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Chapter 6

Optimality conditions in $\mathcal{H}_2$ approximation

A reasonable criterion to obtain a reduced–order model of a high–order original system consists in the minimization of the $\mathcal{H}_2$ norm of the output error, which is the difference between the transfer functions of the original and approximate model. One reason of the continual interest in this approch to model reduction (see, e.g., [1] $\div$ [14]) is the appealing interpretation of the aforementioned norm as an energy. However, finding the $\mathcal{H}_2$–optimal reduced–order model of a complex system is a computationally hard task. Indeed, many of the available techniques are rather difficult to implement (see, e.g., [6], [7], [9]). Also, even when the convergence of the algorithm is ensured, the attainment of the global optimum, which is in general unique [15], is not guaranteed because more local minima may exist, so that the result depends on the initial guess. To avoid these difficulties, resort has been made to procedures that partly constrain the structure of the approximant [16] $\div$ [18] or refer to suitably modified performance indices [10] which, however, alter substantially the original problem.

The rational $\mathcal{H}_2$ approximation problem is a classic topic in approximation theory [20]. In particular, it has been shown that the best approximating function must satisfy some necessary conditions that can be expressed in terms of *interpolation constraints*. In the context of circuit and system theory, these conditions have been derived in [21] and [1] for SISO systems, even if they were already known in classical analysis since the 1920s [22]. These conditions have been extended to MIMO

systems in [16] where, however, attention is limited to the case in which the poles of the approximating function are fixed.

This chapter is organized as follows. Section 6.1 states the $\mathcal{H}_2$ approximation problem. Necessary conditions of optimality for the general MIMO case are derived along the lines of [23] in Section 6.2. In Section 6.3, these conditions are expressed in a compact form that is suggestive of an efficient search algorithm [23]. Section 6.3 shows that this procedure can conveniently be translated into a fixed–point algorithm [10] ÷ [12] whose convergence is discussed in Section 6.5. The treatment is limited to frequency–domain input/output representations. Its extension to state–space models can be found in [13] and [14].

6.1 Problem statement

Consider a continuous–time linear system with m_o outputs and m_i inputs and denote by

$$F(s) = [f_{hi}(s)], \;\; h = 1, \ldots, m_o; \;\; i = 1, \ldots, m_i \tag{6.1}$$

its real rational transfer matrix which is assumed to belong to the real Hardy space $\mathcal{H}_2$ ($f_{hi}(s)$ stable and strictly proper). Indicating with $d_F(s)$ the least common denominator of all $f_{hi}(s)$, with $\deg[d_F(s)] = n$, matrix (6.1) can be written as

$$F(s) = \frac{N_F(s)}{d_F(s)}, \tag{6.2}$$

where $N_F(s) = [n^F_{hi}(s)]$ is an $m_o \times m_i$ polynomial matrix whose elements $n^F_{hi}(s)$ have degree, at most, $n-1$.

Similarly, denote the $m_o \times m_i$ transfer matrix $G(s)$ of the reduced–order system by

$$G(s) = [g_{hi}(s)] = \frac{N_G(s)}{d_G(s)}, \tag{6.3}$$

where $\deg[d_G(s)] = r < n$ and $N_G(s) = [n^G_{hi}(s)]$ is an $m_o \times m_i$ polynomial matrix whose elements have degree, at most, $r-1$. By assuming for technical simplicity that the r roots p_k of $d_G(s)$ are simple (but not necessarily real), $G(s)$ can be expanded into partial fractions as

$$G(s) = \sum_{k=1}^{r} \frac{R_k}{s - p_k}, \tag{6.4}$$

where $R_k = [r_{hi}^k] \in \mathbb{C}^{m_o \times m_i}$ and

$$g_{hi}(s) = \sum_{k=1}^{r} \frac{r_{hi}^k}{s - p_k}. \tag{6.5}$$

Consider the $m_o \times m_i$ approximation error matrix:

$$E(s) = [e_{hi}(s)] := F(s) - G(s) = [f_{hi}(s) - g_{hi}(s)] \tag{6.6}$$

and define the scalar product of two $m_o \times m_i$ rational matrices $A(s)$ and $B(s)$ as:

$$< A(s), B(s) >:= \frac{1}{2\pi\jmath} \int_{-\jmath\infty}^{+\jmath\infty} \mathrm{tr}[A(s)B^*(-s^*)]ds =$$
$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}[A(\jmath\omega)B^*(\jmath\omega)]d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}[A^*(\jmath\omega)B(\jmath\omega)]d\omega, \tag{6.7}$$

where the asterisk denotes complex conjugate transpose. This product induces in $\mathcal{H}_2$ the norm:

$$\|A(s)\| = \sqrt{< A(s), A(s) >}. \tag{6.8}$$

The $\mathcal{H}_2$ approximation problem can now be stated as follows.

Approximation problem: Find a reduced–order transfer matrix $G(s)$ that minimizes the index

$$J := \|E(s)\|^2 \tag{6.9}$$

with respect to the parameters r_{hi}^k and p_k or, alternatively, to the coefficients of the polynomials $n_{hi}^G(s)$ and $d_G(s)$.

6.2 Necessary conditions

The index to be minimized can be written as

$$J = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}[E^*(\jmath\omega)E(\jmath\omega)]d\omega =$$
$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_h \sum_i e_{hi}^*(\jmath\omega)e_{hi}(\jmath\omega) = \sum_h \sum_i J_{hi}, \tag{6.10}$$

where

$$J_{hi} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |e_{hi}(\jmath\omega|^2 d\omega. \tag{6.11}$$

By Parseval's theorem

$$J_{hi} = \int_{0}^{\infty} |LT^{-1}[e_{hi}(s)]|^2 dt, \tag{6.12}$$

where $LT^{-1}[\cdot]$ denotes the inverse Laplace transform.

The partial derivative of J with respect to r_{hi}^k is

$$\begin{aligned}\frac{\partial J}{\partial r_{hi}^k} = \frac{\partial J_{hi}}{\partial r_{hi}^k} = \frac{1}{2\pi\jmath} \int_{-\jmath\infty}^{+\jmath\infty} \left[-\frac{1}{s-p_k} e_{hi}^*(-s^*) + e_{hi}(s) \frac{1}{s+p_k^*} \right] ds \\ = \frac{1}{\pi\jmath} \int_{-\jmath\infty}^{+\jmath\infty} e_{hi}(s) \frac{1}{s+p_k^*}\, ds. \end{aligned} \tag{6.13}$$

Equating (6.13) to zero and using Cauchy's integral formula leads to

$$e_{hi}(-p_k^*) = f_{hi}(-p_k^*) - g_{hi}(-p_k^*) = 0 \tag{6.14}$$

which holds for all k, h and i, so that

$$F(-p_k^*) - G(-p_k^*) = O, \;\; k = 1, \ldots, r, \tag{6.15}$$

where O is the zero matrix. Relation (6.15) clearly amounts to r interpolation constraints at the opposites of the poles of the approximant.

The partial derivative of J with respect to p_k is

$$\begin{aligned}\frac{\partial J}{\partial p_k} = \frac{1}{2\pi\jmath} \sum_h \sum_i \int_{-\jmath\infty}^{+\jmath\infty} \left[\frac{r_{hi}^k}{(s-p_k)^2} e_{hi}^*(-s^*) + e_{hi}(s) \frac{r_{hi}^{k*}}{(-s-p_k^*)^2} \right] ds \\ = \frac{1}{\pi\jmath} \sum_h \sum_i \int_{-\jmath\infty}^{+\jmath\infty} \frac{r_{hi}^k}{(s-p_k)^2} e_{hi}^*(-s^*)\, ds. \end{aligned} \tag{6.16}$$

Equating (6.16) to zero and using the residue theorem leads to

$$2 \sum_h \sum_i \left[f_{hi}'^*(-p_k^*) - g_{hi}'^*(-p_k^*) \right] r_{hi}^k = 0, \tag{6.17}$$

where the expression between square brackets corresponds to the derivative of $e^*(-s^*)$ at $s = p_k$. Relation (6.17) holds for all k, so that

$$\operatorname{tr}\left[F'^*(-p_k^*) - G'^*(-p_k^*) \right] R_k = 0, \;\; k = 1, \ldots, r \tag{6.18}$$

where, again, the prime denotes derivative.

The following proposition summarizes the previous results that extend to MIMO systems the conditions derived in [1].

Proposition 6.2.1 *If the roots of $d_G(s)$ are simple, necessary conditions for the transfer matrix $G(s)$ to be the optimal rational $\mathcal{H}_2$ approximant of $F(s)$ are (6.15) and (6.18).*

The unknowns in (6.15) and (6.18) are the $m_o\, m_i\, r$ residues r_{hi}^k and the r poles p_k. To determine these $r(1 + m_o\, m_i)$ unknowns (or the parameters of an equivalent parametrization), an equal number of scalar equations need be formed. In fact, the matrix equation (6.15) gives rise to $m_o\, m_i\, r$ scalar equations and (6.18) provides the remaining r scalar equations.

If the poles of $G(s)$ are not simple, conditions (6.15) and (6.18) should be modified, respectively, to

$$F^{(i)}(-p_k^*) - G^{(i)}(-p_k^*) = O, \;\; i = 1, \ldots, \mu_k - 1, \;\; k = 1, \ldots, \hat{r}, \quad (6.19)$$

where the superscript (i) denotes ith derivative, μ_k is the multiplicity of p_k, and $\hat{r}$ is the number of distinct poles, and

$$\mathrm{tr}\big[F^{(\mu_k)*}(-p_k^*) - G^{(\mu_k)*}(-p_k^*)\big] R_{k,\mu_k} = O, \;\; k = 1, \ldots, \hat{r}, \quad (6.20)$$

where R_{k,μ_k} is the coefficient of $1/(s - p_k)^{\mu_k}$ in the partial fraction expansion of $G(s)$ replacing (6.4).

6.3 Compact form

The necessary conditions of optimality in Proposition 6.2.1 can be converted into another set of equivalent conditions that refer to a different parametrization of $G(s)$, that is, to the coefficients of its numerator polynomials $n_{hi}^G(s)$ and to those of its monic denominator polynomial $d_G(s)$.

Taking into account (6.2) and (6.3), conditions (6.15) imply

$$N_F(-p_k^*) d_G(-p_k^*) - N_G(-p_k^*) d_F(-p_k^*) = O, \;\; k = 1, \ldots, r. \quad (6.21)$$

In other words, every entry of the polynomial matrix at the numerator of the error function (6.6), that is,

$$N_E(s) := N_F(s) d_G(s) - N_G(s) d_F(s), \quad (6.22)$$

is a polynomial of degree at most $n+r-1$ admitting $d_G^*(-s^*)$ as a factor, so that

$$N_E(s) = Q_1(s)d_G^*(-s^*), \tag{6.23}$$

where $Q_1(s)$ is an $m_o \times m_i$ matrix of polynomials whose degree is at most $n-1$. Similarly, conditions (6.18) imply

$$\mathrm{tr}[N_E'^*(-p_k^*)d_E^*(-p_k^*) - N_E^*(-p_k^*)d_E'^*(-p_k^*)]R_K = 0, \ \ \forall k, \tag{6.24}$$

where $d_E(s) := d_F(s)d_G(s)$.
Equation (6.23) implies $N_E^*(-p_k^*) = O$, so that (6.24) reduces to

$$\mathrm{tr}N_E'^*(-p_k^*)R_k = 0, \ \ \forall k. \tag{6.25}$$

Since the derivative of (6.23) is

$$N_E'(s) = Q_1'(s)d_G^*(-s^*) + Q_1(s)d_G'^*(-s^*) \tag{6.26}$$

and $d_G(p_k) = 0$ with $d_G'(p_k) \neq 0$ (p_k is a simple pole of $G(s)$), eqn.s (6.25) lead to

$$\mathrm{tr}Q_1^*(-p_k^*)R_k = 0, \ \ \forall k. \tag{6.27}$$

Therefore

$$\mathrm{tr}Q_1^*(-p_k^*)N_G(p_k) = 0, \ \ \forall k, \tag{6.28}$$

because

$$R_k = \frac{1}{d_G^k(p_k)}N_G(p_k) \tag{6.29}$$

with

$$d_G^k(s) = \prod_{\ell=1,\ell\neq k}^{r}(s-p_\ell). \tag{6.30}$$

It follows that the polynomial $\mathrm{tr}Q_1^*(s)N_G(-s^*)$ must have $d_G(-s^*)$ as a factor, that is,

$$\mathrm{tr}Q_1^*(s)N_G(-s^*) = q_2(s^*)d_G(-s^*), \tag{6.31}$$

where $q_2(s^*)$ is a polynomial of degree at most $n-2$.

Equations (6.23) and (6.31) are valid in the general case of polynomials with complex coefficients. In the case of polynomials with real coefficients, they can be rewritten in the simpler form:

$$N_E(s) = Q_1(s)d_G(-s), \tag{6.32}$$

$$\mathrm{tr}Q_1^T(s)N_G(-s) = q_2(s)d_G(-s), \tag{6.33}$$

to which reference is made in the sequel.

On the basis of the previous considerations, the following proposition can be stated.

Proposition 6.3.1 *If the roots of $d_G(s)$ are simple, necessary conditions for the transfer matrix $G(s)$ to be the optimal real rational $\mathcal{H}_2$ approximant of $F(s)$ are (6.32) and (6.33).*

Note that in the scalar case the two conditions (6.32) and (6.33) can be merged into the single polynomial equation:

$$n_F(s)d_G(s) - n_G(s)d_F(s) = q(s)d_G^2(-s), \tag{6.34}$$

where $q(s)$ is a polynomial of degree at most $n - r - 1$.

6.4 Fixed–point algorithm

The alternative optimality conditions derived in Section 6.3 are expressed by the simple polynomial identities (6.32) and (6.32) or (6.34). This is achieved, however, by increasing the number of unknowns.

By equating the coefficients of the equal powers of s at both sides of these identities, a set of $(n + r)(m_o\, m_i + 1) - 1$ equations in the same number of unknowns is obtained. Precisely, in the multivariable case, the unknowns are: (i) the r coefficients of the monic polynomial $d_G(s)$, (ii) the $r\, m_o\, m_i$ coefficients of the polynomial matrix $N_G(s)$, (iii) the $n\, m_o\, m_i$ coefficients of the polynomial matrix $Q_1(s)$, and (iv) the $n - 1$ coefficients of polynomial $q_2(s)$.

Since the unknowns appear nonlinearly in the considered equations, an efficient procedure for their solution is required. To this purpose, resort can be made to the following iterative scheme.

Denoting by superscript $(h + 1)$ the quantities to be computed in the current $(h + 1)$th iteration and by superscript (h) the quantities computed in the preceding hth iteration (or guessed at the beginning of the procedure), the basic equation of the aforementioned iterative scheme is:

$$N_F(s)d_G^{(h+1)}(s) - N_G^{(h+1)}(s)d_F(s) = Q_1^{(h+1)}(s)d_G^{(h)}(-s), \tag{6.35}$$

$$\mathrm{tr}Q_1^{T(h+1)}(s)N_G^{(h)}(-s) = q_2^{(h+1)}(s)d_G^{(h)}(-s), \tag{6.36}$$

which give rise to linear equations at each iteration. Essentially, at every iteration eqn.s (6.35) and (6.36) allow us to determine the "new" denominator $d_G^{(h+1)}(s)$ from the "old" denominator $d_G^{(h)}(s)$ only.

Algorithm (6.35)–(6.36) can be formulated [7], [10] in a more suggestive form as:

$$d^{(h+1)} = \Phi\big(d^{(h)}\big), \tag{6.37}$$

where $d^{(i)}$, $i = h, h+1$, is a vector formed from the coefficients of the monic polynomial $d_G^{(i)}(s)$, and $\Phi : \mathbb{R}^r \to \mathbb{R}^r$ is a continuously differentiable function. In this way, the solution of the approximation problem corresponds to a fixed point $\hat{d}$ of Φ, that is,

$$\hat{d} = \Phi(\hat{d}). \tag{6.38}$$

By indicating with $p^{(i)}$ the properly–ordered vector consisting of the roots of $d_G^{(i)}(s)$, there exists a one–to–one mapping $\Gamma : \mathbb{R}^r \to \mathbb{C}^r$ such that $p^{(i)} = \Gamma\big(c^{(i)}\big)$. Therefore, (6.37) is equivalent to

$$p^{(h+1)} = \Psi\big(p^{(h)}\big), \tag{6.39}$$

where $\Psi = \Gamma \circ \Phi \circ \Gamma^{-1} : \mathbb{C}^r \to \mathbb{C}^r$ is a continuously differentiable function whose fixed points $\hat{p}$ are given by $\hat{p} = \Gamma(\hat{d})$.

6.5 Algorithm convergence

Concerning the convergence of the fixed–point algorithm (6.39), the following proposition is proved in [7].

Proposition 6.5.1 *Let*

$$J_\Psi(p) := \partial\Psi/\partial p \tag{6.40}$$

be the Jacobian of the function in (6.39). Then:
(i) at every fixed point $\hat{p}$ of Ψ the eigenvalues of (6.41) are real,
(ii) at a fixed point of Ψ that does not correspond to a minimum of the index (6.9), at least one eigenvalue is greater than 1, so that such a point is repelling for (6.39), and
(iii) at a fixed point of Ψ corresponding to a minimum of (6.9), every eigenvalue is less than 1 (but not necessarily greater than -1).

The convergence properties of (6.37) are the same as those of (6.39) because $J_\Psi(\hat{p})$ is similar to:

$$J_\Phi(\hat{d}) := \partial\Phi/\partial d|_{d=\hat{d}} \tag{6.41}$$

and has the same eigenvalues.

Point (iii) of Proposition 6.5.1 implies that there may be fixed points corresponding to minima of (6.9) where (6.37) does not converge. To avoid this problem, Φ can be replaced by another continuously differentiable function Φ_m with the following properties: a) it has the same fixed points $\hat{d}$ as Φ, b) all of the eigenvalues of the Jacobian of Φ_m at every $\hat{d}$ have magnitude less than 1, and c) Φ_m can easily be obtained from Φ without *a priori* information about $\hat{d}$. Functions with these properties can be formed in a systematic way according to [11] and [12]. A simpler but heuristic procedure to find a function of this kind is based on the following proposition that draws on [10].

Proposition 6.5.2 *The function $\Phi_\alpha : \mathbb{R}^r \to \mathbb{R}^r$ related to Φ in (6.37) by means of*

$$\Phi_\alpha(d) = \alpha\Phi(d) + (1-\alpha)d \tag{6.42}$$

has the same fixed points $\hat{d}$ as Φ and, for suitably small positive values of α, all of the eigenvalues of the Jacobian of Φ_α at every $\hat{d}$ have magnitude less than 1.

Proof Proving that the fixed points $\hat{d}$ of Φ and Φ_α are the same is trivial. Concerning the eigenvalues of

$$J_\alpha(\hat{d}) := \partial\Phi_\alpha/\partial d|_{d=\hat{d}}\,, \tag{6.43}$$

from (6.42) we have

$$J_\alpha(\hat{d}) = \alpha J_\Phi(\hat{d}) + (1-\alpha)I = \alpha[J_\Phi(\hat{d}) - I] + I. \tag{6.44}$$

Since, according to Proposition 6.5.1, the spectrum $\sigma[J_\Phi(\hat{d})]$ of $J_\Phi(\hat{d})$ is contained in the interval $[\lambda_m, 1)$, where λ_m is the smallest (real) eigenvalue of $J_\Phi(\hat{d})$, it follows that

$$\sigma[J_\Phi(\hat{d}) - I] \subset [\,\lambda_m - 1, 0), \tag{6.45}$$

so that

$$\sigma\big\{\alpha[J_\Phi(\hat{d}) - I]\big\} \subset [\,\alpha(\lambda_m - 1), 0), \;\; \alpha > 0, \tag{6.46}$$

and from (6.44)

$$\sigma[J_\alpha(\hat{d})] = \sigma\big\{\alpha[J_\Phi(\hat{d}) - I] + I\big\} \subset [\,\alpha(\lambda_m - 1) + 1, 1). \tag{6.47}$$

Therefore, if

$$0 < \alpha < \alpha_m := \frac{2}{1 - \lambda_m}, \tag{6.48}$$

all of the eigenvalues of $J_\alpha(\hat{d})$ have magnitude less than 1. □

An immediate consequence of Proposition 6.5.2 is that the algorithm:

$$d^{(h+1)} = \Phi_\alpha\big(d^{(h)}\big), \tag{6.49}$$

converges to every $\hat{d}$ from a suitable neighbourhood of $\hat{d}$ if α is suitably small. It can be shown [14] that (6.49) is a special case of both Mann's and Ishikawa's iterations [27], [28]. Computational details of this algorithm as well as its application to some benchmark examples are provided in [14], where the procedure is extended to state–space models, too.

6.6 Concluding remarks

Necessary conditions for a reduced–order model to be the $\mathcal{H}_2$–optimal approximant of a MIMO system have been derived in terms of interpolation constraints, thus extending the results valid in the SISO case [1]. These conditions have then been expressed in a compact form that only entails polynomial identities. This compact form is suggestive of a numerical algorithm for finding the optimal reduced–order model.

This algorithm can be translated into a fixed–point algorithm which is not always convergent. To avoid this problem, a modified algorithm, easily obtainable from the original one, has been proposed.

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Often, short papers tend to be sharper than longer works because they focus on a single theme without lingering on unessential aspects, thus showing clearly the signicance of a contribution or an idea. The author of this book had the privilege of collaborating for over a quarter of a century with Antonio Lepschy (1931-2005), a recognized leader of the Italian control community.

Lepschy had a liking for the brief paper format, so that many results obtained by his research team were published in this way. The present compilation tells a few of these short stories, duly updated, trying to preserve their original flavour.

Umberto Viaro (http://umbertoviaro.blogspot.com/) has been professor of System and Control Theory at the University of Udine, Italy, since 1994. His 25-year-long collaboration with Antonio Lepschy resulted in more than 100 joint papers and two books. An essential role in this research activity was played by Wiesław Krajewski of the Systems Research Institute, Polish Academy of Sciences. The current research interests of Umberto Viaro concern optimal model reduction, robust control, switching and LPV control. He is the author or coauthor of 4 books and about 180 research papers.

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