



POLSKA AKADEMIA NAUK
Instytut Badań Systemowych

**ROZWÓJ I ZASTOSOWANIA
METOD ILOŚCIOWYCH
I TECHNIK INFORMATYCZNYCH
WSPOMAGAJĄCYCH PROCESY
DECYZYJNE**

Redakcja:

Jan Studziński
Ludostław Drelichowski
Olgierd Hryniewicz

**ROZWÓJ I ZASTOSOWANIA
METOD ILOŚCIOWYCH
I TECHNIK INFORMATYCZNYCH
WSPOMAGAJĄCYCH PROCESY
DECYZYJNE**

Redakcja:

Jan Studziński

Ludosław Drelichowski

Olgierd Hryniewicz

Wydanie tej publikacji było możliwe dzięki pomocy finansowej
MINISTERSTWA NAUKI I SZKOLNICTWA WYŻSZEGO.

Książka zawiera wybór artykułów poświęconych omówieniu aktualnego stanu badań w kraju w zakresie rozwoju i zastosowań metod, modeli, technik i systemów informatycznych w procesach podejmowania decyzji. Kilka artykułów przedstawia rezultaty projektów badawczych finansowanych przez Ministerstwo Nauki i Szkolnictwa Wyższego i realizowanych przez polskie instytucje badawcze.

Recenzenci:

Prof. Olgierd Hryniewicz

Prof. Andrzej Straszak

Dr hab. Jan Studziński

Komputerowa edycja tekstu: Anna Gostyńska

© Instytut Badań Systemowych, Warszawa 2006

Wydawca: Instytut Badań Systemowych PAN
Newelska 6, PL 01-447 Warszawa

Sekcja Informacji Naukowej i Wydawnictw
e-mail: biblioteka@ibspan.waw.pl

ISBN 83-894-7506-5

9788389475060

ISSN 0208-8029



**ROZWÓJ I ZASTOSOWANIA
METOD ILOŚCIOWYCH I TECHNIK
INFORMATYCZNYCH
WSPOMAGAJĄCYCH PROCESY
DECYZYJNE**

Instytut Badań Systemowych • Polska Akademia Nauk
Seria: Badania Systemowe
Tom 49

Redaktor Naukowy:
Prof. Jakub Gutenbaum

Warszawa 2006



ALGEBRAIC ASPECTS OF GENERALIZED NETS*

Maciej KRAWCZAK

Systems Research Institute, Polish Academy of Sciences
Warsaw School of Information Technology
<krawczak@ibspan.waw.pl>

Abstract: *The paper considers the so-called generalized nets as an extension of Petri nets. First the basic of the theory of generalized nets is introduced. Here is possible to consider the algorithmic, algebraic and operator aspects of the theory of generalized nets. Here we emphasized the algebraic aspects of the theory only.*

Keywords: Modeling, generalized nets, knowledge representation, system science.

1. Introduction

K. T. Atanassov (1991) in 1982 proposed a new definition of nets for modeling and analyzing various kinds of dynamic systems, the nets are called *generalized nets*. In several papers it was shown that existing *Petri nets* were particular cases of generalized nets. The conception of generalized nets is based on developing a relation *place – transition*.

Generalized nets are characterized by:

- a static structure,
- dynamical elements called *tokens*,
- temporal components.

The static structure of generalized nets is characterized by *transitions*. Tokens are described by changeable *characteristics*, and characteristics of tokens play a roll of *memory* of the nets. There three global temporal constants: the initial moment in which the net starts functioning, the elementary time-step of the process, and the duration of functioning.

Generalized nets can be used for:

- comparing different types of nets as mathematical objects,
- investigating properties of generalized nets and transfer them to other nets,
- modeling in details real processes.

* Praca wykonana częściowo w ramach projektu badawczego Nr 1H02B 03828 pt.: „Zastosowanie metod sztucznej inteligencji do zarządzania długiem publicznym”.

The theory of generalized nets, by analogy with the theory of Petri nets, can be divided into two basic fields - a *special* and a *general* theory of generalized nets. The special theory of generalized nets concerns both the definitions and the properties of generalized nets as well as modifications of generalized nets. The general theory of generalized nets deals with different aspects like: algebraic, logical, operation, program, methodological and topological. A list of 353 scientific works related to generalized nets as a review and bibliography on generalized nets theory and applications can find in Radeva, Krawczak, Choy (2002).

2. Generalized Nets Methodology

The first basic difference between generalized nets and the ordinary Petri nets is the place – transition relation (Atanassov, 1991). In the theory of generalized nets the transitions are objects of a very complex nature. The places are marked by Υ , and the transitions by \bigcirc . Generalized nets contain tokens, which are transferred from place to place. Every token bears some information, which is described by token's characteristic, and any token enters the net with an initial characteristic. After passing a transition the tokens' characteristics are modified.

The transition has input and output places, as it is shown in Figure 1

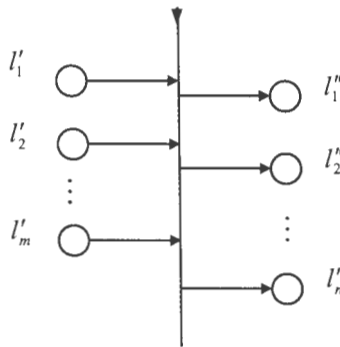


Figure 1. A generalized net transition.

Formally, every transition is described by a seven-tuple

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle \quad (1)$$

where:

- $L' = \{l'_1, l'_2, \dots, l'_m\}$ is a finite non empty set of the transition's input places,
- $L'' = \{l''_1, l''_2, \dots, l''_n\}$ is a finite non empty set of the transition's output places,
- t_1 is the current time of the transition's firing,

- t_2 is the current duration of the transition active state,
- r is the transition's *condition* determining which tokens will pass (or transfer) from the transition's inputs to its outputs; it has the form of an *index matrix* described in Atanassov (1987)

$$r = \begin{array}{c|cccc} & l_1'' & \dots & l_j'' & \dots & l_n'' \\ \hline l_1' & r_{11} & \dots & r_{1j} & \dots & r_{1n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ l_i' & r_{i1} & \dots & r_{ij} & \dots & r_{in} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ l_m' & r_{m1} & \dots & r_{mj} & \dots & r_{mn} \end{array}$$

where r_{ij} is a predicate that corresponds to the i -th input and the j -th output places, $1 \leq i \leq m$, $1 \leq j \leq n$; when its truth value is *true*, a token is allowed to pass the transition from the i -th input place to the j -th output place,

- M is an index matrix of the capacities of transition's arcs:

$$M = \begin{array}{c|cccc} & l_1'' & \dots & l_j'' & \dots & l_n'' \\ \hline l_1' & m_{11} & \dots & m_{1j} & \dots & m_{1n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ l_i' & m_{i1} & \dots & m_{ij} & \dots & m_{in} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ l_m' & m_{m1} & \dots & m_{mj} & \dots & m_{mn} \end{array}$$

where $m_{ij} \geq 0$ are natural numbers;

- \square is an object of a form similar to a Boolean expression, it may contain as variables the symbols that serve as labels for transition's input places, and \square is an expression built up from variables and the Boolean connectives \wedge and \vee whose semantics is defined as follows $\wedge (l_{i1}, l_{i2}, \dots, l_{iu})$ - every place $l_{i1}, l_{i2}, \dots, l_{iu}$ must contain at least one token, $\vee (l_{i1}, l_{i2}, \dots, l_{iu})$ - there must be at least one token in all places $l_{i1}, l_{i2}, \dots, l_{iu}$, where $\{l_{i1}, l_{i2}, \dots, l_{iu}\} \subset L'$; when the value of a type (calculated as a Boolean expression) is *true*, the transition can become active, otherwise it cannot.

The following ordered four-tuple

$$E = \left\langle \langle A, \pi_A, \pi_L, c, f, \Theta_1, \Theta_2 \rangle, \langle K, \pi_K, \Theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \right\rangle \quad (2)$$

is called *generalized net* if the elements are described as follows:

- A is a set of transitions,
- π_A is a function giving the priorities of the transitions, i.e. $\pi_A : A \rightarrow N$, where $N = \{0, 1, 2, \dots\} \cup \{\infty\}$,
- π_L is a function giving the priorities of the places, i.e. $\pi_L : L \rightarrow N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i -th projection of the n -dimensional set, where $n \in N$, $n \geq 1$ and $1 \leq i \leq n$ (obviously, L is the set of all generalized nets places),
- c is a function giving the capacities of the places, i.e. $c : L \rightarrow N$,
- f is a function that calculates the truth values of the predicates of the transition's conditions (the function f have the value *false* or *true*, i.e. a value from the set $\{0, 1\}$),
- Θ_1 is a function giving the next time-moment when a given transition Z can be activated, i.e. $\Theta_1(t) = t'$, where $pr_3 Z = t$, $t' \in [T, T + t^*]$ and $t \leq t'$; the function value is calculated at the moment when the transition terminates its functioning,
- Θ_2 is a function giving the duration of the active state of a given transition Z , i.e. $\Theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$; the value of this function is calculated at the moment when the transition starts its functioning,
- K is the set of the generalized net's tokens; in some cases, it is convenient to consider it as a set of the form $K = \bigcup_{l \in Q^I} K_l$

where K_l is the set of tokens that enter the net from place l , and Q^I is the set of all input places of the net,

- π_K is a function giving the priorities of the tokens, i.e. $\pi_K : K \rightarrow N$,
- Θ_K is a function giving the time-moment when a given token can enter the net, i.e. $\Theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$,
- T is the time-moment when the generalized net starts functioning; this moment is determined with respect to a fixed (global) time-scale,
- t^0 is an elementary time-step, related to the fixed (global) time-scale,

- t^* is the duration of the generalized net functioning,
- X is the set of all initial characteristics the tokens can receive on entering the net,
- Φ is a characteristic function that assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition,
- b is a function giving the maximum number of characteristics a given token can receive, i.e. $b: K \rightarrow N$; for example, if $b(\alpha) = 1$ for some token α , then this token will enter the net with some initial characteristic and subsequently it will keep only its current characteristic; when $b(\alpha) = \infty$ the token α will keep all its characteristics; when $b(\alpha) = k < \infty$ the token α will keep its last k characteristics (the characteristics older than the last k will be forgotten); in the general case every token α has $b(\alpha) + 1$ characteristics on leaving the net.

A given generalized net may lack some of the above components. In these cases, any missing component will be omitted. The generalized nets of this kind form a special class and is called *reduced generalized nets*.

3. Algebraic Approach to GNs

For two given transitions Z_1 and Z_2 we will define

$$Z_1 = Z_2 \text{ iff } (\forall i: 1 \leq i \leq 7)(pr_i Z_1 = pr_i Z_2);$$

$$\begin{aligned} Z_1 \subset Z_2 \text{ iff } & (\forall i: 1 \leq i \leq 2)(pr_i Z_1 \subset pr_i Z_2) \\ & \& (\forall i: 3 \leq i \leq 4)(pr_i Z_1 = pr_i Z_2) \\ & \& (\forall i: 5 \leq i \leq 6)(pr_i Z_1 \subset pr_i Z_2) \\ & \& (pr_7 Z_1 \subset_2 pr_7 Z_2), \end{aligned}$$

where:

- \subset_1 is a relation of inclusion over index matrices and if $A = [K_1, L_1, \{a_{i,j}\}]$, $B = [K_2, L_2, \{b_{i,j}\}]$, then $A \subset_1 B$ iff $(K_1 \subset K_2) \& (L_1 \subset L_2) \& (\forall i \in K_1)(\forall j \in L_1)(a_{i,j} = b_{i,j})$;
- \subset_2 is a relation of inclusion over Boolean expressions and, for two such expressions a and b iff the expression a is obtained after removing a part of the arguments of b and the logical operations associated to them.

We will define four operations over the transitions

$$Z_i = \langle L_1^i, L_2^i, t_1^i, t_2^i, r^i, M^i, \square^i \rangle, > (i = 1, 2)$$

The following statement must necessarily hold: if place $l \in pr_1 Z_i \cap pr_2 Z_i$ and $l \in pr_s Z_{3-i}$, then $l \in pr_{3-s} Z_{3-i}$ for $1 \leq i \leq 2$, $1 \leq s \leq 2$. These operations are

- a) a *union* (the necessary conditions for this operation are $t_j^1 = t_j^2$ ($j=1,2$), and if $l \in pr_s Z_i$, and it is not allowed that $l \in pr_{3-s} Z_{3-i}$ for $1 \leq i \leq 2$, $1 \leq s \leq 2$:

$$Z_1 \cup Z_2 = \langle L_1^1 \cup L_1^2, L_2^1 \cup L_2^2, t_1^1, t_2^1, r^1 + r^2, M^1 + M^2, \vee(\square^1, \square^2) \rangle;$$

- b) an *intersection* (with the above conditions):

$$Z_1 \cap Z_2 = \langle L_1^1 \cap L_1^2, L_2^1 \cap L_2^2, t_1^1, t_2^1, r^1 \times r^2, M^1 \times M^2, \wedge(\square^1, \square^2) \rangle;$$

- c) a *composition* (with the above condition and with the condition

$$L_1^1 \cap L_1^2 = L_2^1 \cap L_2^2 = \emptyset):$$

$$Z_1 \circ Z_2 = \langle L_1^1 \cup (L_1^2 - L_2^2), L_2^1 \cap (L_2^2 - L_1^2), t_1^1, t_2^1 + t_2^2, r^1 \cdot r^2, M^1 \cdot M^2, \vee(\square^1, \square^2) \rangle,$$

where $\overline{\square}$ can be obtained from \square after removing all its arguments whose identifiers are elements of the set $L_2^1 \cup L_1^2$.

It is possible that $L_1^1 \cap L_1^2 = \emptyset$ and/or $L_2^1 \cap L_2^2 = \emptyset$. In this case $Z_1 \cap Z_2 Z_\emptyset$ is the empty transition, i.e., a transition without places (some other components of it such as M, r, \square will also be degenerated).

- d) A *difference* (with the above conditions):

$$Z_1 - Z_2 = \begin{cases} Z_\emptyset & \text{if } L_1^1 \subset L_1^2 \text{ or } L_2^1 \subset L_2^2 \\ Z_1 - Z_2 = \langle L_1^1 - L_1^2, L_2^1 - L_2^2, t_1^1, t_2^1, r^1 - r^2, \text{ where } \square^1 / \square^2 \text{ results from} \\ & M^1 - M^2, \square^1 / \square^2 \rangle, \text{ otherwise} \end{cases}$$

\square^1 after removing all its arguments whose identifiers are element of the set $L_1^1 \cap L_2^2$.

The operations described below do not exist elsewhere in the Petri net theory. The can be transferred to practically all other types of Petri nets. These operations are useful for constructing generalized net models of real processes.

Before introducing the different generalized net operations, we will formulate some appropriateness conditions for the arguments of these operations. We will assume that if a place participates simultaneously in two generalized nets, then in both of them it has:

- a) equal capacities,
- b) the same characteristic functions,

- c) equal possibility to host tokens from K , that is, if in the first net (which is a model of some process) the tokens which are elements of $K' \subset K$ pass through the place and in the second net tokens from the set $K'' \subset K$ can also pass through this place, then $K' = K''$,
- d) equal values of the priority functions, and
- e) the same number (identification).

Similarly, we will expect that if two places are respectively input and an output one for two transitions in both nets, then:

- a) the capacities of the connecting arcs are equal,
- b) the time for passing from the input to the output place is the same,
- c) the place which is an input (output) one in one of the transitions should play the same role in the other transition.

Let E_1 and E_2 be two generalized nets and let for $1 \leq i \leq 2$:

$$E_i = \langle \langle A_i, \theta_1^i \pi_L^i, c^i, f^i, \theta_2^i \rangle, \langle K_i, \pi_k^i \theta_k^i \rangle, \langle T_i, t_i^o, t_i^* \rangle, \langle X_i, \Phi_i, b_i \rangle \rangle$$

A *union* will be called the object:

$$E_1 \cup E_2 = \langle \langle A_1 \cup A_2, \pi_A^1 \cup \pi_A^2, \pi_L^1 \cup \pi_L^2, c^1 \cup c^2, f^1 \cup f^2, \theta_1^1 \cup \theta_1^2, \theta_2^1 \cup \theta_2^2 \rangle, \langle K_1 \cup K_2, \pi_k^1 \cup \pi_k^2, \theta_k^1 \cup \theta_k^2 \rangle, \langle \min(T_1, T_2), GCD(t_1^o, t_2^o), \max_{1 \leq i \leq 2} \left(T_i + \frac{t_i^* \cdot t_i^o}{GCD(t_1^o, t_2^o)} - \min(T_1, T_2) \right) \rangle, \langle X_1 \cup X_2, \Phi_1 \cup \Phi_2, b_1 \cup b_2 \rangle \rangle$$

$$\langle \theta_2^1 \cup \theta_2^2 \rangle, \langle K_1 \cup K_2, \pi_k^1 \cup \pi_k^2, \theta_k^1 \cup \theta_k^2 \rangle,$$

$$\langle \min(T_1, T_2), GCD(t_1^o, t_2^o), \max_{1 \leq i \leq 2} \left(T_i + \frac{t_i^* \cdot t_i^o}{GCD(t_1^o, t_2^o)} - \min(T_1, T_2) \right) \rangle,$$

$$\langle X_1 \cup X_2, \Phi_1 \cup \Phi_2, b_1 \cup b_2 \rangle \rangle$$

where

$$A_1 \cup A_2 = \bigcup_{i=1}^2 \{ Z \mid (Z \in A_i) \& (\forall Z' \in A_{3-i})(Z \cap Z' = Z_0) \} \cup \bigcup_{i=1}^2 \{ Z \mid (\exists Z'' \in A_i)(\exists Z''' \in A_{3-i})(Z' \cap Z''' \neq Z_0)(Z = Z' \cup Z''') \}.$$

A *composition* of the above nets will be called the object:

$$E_1 \circ E_2 = \begin{cases} E_1, & \text{if } T_2 + t_2^* < T_1 \\ E_2, & \text{if } T_1 \leq T_2 + t_2^* \end{cases}$$

where

$$E_3 = \langle \langle A_1 \cup A_2, \pi_A^1 \cup \pi_A^2, \pi_L^1 \cup \pi_L^2, c^1 \cup c^2, f^1 \cup f^2, \theta_1^1 \cup \theta_1^2, \theta_2^1 \cup \theta_2^2 \rangle, \langle K_1 \cup K_2, \pi_k^1 \cup \pi_k^2, \theta_k^1 \cup \theta_k^2 \rangle, \langle T_1, t_1^o, t_1^* \rangle, \langle X_1, \Phi_1, b_1 \rangle \rangle$$

$$\langle \langle T_1, \text{GCD}(t_1^o, t_2^o), \max_{1 \leq i \leq 2} (T_i + \frac{t_i^* \cdot t_i^o}{\text{GCD}(t_1^o, t_2^o)} - T_1) \rangle, \langle X_1 \cup X_2, \Phi_1 \cup \Phi_2, b_1 \cup b_2 \rangle \rangle$$

A *difference* of the two nets will be called the object:

$$\begin{aligned} E_1 - E_2 = & \langle \langle A_1 - A_2, \pi_A^1 - \pi_A^2, \pi_L^1 - \pi_L^2, c^1 - c^2, f^1 - f^2, \\ & \theta_1^1 - \theta_1^2, \theta_2^1 - \theta_2^2, \langle K_1 - K_2, \pi_K^1 - \pi_K^2, \theta_K^1 - \theta_K^2 \rangle, \\ & \langle T_1, t_1^o, t_1^* \rangle, \langle X_1 - X_2, \Phi_1 - \Phi_2, b_1 \rangle \rangle, \end{aligned}$$

where

$$A_1 - A_2 = \{Z \mid (Z \in A_1) \& (\forall Z' \in A_2)(Z \cap Z' = Z_\theta)\} \cup$$

$\{Z \mid (\exists Z' \in A_1)(\exists Z'' \in A_2)(Z' \cap Z'' \neq Z_\theta) \& (Z = Z' - Z'')\}$, $\pi_A^1 - \pi_A^2$ is obtained from π_A^1 after removing all its arguments whose identifiers are not elements of the set $A_1 - A_2$; $\pi_L^1 - \pi_L^2$ is obtained from π_L^1 after removing all its arguments whose identifiers are not elements of the set $L_1^1 - L_2^1$, etc.

Let us define for a given generalized net E two sets, K of all tokens and X of all initial characteristics:

$$K(E) = \{\alpha \mid (\forall \alpha \in K)(\theta_K(\alpha) < T + t^*)\}, \quad X(E) = \{x_0^\alpha \mid (\forall \alpha \in K(E))(x_0^\alpha \in X)\},$$

and let $X(\alpha)$ be the set of all different characteristics the token α can have initially. Obviously, $X(E) \subset \bigcup_{\alpha \in K} X(\alpha)$.

For a given token $\alpha \in K$ and a given initial characteristic $x \in X(E)$ let

$$\begin{aligned} E(\alpha, x) = & \begin{cases} \langle \alpha, x_{f_m}^\alpha \rangle, & \text{if } \alpha \in K(E) \text{ and } x \in X(\alpha) \\ \langle \alpha, x \rangle, & \text{otherwise} \end{cases}, \\ E(\alpha, x) = & \begin{cases} \langle \alpha, x, x_1^\alpha, x_2^\alpha, \dots, x_{f_m}^\alpha \rangle, & \text{if } \alpha \in K(E) \text{ and } x \in X(\alpha) \\ \langle \alpha, x \rangle, & \text{otherwise} \end{cases}, \end{aligned}$$

where $x_{f_m}^\alpha$ is the final characteristic of the token α in the generalized net E and $x_1^\alpha, x_2^\alpha, \dots, x_{f_m-1}^\alpha$ are the rest of characteristics the token has received during its transfer in the net.

The first generalized net definition is in some sense deductive. Below we will introduce a second definition of a generalized net. In the above sense, the new definition will be of inductive nature.

Let Z be a given object of the following components:

- a) L' – a set of places called input places;
- b) L'' – a set of places called output places;
- c) t_1 – a time-moment taken with respect to some fixed time-scale (with an elementary time-step t^o);
- d) t_2 – a real number which corresponds to the length of the of the time-interval in the above mentioned time-scale;
- e) r – an index matrix having the from

$$r = \begin{array}{c|ccc} & L_1'' & \dots & L_j'' & \dots & L_n'' \\ \hline L_1' & & & & & \\ \vdots & & & & & \\ L_i' & & & r_{i,j} & & \\ \vdots & & & & & \\ L_m' & & & & & \end{array}$$

($r_{i,j}$ – predicate)
($1 \leq i \leq m, 1 \leq j \leq n$)

(i, j) -th element of which is a predicate and corresponds to the i -th input and j -th output place;

- f) M , and index matrix having the form

$$r = \begin{array}{c|ccc} & L_1'' & \dots & L_j'' & \dots & L_n'' \\ \hline L_1' & & & & & \\ \vdots & & & & & \\ L_i' & & & m_{i,j} & & \\ \vdots & & & & & \\ L_m' & & & & & \end{array}$$

($m_{i,j} \geq 0$ – natural number)
($1 \leq i \leq m, 1 \leq j \leq n$)

- g) \square is an object having a from similar to a Boolean expression. Its variables are exactly the names of Z 's input places.

The object described above will be called a *transition*.

The inductive definition of the concept of generalized net is as follows:

1. An object that has the from of a transition is called a generalized net if the following are added to it:
 - a) π_A – a function giving a natural number (transition priority);

- b) π_L – a function giving the priorities of the transition's places;
 - c) c – a function giving the capacities of the transition's places;
 - d) f – a function which calculates the truth values of the predicates of the index matrix r ;
 - e) θ_1 – a function giving the next time-moment when the given transition can be activated, i.e., $\theta_1(t) = t'$, where $t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;
 - f) θ_2 – a function giving the duration of the active state of a given transition, i.e., $\theta_2(t) = t'$, where $t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts its functioning;
 - g) K – a set of tokens;
 - h) π_K – a function giving the priorities of the tokens;
 - i) θ_K – a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K, t \in [T, T + t^*]$;
 - j) T – a time-moment when generalized net starts functioning. This moment is determined with respect to a fixed time-scale and the first value of $t_1 = T$;
 - k) t^o – an elementary time-step related to the fixed time-scale;
 - l) t^* – duration of the net functioning;
 - m) X – a set of all initial characteristics with the token can receive on entering the net;
 - n) Φ – a characteristic function which assigns new characteristics to every token when it makes a transfer from an input to an output place of any transition;
 - o) b – a function giving the maximum number of characteristics, which a given token can receive during its transfer in the net.
2. If E_1 and E_2 are generalized nets, then $E_1 \cup E_2$ is a generalized net.

4. Conclusions

We have described the concept of a generalized nets methodology for modeling discrete event systems, and next the concept of index matrix useful for aggregation as well as for separation of subsystems. We have emphasized the algebraic aspect of this novel methodology.

References

- Atanassov K. (1991) *Generalized Nets*. World Scientific, Singapore, New Jersey, London.
- Atanassov K. (1997) *Generalized Nets and Systems Theory*. „Prof. M. Drinov" Academic Publishing House, Sofia.
- Radeva V., Krawczak M. Choy, E. (2002) Review and Bibliography on Generalized Nets Theory and Applications. *Advanced Studies in Contemporary Mathematics*, **4**, 2: 173-199.
- Atanassov K. (1987) Generalized index matrices. *Comptes Rendus de l'Academie Bulgare des Sciences*, **40**, 11: 15-18.
- Krawczak M. (2003) *Multilayer Neural Systems and Generalized Net Models*. Academic Press EXIT, Warsaw, Poland.
- Krawczak M. (2004) Generalized Nets Modeling Concept: Neural Networks Models. In: Grzegorzewski P., Krawczak M., Zadrozny S. (Eds.): *Soft Computing. Tools, Techniques and Applications*. Akademicka Oficyna Wydawnicza EXIT, Warszawa 2004.

Jan Studziński, Ludosław Drelichowski, Olgierd Hryniewicz
(Redakcja)

**ROZWÓJ I ZASTOSOWANIA METOD ILOŚCIOWYCH
I TECHNIK INFORMATYCZNYCH WSPOMAGAJĄCYCH
PROCESY DECYZYJNE**

Monografia zawiera wybór artykułów dotyczących informatyzacji procesów zarządzania, prezentując aktualny stan rozwoju informatyki stosowanej w Polsce i na świecie. Zamieszczone artykuły opisują metody, modele, techniki i systemy informatyczne stosowane do wspomaganie procesów podejmowania decyzji, a także omawiają zastosowania narzędzi informatycznych w różnych sektorach gospodarki. Kilka prac przedstawia wyniki projektów badawczych Ministerstwa Nauki i Szkolnictwa Wyższego, dotyczących rozwoju metod informatycznych i ich zastosowań.

ISBN 83-894-7506-5
9788389475060
ISSN 0208-8029

W celu uzyskania bliższych informacji i zakupu dodatkowych egzemplarzy
prosimy o kontakt z Instytutem Badań Systemowych PAN
ul. Newelska 6, 01-447 Warszawa
tel. 837-35-78 w. 241 e-mail: biblioteka@ibspan.waw.pl