## COMPUTER AIDED DESIGN OF VIBRATING STRUCTURES ACCOUNTING FOR MATERIAL FATIGUE AND RELIABILITY

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The numerical method for prediction of fatigue life of structure is presented. The finite element modeling and damage parameter calculation are based on algoritm described originally in [1].

As an example the optimization of double – disk rotor shaft system, presented if Figure 1, based on sensitivity analysis followed by the fatigue life estimation of optimized shaft is considered. The objective of optimization is to avoid resonance, which can cause excessive stresses leading to prematural fatigue cracks. Sensitivities of eigenvalues with respect to design variables are calculated by the Direct Differentiation Method (DDM) [2,3]. The optimization is performed with the objective to move natural frequencies as far as possible from resonance frequency. Next the shaft system is dynamically loaded. Fluctuation of stress established from FEM analysis by numerical code FEAP [4] are stored, cycles are counted by rainflow method and next fatigue life estimation is performed. Design parameters are diameters of selected shaft parts.



Figure 1. Double - disk rotor shaft system before and after optimization.

Damage accumulation hypothesis is used in order to take into account all components of dynamic load [5]. An accumulated damage caused by all cycles in block can be calculated.

The idea of the equivalent amplitude stress described in [6] is used, in order to calculate the number of cycles to failure of the analyzed structure loaded by arbitrary non-symmetric load with any stress ratio.

Fatigue durability may be expressed by number of cycles to expected failure N or hours of safety work of the structure [7]. Design life and probability based fatigue factors are calculated as so called Factor of Strength (FOS). It shows how much the component is over or under strength in terms of Finite Element Analysis (FEA) stresses.

The FOS is the ratio of the strength of material to the working stress. The FOS value is calculated using the Goodman mean stress correction. Referring to Figure 2, which shows the Goodman line in coordinates ( $S_a$ ,  $S_m$ ), the FOS is given by

$$FOS = \frac{OB}{OA} \tag{1}$$

Effects of unloading on fatigue life is also calculated [8]. The simplified reliability analysis makes use of fuzzy set approach to the basic fatigue data such as material data, load parameters etc. [8].



Figure 2. Factor of Strength based on Goodman line.  $S_a$  - stress amplitude,  $S_m$  - mean stress,  $S_y$  - yield stress,  $S_u$  - ultimate tensile strength,  $K_f$  - stress concentration factor.

## Short summary

Present work deals with optimization performed on the basis of exact sensitivity. Optimization with objective to avoid resonance allows to eliminate undesirable vibration, noise and considerable increase fatigue life.

Equivalent amplitude stress is very effective tool in fatigue analysis in engineering cases, when fatigue data are provided only for stress ratio R = -1, but also in dynamic fatigue analysis where rainflow algorithm is used.

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