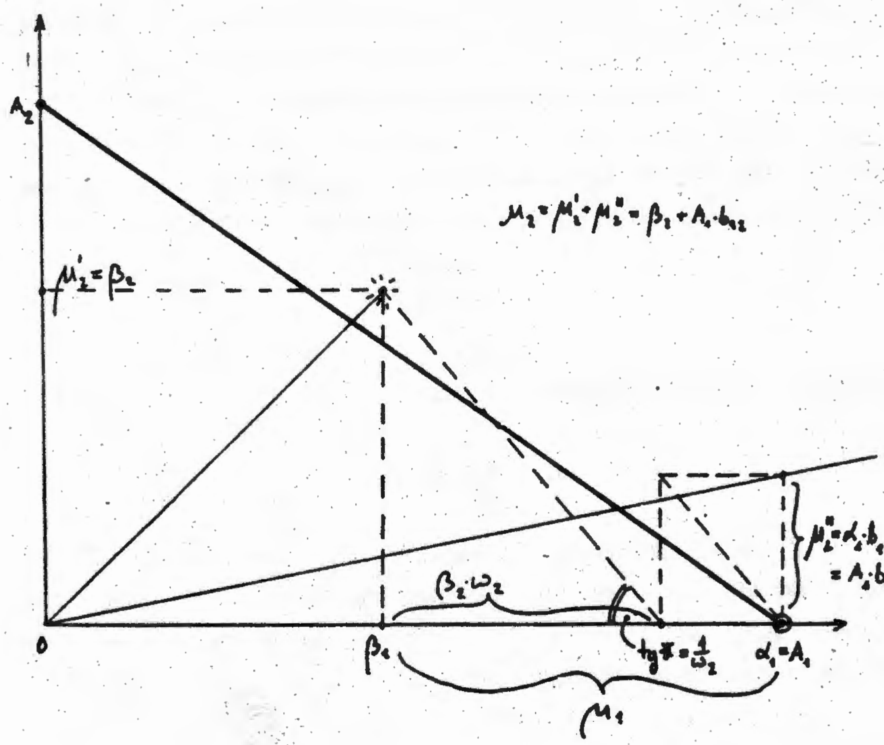

$\mu_2 = \mu_2' + \mu_2'' = \beta_2 + A_1 \cdot b_{12}$
A_2
$\mu_2' = \beta_2$
$\mu_2'' = \alpha_1 \cdot b_{12} =$
$= A_1 \cdot b_{12}$
$\beta_2 \cdot \omega_2$
0
β_2
$tg\,\gamma = \frac{1}{\omega_2}$
$\alpha_1 = A_1$
μ_1

Figure 4.

Thus, quantities β_1 and β_2 satisfy the system of equations (see Fig. 4):

$$\beta_2 = \beta_1\gamma_2 \quad \text{and} \quad \beta_1 + \beta_2\omega_2 + A_1b_{12}\omega_2 = A_1 \ .$$

which, when solved, yields:

$$\beta_1 = A_1 \cdot \frac{1 - \omega_2 b_{12}}{1 + \omega_2\gamma_2}$$

$$\beta_2 = \gamma_2\beta_1 = A_1\gamma_2 \cdot \frac{1 - \omega_2 b_{12}}{1 + \omega_2\gamma_2}$$

with

$$\frac{1}{\omega_2}\cdot\mu_1 = \beta_2 + A_1b_{12} = b_{12} + \gamma_2 \cdot \frac{1 - b_{12}\omega_2}{1 + \gamma_2\omega_2}$$

$$-\mu_2 = A_1(b_{12} + \gamma_2 \cdot \frac{1 - \omega_2 b_{12}}{1 + \omega_2\gamma_2})$$

If, besides this, in the formula given before defining the production magnitude α_1, we substitute β_1 as defined above then we conclude, in fact, that in order to secure the given level of consumption we have to take $\alpha_1 = A_1$

b. Consider now the second case, i.e. when commodity "2" is produced in just such quantity as to secure production of commodity "1". Then, the whole of imports is directed solely to satisfaction of consumption demand:

$$\mu_2 = -\beta_2$$

We shall determine for each case the maximum value of β_1, with

$$\beta_2 = \gamma_2\beta_1$$

Since we have, in general,

$$\mu_1 = \alpha_1 - \alpha_2 b_{21} - \beta_1$$
$$\mu_2 = \alpha_2 - \alpha_1 b_{12} - \beta_2$$

and there is, in our case, $\mu_2 = -\beta_2$, then $\alpha_2 = \alpha_1 b_{12}$.

On the other hand we have (see Fig.5.):

$$\alpha_1 = A_1 - \alpha_2 \cdot \frac{A_1}{A_2}$$

so that by solving this system we get

$$\alpha_1 = \frac{A_2}{b_{12} + \frac{A_2}{A_1}}, \qquad \alpha_2 = \frac{b_{12} A_2}{b_{12} + \frac{A_2}{A_1}}$$

Now, by substituting these quantities in the formula for μ_1 and taking into account that

$$\mu_1 = -\mu_2 \omega_2 = \beta_2 \omega_2$$

we obtain the following equation:

$$\beta_2 \omega_2 = \frac{A_2}{b_{12} + \frac{A_2}{A_1}} - b_{21} \cdot \frac{b_{12} A_2}{b_{12} + \frac{A_2}{A_1}} - \beta_1$$

from which, having introduced $\beta_2 = \beta_1 \gamma_2$ we can determine the quantity we are looking for:

$$\beta_1 = \frac{A_2}{1 + \omega_2 \gamma_2} \cdot \frac{1 - b_{12} b_{21}}{b_{12} + \frac{A_2}{A_1}}$$

The magnitudes of exports and imports shall be equal, respectively:

$$\mu_1 = A_1 \cdot \frac{\omega_2 \gamma_2}{1 + \omega_2 \gamma_2} \cdot \frac{1 - b_{12} b_{21}}{b_{12} + \frac{A_2}{A_1}},$$

$$\mu_2 = -\mu_1 \cdot \frac{1}{\omega_2}$$

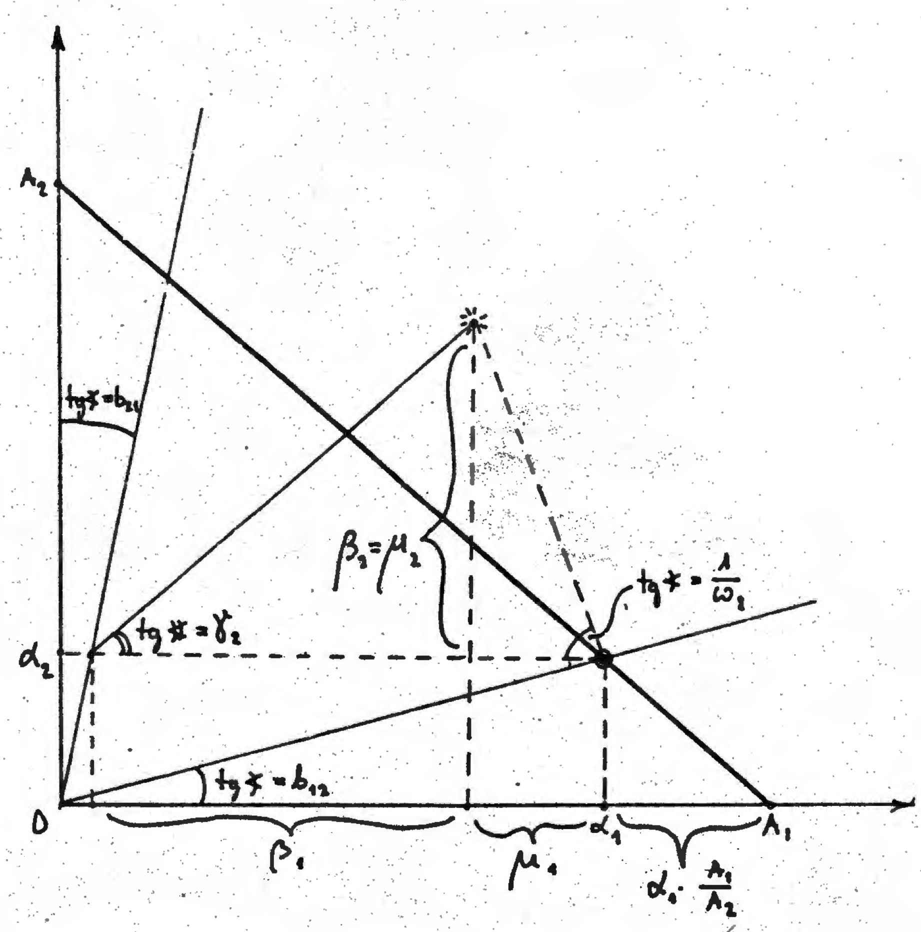


Figure 5

Let us summarize now the possibilities of maximization of consumption, in case of an open economy with unlimited possibilities of exchange. Thus, we can state that:

! the optimal production level for the open economy with unlimited possibilities of exchange is defined by the quantities

$$\alpha_1 = \begin{cases} A_1, & \text{if } \beta_1^{a)} \geq \beta_1^{b)} \\ \dfrac{A_2}{b_{12} + \dfrac{A_2}{A_1}}, & \text{in the opposite case} \end{cases}$$

and

$$\alpha_2 = \begin{cases} 0, & \text{if } \beta_1^{a)} \leq \beta_1^{b)} \\ \dfrac{A_2 b_{12}}{b_{12} + \dfrac{A_2}{A_1}}, & \text{in the opposite case} \end{cases}$$

under the assumption that $\dfrac{C_2}{C_1} \geq \dfrac{A_1}{A_2}$. In the above the quantities $\beta_1^{a)}$ and $\beta_1^{b)}$ are defined with the formulae

$$\beta_1^{a)} = A_1 \cdot \frac{1 - b_{12}\omega_2}{1 + \gamma_2\omega_2}$$

$$\beta_1^{b)} = A_2 \cdot \frac{1}{1 + \gamma_2\omega_2} \cdot \frac{1 - b_{12}b_{21}}{b_{12} + \dfrac{A_2}{A_1}}$$

Comparing of quantities β is not straightforward so that let us better try to establish whether inequality $\beta_1^{a)} > \beta_1^{b)}$ does not hold, incidentally, always. In order to do so we shall calculate the difference $\beta_1^{a)}-\beta_1^{b)}$. After simple transformations we obtain:

$$\beta_1^{a)}-\beta_1^{b)} = \frac{A_1 b_{12} Q}{(1+\gamma_2\omega_2)(b_{12}+\frac{A_2}{A_1})}$$

with

$$Q = 1 - b_{12}\omega_2 - \frac{A_2}{A_1}(\omega_2-b_{21}) = \frac{1}{C_1 A_1}[A_1(C_1-C_2 b_{12}) - A_2(C_2-C_1 b_{21})]$$

The sign of the difference depends upon the sign of expression defining Q. Note that quantities $1-b_{12}\omega_2$ as well as ω_2-b_{21} are always positive, since

$$1-b_{12}\omega_2 = 1-b_{12}\cdot\frac{C_2}{C_1} = \frac{1}{C_1}(C_1-b_{12}C_2)$$

$$\omega_2-b_{21} = \frac{C_2}{C_1} - b_{21} = \frac{1}{C_1}(C_2-b_{21}C_1)$$

The terms $C_1-b_{12}C_2$ and $C_2-b_{21}C_1$, which define the prices of units of commodities "1" or "2" decreased by the cost of one of the components necessary for their production must be positive quantities.

Consequently, the sign of the expression defining Q depends upon the difference:

$$A_1(C_1-b_{12}C_2) - A_2(C_2-b_{21}C_1)$$

Thus, if inequality

$$\frac{C_1-b_{12}C_2}{C_2-b_{21}C_1} > \frac{A_2}{A_1}$$

or

$$\frac{1-\omega_2 b_{12}}{\omega_2-b_{21}} > \frac{A_2}{A_1}$$

holds, then $\beta_1^{a)}>\beta_1^{b)}$ and the optimal value of α_1 is $\overset{*}{\alpha}_1 = A_1$.

Then:

$$\overset{*}{\beta}_1 = A_1 \cdot \frac{1-\omega_2 b_{12}}{1+\omega_2\gamma_2}$$

Let us consider other characteristic cases of possible production strategies. They are as follows:

c. We are producing primarily the commodity "2", and also commodity "1" in such quantities as to ensure its supplies necessary for producing commodity "2". An adequate level of consumption of commodity "1" is ensured through imports. In this situation we have

$$\alpha_1 = \alpha_2 b_{21}$$

$$\alpha_2 = \frac{A_1}{b_{21} + \frac{A_1}{A_2}} = \frac{A_2}{1 + b_{21}\cdot\frac{A_2}{A_1}}$$

and

$$\beta_1^{c)} = \frac{A_1\omega_2}{b_{21} + \frac{A_1}{A_2}}\cdot\frac{1 - b_{12}b_{21}}{1 + \gamma_2\omega_2} =$$

$$= \frac{A_2\omega_2}{1 + b_{21}\cdot\frac{A_2}{A_1}}\cdot\frac{1 - b_{12}b_{21}}{1 + \gamma_2\omega_2}$$

d. We are producing solely the commodity "2". The imports of commodity "1" are used to satisfy the demands related to the necessity of meeting the needs for commodity "2" as well as those resulting from a definite consumption level.

In this situation we have $\alpha_1 = 0$, $\alpha_2 = A_2$, and

$$\beta_1^{d)} = A_2 \cdot \frac{\omega_2 - b_{21}}{1 + \omega_2\gamma_2}$$

By comparing the two above strategies we determine the value of the difference $\beta_1^{c)} - \beta_1^{d)}$, similarly as we did before. After simple transformations we obtain, therefore

$$\beta_1^{c)} - \beta_1^{d)} = Q \cdot \frac{A_2\omega_{21}}{1 + \omega_2\gamma_2}$$

The sign of this difference depends upon the sign of Q.

If we determine in a similar way the difference

$$\beta_1^{b)} - \beta_1^{c)} = Q \cdot \frac{A_1(1 - b_{12}b_{21})}{(1 + \omega_2\gamma_2)(b_{12} + \frac{A_2}{A_1})(b_{21} + \frac{A_1}{A_2})}$$

then we can formulate the following conclusion:

If $Q>0$, that is, if the following inequality

$$\frac{1 - \omega_2 b_{12}}{\omega_2 - b_{21}} > \frac{A_2}{A_1}$$

holds, then

$$\beta_1^{a)} > \beta_1^{b)} > \beta_1^{c)} > \beta_1^{d)}$$

and, in the opposite case

$$\beta_1^{a)} < \beta_1^{b)} < \beta_1^{c)} < \beta_1^{d)}$$

Thus, we have altogether proved the following

Theorem 1.

If the following inequality

$$\frac{1 - \omega_2 b_{12}}{\omega_2 - b_{21}} > \frac{A_2}{A_1}$$

then $\overset{*}{\alpha}_1 = A_1$, and $\overset{*}{\beta}_1 = A_1 \cdot \dfrac{1 - \omega_2 b_{12}}{1+\omega_2\gamma_2}$,

while in the opposite case there is: $\overset{*}{\alpha}_2 = A_2$, and $\overset{*}{\beta}_1 = A_2 \cdot \dfrac{\omega_2 - b_{21}}{1 + \omega_2\gamma_2}$

If inequality becomes equality then every strategy is equally optimal and

$$\overset{*}{\beta}_1 = A_1 \cdot \frac{1 - \omega_2 b_{12}}{1+\gamma_2\omega_2} = A_2 \cdot \frac{\omega_2 - b_{21}}{1 + \omega_2\gamma_2}$$

The proof of this theorem is constituted by the considerations given previously.

Note that if the commodities considered are consumption goods of industrial origin (such as e.g. home appliances - washing machines, refrigerators,...), the economic agent is national economy, and the outside market is the international one, that the demand structure $\hat{\gamma}$ can be interpreted as the structure of consumption of the society of a given country, while Theorem 1 determines the optimal production, exports and imports policy for these commodities.

If population number is equal L then the "living standard", defined as the level P_1 of per capita consumption of commodity "1" (together with the corresponding consumption level of commodity "2"), will be equal

$$P_1 = \frac{\beta_1}{L}$$

It is obvious that we try to make this "standard of living P_1" as high as possible, meaning that we tend to maximize the supplies ensuring satisfaction of demand at the possibly high level.

For an enterprise producing two kinds if semi products, of which eventually a final product is made, Theorem 1 makes it possible to perform a choice of specialization. Thus, namely, it is attained through giving up of production of one of the semi products to the advantage of the production intensity of the other, in such a way as to purchase as much of the non-produced semi-product for the money accruing from the sales of the surplus of the produced semi-product, as to ultimately increase the production of the final good. Since with increase of production of the final product production value as well as profit increase, too, then it is understandable that the growth of final production magnitude of an enterprise is almost always advantageous for an enterprise.

5. MODEL OF COOPERATION OF TWO ECONOMIC ORGANIZATIONS (WITH LIMITED EXCHANGE)

The model considered in the previous chapter assumed that exchange of goods is not anyhow limited (both at the sales end of the good we have in surplus and the purchase end of the good which we are in need of) and that this exchange takes place at the constant price ratio ω_2.

Such a situation does not exist when economic cooperation takes place between two economic organizations having limited capacities of absorbing exports. Exchange between them is then limited by the finite capacities of importing the surpluses of products turned out within the other organization, this limitation being a mutual one. Besides that, in bilateral agreements it is not necessary that existing international or national prices of products in question be valid. It may happen that, for instance, "contractual prices" are introduced, for purposes of keeping financial balances, these prices, in some cases, depending also upon the magnitude of exchange.

Consequently, we can approach the problem of determination of optimal cooperation strategies in a somewhat different manner.

Let us look at Figure 6 (and at Figure 7). In the first "quarter" of the coordinate system (α_1, α_2) the characteristics of

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