

**New Trends in Fuzzy Sets,  
Intuitionistic Fuzzy Sets,  
Generalized Nets and Related Topics  
Volume II: Applications**

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Systems Research Institute  
Polish Academy of Sciences  
Newelska 6, 01-447 Warsaw, Poland  
[www.ibspan.waw.pl](http://www.ibspan.waw.pl)

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# The transfer of tokens in generalized nets with tokens duration of life

**Velin Andonov**

Institute of Biophysics and Biomedical Engineering  
Bulgarian Academy of Sciences,  
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, BULGARIA,  
e-mail: *velin\_andonov@yahoo.com*

## Abstract

In Generalized Nets with tokens duration of life (GNTDL), as defined in [2], some or all of the tokens have initial characteristic a number corresponding to the duration of time for which the token stays in the net. When such token reaches its limit of lifetime it leaves the net and all of its characteristics are lost. We propose different strategies to preserve these characteristics by selecting a token successor for every token with duration of life which reaches its limit of lifetime.

**Keywords:** Generalized nets

## 1 Introduction

In a Generalized net with tokens duration of life (GNTDL) some or all of the tokens possess an initial characteristic which corresponds to the number of elementary time steps for which the token can stay in the net (see Atanassov [2]). It is proved that  $\Sigma_{TDL}$  - the class of all *GNTDL* - is a conservative extension of  $\Sigma$  - the class of the standard GNs. It will be useful to give a formal definition of GNTDL where the duration of life of the tokens is given by a function . GNTDL is the ordered four-tuple

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K, \theta^* \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \rangle,$$

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where all other components except  $\theta^*$  are in the sense of ordinary GN. The new component  $\theta^* : K \mapsto N$  gives the duration of life of the tokens. Here  $N = \{0, 1, 2, \dots\} \cup \{\infty\}$ . Obviously, if  $K_1$  is the set of all tokens which do not have duration of life, as it is in the case of ordinary GN, then for every  $\alpha \in K_1$ ,  $\theta^*(\alpha) = \infty$ .

A token  $\alpha$  at an input place  $l'_i$  of a transition of a *GNTDL* will be transferred to an output place  $l''_j$  if:

- the corresponding predicate  $r_{i,j}$  of the index matrix  $r$  is true;
  - the output place  $l''_j$  is not full;
  - the current capacity of the arc between the input place and the output place is not 0;
  - $TIME - \theta_K(\alpha) < \theta^*(\alpha)$ , where  $TIME$  denotes the current time-moment;
- When  $TIME - \theta_K(\alpha) \geq \theta^*(\alpha)$  the token leaves the net with final characteristic equal to the last characteristic which it has obtained.

## 2 Preserving the characteristics of the tokens in GNTDL

Let  $K_2$  be the set of all tokens which have duration of life. When a token  $\alpha \in K_2$  leaves the net all of its characteristics are lost. Sometimes we need to keep those characteristics. One way to do this is to use a GN with global memory (Atanassov [1]). Here we shall discuss other possible ways of preserving the characteristics of the tokens with duration of life without changing the type of the net. The idea is to look for other tokens with duration of life which can inherit the characteristics of the tokens that leave the net.

First, we should examine the trivial case when the token with duration of life is at input place for the whole net and its only characteristic is the initial. In this case we do not need to look for successor of the token because it does not carry any new information and its initial characteristic is determined by the function  $X$ .

The other possibility is that of a token  $\alpha_i \in K_2$ ,  $1 \leq i \leq |K_2|$  in a place  $l'_i$  which is input for transition  $Z'$  (see Fig. 1) and output for some other transition  $Z$  when  $TIME - \theta_K(\alpha_i) = \theta^*(\alpha_i)$ .

In this case we look for other tokens with duration of life which are in the same input place  $l'_i$ . To determine the successor of  $\alpha$  we follow one of the two scenarios:

- (A) the successor is the token with the highest priority in the place;
- (B) the successor is the token with greatest duration of life in the place;

If there are no tokens with duration of life in the place, then the successor can

be selected among the tokens with duration of life in the input places of  $Z_i$ .

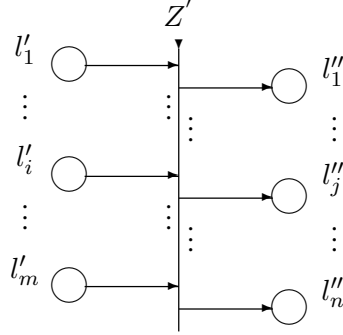


Fig. 1.

## 2.1 Preserving the characteristics with direct contact between the tokens

If there are other tokens with duration of life in the input places of  $Z_i$  which can be transferred to output place at the current time moment, then we choose the successor of the token  $\alpha$  following either rule  $A$  or  $B$ . First we look for possible successors in the place  $l'_i$  and if there are no tokens with duration of life in this place then we look for successors among all other input places of the transition. If we follow rule  $A$ , then we choose for successor the token with the highest priority in the input places by order of their priorities. In case we follow rule  $B$ , the successor will be the token with greatest duration of life among all tokens in all input places. Let  $\alpha_s$  be the token which succeeds the token  $\alpha$ . Then as it enters the output place  $l''_j$  it will obtain the characteristic

$$\langle \alpha_s, x_{cur}^\alpha, l''_j \rangle$$

where  $x_{cu}^\alpha$  is the current characteristic of the token with duration of life  $\alpha$  and  $l''_j$  gives information about the place where the succession takes place.

## 2.2 Preserving the characteristics with virtual contact

If there are no other tokens with duration of life in the place  $l'_i$  that can be transferred at the current time moment, we look for such tokens in all other input places of the transition. If we cannot find a token which can succeed the token  $\alpha$  at the current time step, then the characteristics of the token will be lost on the next step. One way to prevent this is to use some token as a mediator. The mediator token will obtain the characteristics of the token  $\alpha$  and when another token with duration

of life makes transfer from input to output place of the transition it will receive the characteristics of the mediator. In order to accomplish this, to every transition

$$Z_i = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

of a GNTDL we add a place  $l_i^*$ . The mediator token stays at place  $l_i^*$  of the transition with no initial characteristic. We denote by  $Z_i^*$  the new transition.

$$Z_i^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \square^* \rangle,$$

(see Fig. 2) where  $t_1$  and  $t_2$  are as above and

$$L'^* = L' \cup \{l_i^*\},$$

$$L''^* = L'' \cup \{l_i^*\},$$

$$\square^* = \square.$$

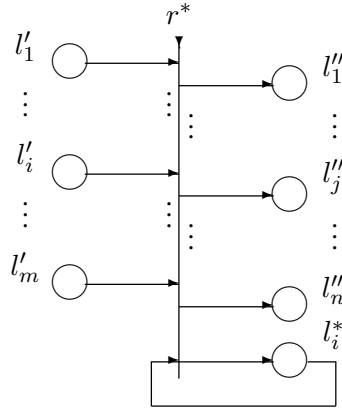


Fig. 2.

If

$$r = pr_5 Z_i = [L', L'', \{r_{l_i, l_j}\}]$$

is the index matrix of the transition's conditions, then

$$r^* = pr_5 Z_i^* = [L' \cup \{l_i^*\}, L'' \cup \{l_i^*\}, \{r_{l_i, l_j}^*\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(r_{l_i, l_j}^* = r_{l_i, l_j}),$$

and

$$(\forall l_i \in L')(\forall l_j \in L'')(r_{l_i, l_j}^* = r_{l_i^*, l_j} = \text{"false"}),$$

$r_{l_i^*, l_i^*}^*$  = “there is a token  $\alpha$  at an input place of the transition for which  $TIME - \theta_K(\alpha) = \theta^*(\alpha)$ ”. When the mediator token enters place  $l_i^*$  it obtains as characteristic the name and the characteristics of the token with duration of life  $\alpha$ . If

$$M = pr_6 Z_i = [L', L'', \{m_{l_i, l_j}\}]$$

is the index matrix of the capacities of the arcs, then

$$M^* = pr_6 Z_i^* = [L' \cup \{l_i^*\}, L'' \cup \{l_i^*\}, \{m_{l_i^*, l_j^*}^*\}],$$

where

$$\begin{aligned} (\forall l_i \in L') (\forall l_j \in L'') (m_{l_i, l_j}^* &= m_{l_i, l_j}), \\ (\forall l_i \in L') (\forall l_j \in L'') (m_{l_i, l_i^*}^* &= m_{l_i^*, l_j} = 0), \\ m_{l_i^*, l_i^*}^* &= 1 \end{aligned}$$

$$\pi_{L_i}^* = \pi_L \cup \pi_{\{l_i^* | Z_i \in A\}},$$

where function  $\pi_{\{l_i^* | Z_i \in A\}}$  determines the priorities of the new places, that are elements of the set  $\{l_i^* | Z_i \in A\}$  and the priorities of the  $l_i^*$  places for every transition  $Z_i \in A$  are smaller than the priorities of all other places of the transition  $Z_i$ .

$$c_L^* = c \cup c_{\{l_i^* | Z_i \in A\}},$$

where the function  $c_{\{l_i^* | Z_i \in A\}}$  is defined as

$$c_{\{l_i^* | Z_i \in A\}}(l_i^*) = 1$$

for all place  $l_i^*$ .

Let

$$E^* = \langle \langle A^*, \pi_A, \pi_L^*, c, f, \theta_1, \theta_2 \rangle, \langle K^*, \pi_K^*, \theta_K^* \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi^*, b \rangle \rangle,$$

where

$$K^* = K \cup_{l_i^*} K_{l_i^*}$$

Here  $K_{l_i^*}$  is the set of the mediator tokens that enter the net through the places  $l_i^*$ .

$$\theta_K^* = \theta_K \cup \theta_{\{l_i^* | Z_i \in A\}},$$

where function  $\theta_{\{l_i^* | Z_i \in A\}}$  determines that each mediator token  $\alpha_i^*$  stays in the initial time-moment  $T$  in its place.



$$\Phi^* = \Phi \cup \Phi_{\{l_i^* | Z_i \in A\}},$$

where function  $\Phi_{\{l_i^* | Z_i \in A\}}$  determines the characteristics of the mediator tokens  $\alpha_i^*$  in the form

$$\Phi_{\{l_i^* | Z_i \in A\}}(l_i^*) = \langle \alpha'', x_{cur}^\alpha, l_i' \rangle$$

where  $x_{cu}^\alpha$  is the current characteristic of the token with duration of life  $\alpha$ . The time components of  $E^*$  remain the same as in  $E$ .

The problem of choosing successors for tokens with duration of life will be discussed in the next section. In this framework we suppose that the successor of the token  $\alpha$  is a token  $\alpha_s$  which enters some input place for the transition  $Z_i$ . If this input place is the same place where the token  $\alpha$  left the net, then it obtains the characteristics of the token  $\alpha$  which are preserved in the second component of the mediator token  $\alpha_i^*$ . If the successor of  $\alpha$  is in some other input place of the transition, then it will obtain the characteristics preserved in  $\alpha_i^*$  when it transfers to some output place  $l_j''$ .

### 3 Generalized Net that determines the successors of the tokens with duration of life

For a given GNTDL

$$E^* = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K, \theta^* \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi, b \rangle \rangle,$$

we construct an ordinary GN  $E$  that determines which of the tokens will inherit the characteristics of the tokens with duration of life.

Let the tokens from the set  $K_2$  of  $E^*$  be denoted by  $\alpha_i$ , where  $1 \leq i \leq n$  and  $n = |K_2|$ . We denote the tokens of the net  $E$  by  $\beta_1, \beta_2, \dots, \beta_n$ . Every token  $\beta_j \in pr_1 pr_2 E$  corresponds to a token  $\alpha_j \in K_2$ . The tokens  $\beta_j$  enter GN  $E$  at place  $l_1$  with initial characteristics  $x_{\beta_{io}} = \langle \theta^*(\alpha_j), l_j \rangle$  where  $l_j$  denotes the input place in which the corresponding  $\alpha_j$  token enters  $E^*$ .

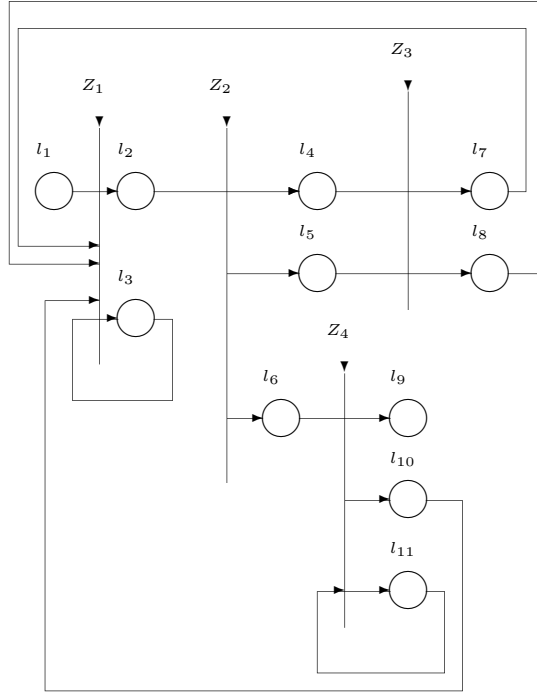


Fig.3 Generalized net that selects the successor tokens

$$Z_1 = \langle \{l_1, l_3, l_7, l_8, l_{10}\}, \{l_2, l_3\}, \begin{array}{c|cc} & l_2 & l_3 \\ \hline l_1 & \text{false} & \text{true} \\ l_3 & W_{3,2} & W_{3,3} \\ l_7 & \text{false} & \text{true} \\ l_8 & \text{false} & \text{true} \\ l_{10} & \text{false} & \text{true} \end{array} \rangle,$$

where

$W_{3,2}$  = "there is a token  $\alpha_i \in K_2$  for which  $TIME - \theta_K(\alpha_i) = \theta^*(\alpha_i)$ "  
 When truth-value of the predicate  $W_{3,2} = \text{true}$  the  $i$ -th token  $\beta_i$  enters place  $l_2$  without obtaining new characteristic.

$$W_{3,3} = \neg W_{3,2}$$

At place  $l_3$  every token  $\beta_i$  obtains as characteristic the current position of the corresponding token  $\alpha_i$  and its remaining duration of life  $\theta^* - TIME + \theta_K(\alpha_i)$ .

$$Z_2 = \langle \{l_2\}, \{l_4, l_5, l_6\}, \begin{array}{c|ccc} & l_4 & l_5 & l_6 \\ \hline l_2 & W_{2,4} & W_{2,5} & W_{2,6} \end{array} \rangle,$$

where

$W_{2,4}$  = "there are tokens with duration of life at the current place of the token  $\alpha_i$  which can be transferred to output place at the current time step"

$W_{2,5} = \neg W_{2,4}$  & "there are tokens with duration of life in the other input places of the same transition"

$W_{2,6} = \neg W_{2,4} \& \neg W_{2,5}$

At place  $l_4$  the token  $\beta_i$  obtains as characteristic the label of the place where the corresponding token  $\alpha_i$  is at the current time moment. At place  $l_5$  the token  $\beta_i$  obtains as characteristic the labels of the input places of the transition where the corresponding token  $\alpha_i$  is at the current time moment.

$$Z_3 = \langle \{l_4, l_5\}, \{l_7, l_8\}, \begin{array}{c|cc} & l_7 & l_8 \\ l_4 & W_{4,7} & W_{4,8} \\ l_5 & W_{5,7} & W_{5,8} \end{array} \rangle,$$

where

$W_{4,7}$  = "the successor should be determined by rule A "

$W_{4,8}$  = "the successor should be determined by rule B "

$W_{5,7}$  = "the successor should be determined by rule A "

$W_{5,8}$  = "the successor should be determined by rule B "

At place  $l_7$  the tokens obtain as characteristic the label of the token with the highest priority among the places in the last characteristic of  $\beta_i$ . At place  $l_8$  the tokens obtain as characteristic the label of the token with the greatest remaining duration of life among the places in the last characteristic of  $\beta_i$ .

$$Z_4 = \langle \{l_6, l_{11}\}, \{l_9, l_{10}, l_{11}\}, \begin{array}{c|ccc} & l_9 & l_{10} & W_{11} \\ l_6 & W_{6,9} & false & W_{6,11} \\ l_{11} & false & W_{11,10} & W_{11,11} \end{array} \rangle,$$

where

$W_{6,9}$  = "there is no mediator token in the current transition"

$W_{6,11} = \neg W_{6,9}$

$W_{11,10}$  = "there is a token with duration of life at the input places of the transition where the corresponding token  $\alpha_i$  is at the current time moment"

$W_{11,11} = \neg W_{11,10} \& "TIME < T + t^*"$

$W_{11,9} = \neg W_{11,11} \& \neg W_{11,10}$

In place  $l_9$  the token obtains the characteristic "there is no successor for the token". In place  $l_{11}$  it does not obtain any characteristic and in place  $l_{10}$  it obtains as characteristic the label of the place where the token with duration of life is at the current time moment.

## References

- [1] Atanassov K., Generalized Nets, World Scientific, Singapore, 1991.
- [2] Atanassov K., On Generalized Nets Theory, Prof. Marin Drinov Publishing House, Sofia, 2007.

**The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.**

**It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:**

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**The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.**

**We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.**

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