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# WSPOMAGANIE INFORMATYCZNE ROZWOJU SPOŁECZNO-GOSPODARCZEGO I OCHRONY ŚRODOWISKA

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Książka wydana dzięki dotacji KOMITETU BADAŃ NAUKOWYCH

Książka zawiera wybór artykułów poświęconych omówieniu aktualnego stanu badań w kraju w zakresie rozwoju modeli, technik i systemów zarządzania oraz ich zastosowań w różnych dziedzinach gospodarki narodowej. Wyodrębnioną grupę stanowią artykuły omawiające aplikacyjne wyniki projektów badawczych i celowych KBN.

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# MODELING OF LAKE'S AQUATIC ECOSYSTEMS

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*In the paper modeling problem of ecological water systems in lake is presented. The model describes interactions between components of aquatic ecosystem (phytoplankton and zooplankton) and wastewater conditioning these components evolution. It is assumed, that a growth of plant and animal organisms in lakes depends on the pollutant quality coefficient such as various forms of phosphorus, nitrogen and dissolved oxygen.*
*The model consists of a number of submodels describing variations of some selected quality coefficient as well as variations in phytoplankton and zooplankton populations.*
*The following processes are considered: growth and mortality of organisms, assimilation of nonorganic nutrients in growth processes and organic nutrients sedimentation. Phytoplankton and zooplankton changes are described by the system of ordinary differential equations with growth and respiration coefficients as parameter. Submodels of concentration changes of water quality indices reflect both physical and biochemical processes in water.*

**Keywords:** Ecological water system, modeling of lake, differential equations, control problem.

## 1. The basic assumptions of ecological model of lake

The ecological models of lake compose of two interconnected components: a submodel describing biochemical processes and submodel describing water transportation of water.

The biochemical submodels reflects variations in phyto- and zooplankton populations and in the concentration of such substances as phosphorus, nitrogen etc. The biochemical processes are related to the growth and mortality of organisms, nonorganic nutrients assimilation and sedimentation.

The water transportation submodel reflects the structure of the spatial partition of lake. This structure has the form of the sequence of layers (segments), homogeneous from the point of view of pollutant concentration.

For the layer - division it is necessary to determine the morphometric parameter of each segment and the mixed and diffussion coefficients.

This division can be illustration as in Figure1.

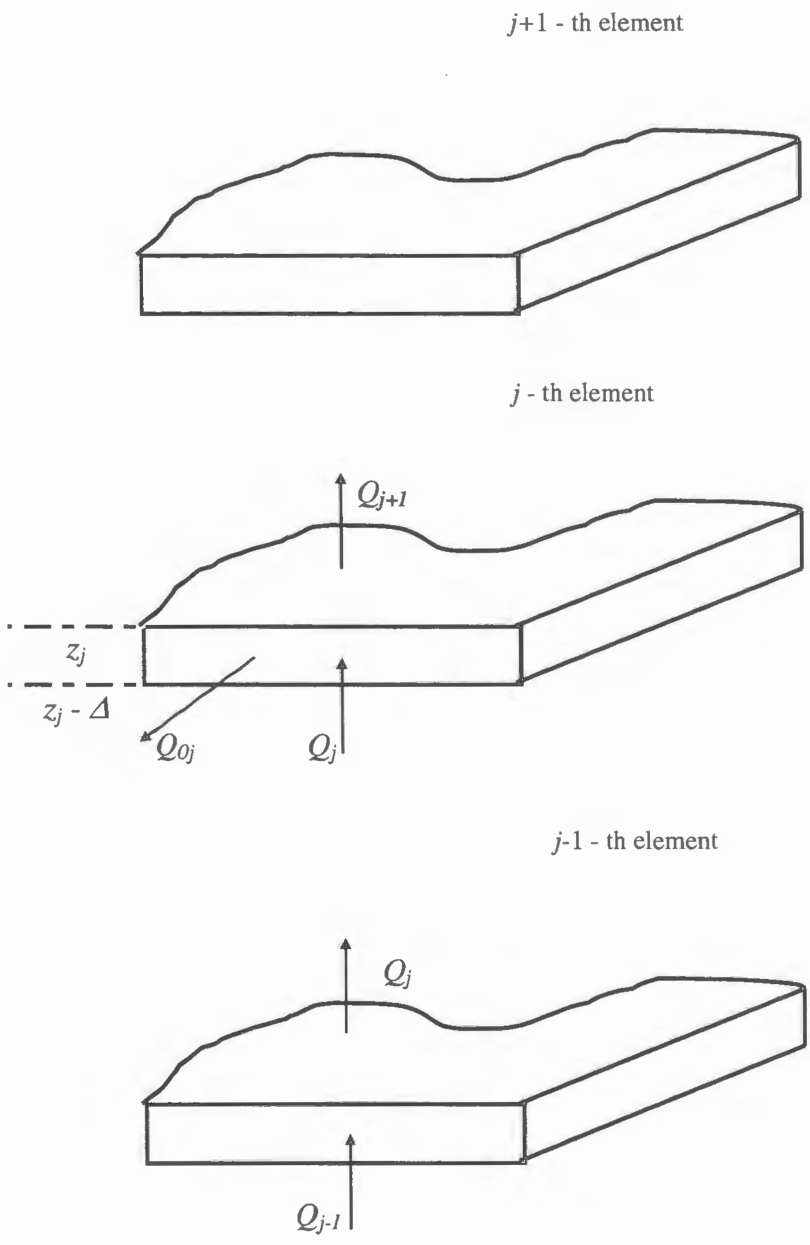


**Figure 1**. The layer - division of lake

The mass conservation equation for a discrete thickness $\Delta$ z is of the form

$$\frac{\partial v_j}{\partial t} = Q_j - Q_{j+1} - Q_{ij} + Q_{0j} \tag{1}$$

where:

$v_j$ - volume of the $j$-th element,

$Q_j$ – vertical inflow to the $j$-th segment,

$Q_{j+1}$ – vertical outflow from the $j$-th segment, which is the inflow to the $j$+1-th segment,

$Q_{ij}$ – flow advected to the control volume in the horizontal plane,

$Q_{0j}$ – flow advected from the control volume in the horizontal plane,

$z_j$ – depth of layer,

and all flows are functions of time.

Based on this relation the general mass conservation equation for the $j$-th element of a lake layer is

$$\frac{\partial (v_j C_j)}{\partial t} = \underbrace{-Q_j C_j + Q_{j+1} C_{j+1}}_{\text{advection}} + \underbrace{\left(E_a \frac{\partial C}{\partial Z}\right)_j - \left(E_a \frac{\partial C}{\partial Z}\right)_{j+1}}_{\text{diffusion}} + \underbrace{C_j \frac{\partial v_j}{\partial t}}_{\text{volume change}} + \underbrace{V_j \frac{\partial C}{\partial t}}_{\text{local derivative}} \tag{2}$$

where:

$C_j$ – concentration of any quality constituent that moves with the fluid mass

$\frac{\partial C}{\partial t}$ – local derivative represents all processes other than advection and diffusion that act to change the concentration in the volume $v_j$.

In ecological model under consideration we assume that the body of water is ideally mixed, i.e. the concentration of a substance in the body can be considered to be completely homogeneous and diffusion process can be neglected.

These assumption can concern either one of the layer of a lake or lakes in which the extent of turbulence due to wind, tides or other forces is so great that for practical purposes the concentration is homogeneous. In the second case we assume that a lake is a nonstratified, well mixed reservoir, in which concentration changes of constituents being considered depend only on time.

It is assume that the growth of plants and organisms in lake depends on the pollutant quality coefficients such as phosphorus and nitrogen. In the presented model the following processes are considered: zooplankton grazing, respiration, nutrients sedimentation and biodegradation.

## 2. The structure of ecological model of lake

Under above assumptions the model describing variation of nitrogen and phosphorus concentration as well as variations in phytoplankton and zooplankton populations has the form of following differential equations:

$$\frac{dA}{dt} = (\mu_a - r_a - s_a - m_a)\cdot A - \frac{\mu_z \cdot Z}{d} - \frac{Q\cdot A}{V} \tag{3}$$

$$\frac{dZ}{dt} = (\mu_z - r_z - m_z)\cdot Z - \frac{\mu_f \cdot F}{d} - \frac{Q\cdot Z}{V} \tag{4}$$

$$\frac{dP}{dt} = \frac{Q_d \cdot P_d}{V} + Z_p + \alpha_s \cdot P - \frac{(\mu_a - r_a)\cdot A}{\alpha_p} + \frac{r_z \cdot Z}{\alpha_p} - \frac{Q\cdot P}{V} + k_4 \cdot dp \cdot k_6 \tag{5}$$

$$\frac{dN}{dt} = \frac{Q_d \cdot N_d}{V} + Z_n + \beta_s \cdot N - \frac{k_d \cdot N}{V} - \frac{(\mu_a - r_a)\cdot A}{\beta_p} + \frac{r_z \cdot Z}{\beta_p} - \frac{Q\cdot N}{V} + k_4 \cdot dn \cdot k_6 \tag{6}$$

where:

$A$ – phytoplankton concentration,

$Z$ – zooplankton concentration,

$P$ – phosphorus dissolved in lake water,

$N$ – nitrogen dissolved in lake water,

$Q$ – rate of total outflow from the lake,

$V$ – the lake volume,

$F$ – fish concentration,

$Q_d$ – rate of total inflow to the lake.

The parameters are:

$m_a$ – mortality of phytoplankton,

$m_z$ – mortality of zooplankton,

$r_a$ – respiration rate of phytoplankton,

$r_z$ – respiration rate of zooplankton,

( $r_z$ and $r_a$ depend on the temperature),

$s_a$ – setting rate,

$d$ – conversion factor phytoplankton – zooplankton biomass,

$\alpha_p$ – phosphorus content in plankton,

$\beta_p$ – nitrogen content in plankton,

$\alpha_s$ – parameter associated with sedimentation of phosphorus,

$\beta_s$ – parameter associated with sedimentation of nitrogen,

$k_4$ – biodegradation coefficient of detritus,

$dn$ – detritus nitrogen in lake water,

$dp$ – detritus phosphorus in lake water,

$k_6$ – temperature coefficient for biodegradation,

$k_d$ – a constant describing denitrification rate,

$P_d$ – phosphorus in streams going to the lake,

$N_d$ – nitrogen in streams going to the lake.

The coefficient $\mu_a$ is the growth rate of phytoplankton and it depends on the nitrogen and phosphorus concentration, on temperature and light.

The nonlinear relationship between nitrogen and phosphorus concentrations and the coefficient $\mu_a$ takes the form

$$\mu_a = \mu_{a\max}(T, L) \cdot \frac{N}{k_n + N} \cdot \frac{P}{k_p + P} \tag{7}$$

where:

$k_n$ – a constant describing nitrogen uptake,

$k_p$ – a constant describing phosphorus uptake.

The coefficient $\mu_z$ is growth rate of zooplankton and it depends on temperature and phytoplankton concentration. This nonlinear function reflecting this dependence is given by

$$\mu_z = \mu_{z\max}(T, L) \cdot \frac{A}{k_a + A} \tag{8}$$

where:

$k_a$ – a constant describing feeding rate of zooplankton.

The coefficient $\mu_f$ – describing the growth rate of fish population is of the following form

$$\mu_f = \mu_{f\max}(T) \cdot \frac{Z}{k_z + Z} \tag{9}$$

$k_z$ – a constant for feeding rate of fish,

$Z_p$, $Z_n$ – waste load discharged to the lake.

As a result from the system of equations (3) - (6) the model describing aquatic ecosystem is of nonlinear form and with respect to interconnection between some model variables can be decomposed into three submodels:

a) a submodel describing variations in phytoplankton population,
b) a submodel describing variations in zooplankton population,
c) a submodel describing variations of nitrogen and phosphorus concentrations.

The submodels are interconnected as shown in Fig.2.

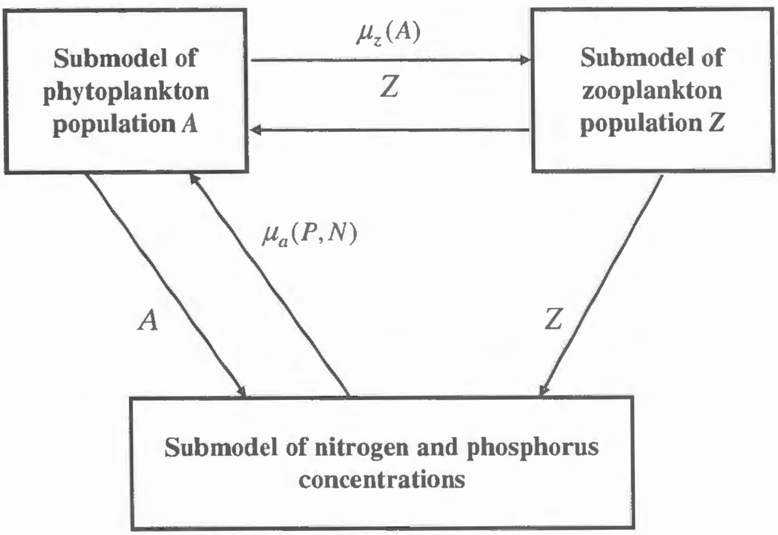


**Figure 2.** Interconnections between submodels in aquatic ecosystem of lake

The global model of aquatic ecosystem in lake composed of these submodels is given by the system of ordinary nonlinear differential equations (3) – (6).

## 3. The formulation of ecological control problem

As follows from the equations (3)–(6) the interconnection between submodels is realized, among other, by the growth coefficients $\mu_a, \mu_z$, where the coefficient $\mu_a$ is a function of nitrogen and phosphorus concentrations and $\mu_z$ is function of the phytoplankton population. Each submodel is described by the appropriate equations i.e. the submodel of phytoplankton population is given by the equation (3), the submodel of zooplankton population is given by the equation (4), whereas equations (5)–(6) describe the submodel of nitrogen and phosphorus concentrations.

Now, we distinguish in above model the state variables, the decision variables (control variables) and interconnection variables.

Let the state variable for the subsequent submodels be:

$x_1$ – the phytoplankton population A,

$x_2$ – the zooplankton population Z,

$x_3 = [P, N]^T$ – phosphorus and nitrogen concentrations,

and let the decision (control ) variables be:

$u_2$ – the fish concentration F,

$u_3 = [Z_p, Z_n]^T$ – waste loads (phosphorus and nitrogen) discharged to the lake.

Interconnection variables between submodels are as follows:

$$m_1 = g_1(x_2, x_3) \tag{10}$$

$$m_2 = g_2(x_1) \tag{11}$$

$$m_3 = g_3(x_1, x_2) \tag{12}$$

The functions $g_i()$ , $I$ =1,2,3, are nonlinear functions of phytoplankton and zooplankton populations and of phosphorus and nitrogen concentrations.

Assuming this notation we can express the system (3)–(6) in the following form:

- the system of state equations for each submodel:

$$\dot{x}_i = f_i(x_i, u_i, m_i) \qquad i = 1,\ 2,\ 3 \tag{13}$$

where: $f_i(\cdot,\cdot,\cdot)$ a nonlinear function

- the system of interconnection equations:

$$m_i = g_i(x_j) \qquad j \neq i, \quad i, j = 1,2,3 \tag{14}$$

The control problem consists in determining such strategy for waste loads discharges, which quarantees ecological equilibrium in the lake. This problem with the appropriate formulated performance functional can be considered as optimization problem.

Because there are interconnection between particular submodels it is quite natural to apply multilevel method of problem solving.

If the functions describing state equations, interconnection and performance functional satisfy appropriate conditions i.e. convexity, differentiality, then to solve this problem we can apply the two-level coordination method based on the augmented Lagrange functional associated with interconnection equations.

This leads to a two-level problem with following structure:

- the first-level problem consists in independent minimization of the local functional with respect to state and decision variables for fixed values of multipliers (associated with interconnection equations of form (14) ) and interconnection variables m,
- the task of the second level consists in determining such values of multipliers and interaction variables m, which guarantee global optimum for the global problem.

If a lake is treated as a sequence of layers with homogeneous pollutants concentration then the multilevel structure of the problem is more complicated. It is the result of interconnections between particular layers. Each lake layer is modeled as a system of interconnected submodels and the water quality problem is solved by the two-level optimization method.

The model of the whole lake should consist of the ecosystem models obtained for particular lake layers and should reflect both advection and diffusion processes.

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