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Abstract
Stochastic averaging methods have been attracting more and more attention of researchers dealing with various engineering problems during last decade. A considerable number of applications have been based on the, so called, weakly correlated random fields theory (WCRFT), developed by Vom Scheidt at al. Independently, in many scientific centers another approach, originated by Hasminskij has been developed. In this paper we show, that both these approaches lead to exactly the same first order approximation when applied to an analysis of asymptotic distribution of solutions of stochastic initial value problems (treated in terms of Volterra equations). This fact supports a conjecture that the fundamental assumption of the WCRFT (see sec. 1, 2) can be, at least in many important cases, violated without loss of the properties of the first approximation. In spite of many applications, already worked out and those not undertaken yet, such an extension of WCRFT seems to be of a great importance.

1. Introduction

Randomnes of various technological phenomena or, at least, purposefulness of stochastic modelling of many important engineering systems is not questionable nowadays. However, the questions arise always when one has to decide which quantities are to be taken into account and which can be omitted during analysis and modelling. Even the smallest observable (or, even nonobservable) quantities may, in some situations, significantly influence a model accuracy. On the other hand, a really efficient model must be parsimonious. Thus, it is of a great importance to know if some factors entering the model can be neglected (if this neglection does not lead to the loss of important information about the system under consideration).

One of such situations arises where random processes acting in the system are "weakly correlated". The correlations observed and estimated from real recorded data often indicate that the amount of correlation between the values of a certain random function is small provided that the points at which the measurements were made are not closer than by a finite "correlation length", ε. However, such an estimation can never justify any rigorous distinction between the following two hypotheses: H_0 - the correlation is zero, and, H_1 - the correlation is small. Thus, to conclude whether the correlation is negligible, one has to analyse the effects which might be lost by dropping small (but non-zero) correlations.

The theory of weakly correlated random fields (WCRFT) developed by Vom Scheidt, Purkert and others in seventies and eighties proved to be very useful in many branches of modern engineering analyses. Dozens of practical problems were approached through this theory (cf.[1-4]). The main assumption lying behind this theory is, roughly, the neglection of all moments concerning values (of a random field) taken at such ansambles of points which can be split into subsets distant by more than ε from each other. Most of the applications mentioned show good qualitative and quantitative agreement with experiments performed on real structures. Some of the theoretical predictions, however, are only of qualitative character. In both cases it would be desirable to have the proof of evidence that it is not the "finite correlation assumption" which is the source of potential modelling discrepancies.

Similar problems connected with the, so called, stochastic averaging procedures became more and more attractive during last years (cf. [5–25] for different aspects undertaken). Besides of the above mentioned theory the most popular approach is based on the Hasminskij's paper [5]. As this approach (HA) does not require that the correlation vanishes outside a finite correlation interval and, alike the WCRFT, ends with the asymptotic result for probability distributions it has been found suitable for the analysis of effects introduced (or lost) by assuming zero correlation outside the finite correlation interval.

In this paper it is shown that the application of both theories to an analysis of asymptotic distribution of solutions of stochastic initial value problems (treated in terms of Volterra equations) leads to exactly the same first order approximation.

Similar comparison can be performed also for Fredholm equations (see [4] for the treatment of such equations through WCRFT) or, for initial-boundary problems (cf. [1], [2] for WCRFT analyses).

Also other theories can be compared with WCRFT in this way. However, as they are formulated in terms of convergence in near-mean-square norm (see Mac Shane [8, 9]), in probability or with probability one (see e.g. Wong-Zakai [6, 7], Sussman [13]) the analysis would not be straightforward. Anyway, the first order approximate models resulting from all these approaches would be equivalent to that obtained through HA.

2. Fundamental Limit Theorem of WCRFT

The essential results of the WCRFT, as summarized in the Introduction to the monograph [3], are contained in the Chapter 2 of [3], and are as follows. First, the definition of a weakly correlated field is given and some existence theorems are proved. For instance, for each given $\varepsilon > 0$ a weakly correlated random field exists having the correlation length ε. Furthermore, weakly correlated fields exist possessing sufficiently smooth realizations which are bounded almost surely. Finally, weakly correlated connected vector fields are also introduced. As it is underlined, all further results are based upon the limit theorems for linear functionals of weakly correlated fields proved in section 2.1.2 of [3]. In what follows the definition of weakly correlated random field is given and the limit theorem which is a prototype of a series of limit theorems obtained from this by generalizations or specializations will be presented. In fourth section, this theorem will be proven with more general assumptions, in one dimensional case, with the help of ideas leading to the result of Hasminskij [5].

Let us start from some definitions (the first definition is slightly changed with respect to the original definition from [3] to avoid an error in the proof of subsequent lemma; the modifications follow the general outline taken in the subsequent monograph of Vom Sheidt, being under publication procedure).

DEFINITION. *Let* $\{x_i,\ i \in I\}$, $I = \{1, 2, \ldots, k\}$ *be a finite set of points from* R^n *and* $\varepsilon > 0$ *an arbitrary real number. A subset*

$$\{x_i,\ i \in \bar{I}\}, \qquad \bar{I} = \{i_1, i_2 \ldots, i_l\}$$

is said to be ε-adjoining *if*

$$\forall i \in \bar{I}\ \exists j \in \bar{I} \quad \text{s.t.} \qquad |x_i - x_j| \leq \varepsilon.$$

The subset $\bar{I}$ *is said to be* maximum ε-adjoining *relative to* $\{x_i,\ i \in I\}$ *if it is* ε*-adjoining but the subset* $\{x_i,\ i \in \bar{I}\} \cup \{x_r\}$ *is not* ε*-adjoining for every* $x_r \in \{x_i,\ i \in I \setminus \bar{I}\}$.

The following lemma can be proven likewise lemma 2.1 from [3].

LEMMA

Every finite set $\{x_i,\ i \in I\}$ of points from R^n decomposes uniquely into mutually exclusive maximum ε-adjoining subsets $\{x_i,\ i \in I_j\}$, $j = 1, 2, \ldots, p = p(\{x_i,\ i \in I\})$.

DEFINITION. *A random field* $f_\varepsilon(x, \omega)$, $x \in \mathcal{D} \subset R^n$, *with* $\mathbf{E} f_\varepsilon \equiv 0$, *is called* weakly correlated *with correlation length* ε *if the relation*

$$\mathbf{E} \prod_{i \in I} f_\varepsilon(x_i) = \prod_{j=1}^{p} \mathbf{E} \prod_{i \in I_j} f_\varepsilon(x_i)$$

is satisfied for all k*-th moments,* $k = 2, 3, \ldots,$ *where* $I = \{1, 2, \ldots, k\}$ *and*

$$\{x_i,\ i \in I_1\}, \quad \{x_i,\ i \in I_2\}, \ \ldots, \ \{x_i,\ i \in I_p\}$$

with $\bigcup_{j=1}^{p} I_j = I$ *denotes the decomposition of* $\{x_i,\ i \in I\}$ *into the maximum* ε*-adjoining subsets.*

It can be easily deduced from the above definition that, in particular, the correlation function of the weakly correlated random field must necessarily vanish outside the ε-tube surrounding the diagonal: $x_1 = x_2$.

Now, the fundamental limit theorem of WCRFT, the theorem 2.7 from [3] is the following.

THEOREM. *Let* $(f_\varepsilon(x,\omega))_{\varepsilon\downarrow 0}$ *be a sequence of weakly correlated fields on a bounded domain* $\mathcal{D} \subset R^n$ *with smooth boundary, and let* f_ε *possess continuous sample functions a.s. and*

$$\mathbf{E}|f_\varepsilon^k(x,\omega)| \leq c_k < \infty$$

for all $k \geq 1$. *The intensity of the weakly correlated field is defined by*

$$a(x) = \lim_{\varepsilon\downarrow 0} \frac{1}{\varepsilon^n} \int\limits_{K_\varepsilon(0)} \mathbf{E} f_\varepsilon(x) f_\varepsilon(x+y)\, dy.$$

Then the convergence in distribution, that is

$$\lim_{\varepsilon\downarrow 0} \left(r_{1\varepsilon}(x,\omega), r_{2\varepsilon}(x,\omega), \ldots, r_{l\varepsilon}(x,\omega)\right) = \left(\xi_1(x,\omega), \xi_2(x,\omega), \ldots, \xi_l(x,\omega)\right)$$

is obtained for the random fields

$$r_{i\varepsilon}(x,\omega) = \frac{1}{\sqrt{\varepsilon^n}} \int\limits_{\mathcal{D}} h_i(x,y) f_\varepsilon(y)\, dy, \qquad i = 1,2,\ldots,l; \quad x \in \mathcal{G},$$

where $h_i : \mathcal{G} \times \mathcal{D} \subset R^p \times R^n \to R$ *are square integrable over* $\mathcal{D}$ *functions of the second variable,* y*, for any fixed value of the first variable,* $x \in \mathcal{G}$*, where* $\mathcal{G}$ *is an arbitrary subset of* R^p*. Moreover, the random vector field* $(\xi_1(x,\omega), \xi_2(x,\omega), \ldots, \xi_l(x,\omega))$ *is a Gaussian vector field with mean 0 and correlation relations*

$$\mathbf{E}\,\xi_i(x)\xi_j(y) = \int_{\mathcal{D}} h_i(x,z)h_j(y,z)a(z)\,dz$$

for $i, j = 1, 2, \ldots, l$.

3. Stochastic Initial Value Problem.

Consider the following stochastic initial value problem (cf. [5]).

$$Y'(t) = \varepsilon F(Y(t), t, w, \varepsilon), \qquad Y(0) = Y_0, \tag{1}$$

where $Y_0 = Y(\omega)$, $Y = Y(t,\omega) \in R$, $F : R \times \Omega \times [0;\infty) \to R^n$, $\omega \in \Omega$, $\{\Omega, \mathcal{F}, \mathcal{P}\}$ is a complete probability space.

A1. Suppose that for $\varepsilon \to 0$ the function F can be decomposed in the following form, uniformly in y, t.

$$F(y,t,\omega,\varepsilon) = F^0(y,t,\omega) + \varepsilon F^1(y,t,\omega) + o(\varepsilon), \tag{2}$$

where F^0, F^1 are uniformly bounded together with their partial derivatives with respect to y up to the second order, and, for each fixed y, as functions of (t,ω) are measurable stochastic processes.

Define:

$$\phi^i(y) = \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} \mathbf{E}[F^i(y,s,\omega)]\,ds, \qquad i = 0,1$$

$$K(y) = \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} ds \int_{t-T}^{s} \mathbf{E}[\frac{\partial F^0}{\partial y}(y,s,\omega)F^0(y,r,\omega)]\,dr$$

$$A(y) = a_{ij}(y)$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} ds\int_t^{t+T} \mathbf{E}[F^0(y,s,\omega)F^{0T}(y,r,\omega)]\,dr$$

A2. It is assumed that the above limits exist, the convergence is uniform with respect to (t,y).

A3. Let also the following assumptions be satisfied.

$$\phi^0 \equiv 0$$

A4. Both integrals

$$\int_t^{t+T} \mathbf{E}\,[\,F^0(y,s,\omega)\,]\,ds$$

and

$$\int_t^{t+T} \frac{\partial}{\partial y}\mathbf{E}\,[\,F^0(y,s,\omega)\,]\,ds$$

are uniformly (in (y,T)) bounded.

A5. $\exists (T_n) \quad \exists p, q > 0$ s.t.

$$T_n < T_{n+1} < pq^n \qquad \forall n = 1, 2, \ldots,$$

and

$$\lim_{n\to\infty} \delta(T_n) = \lim_{n\to\infty} \sup_{x\in R^n, t>T_n} \left| T_n^6 \int_t^{t+T_n} \mathbf{E}[F^0(y, s, \omega)]\, ds \right| = 0.$$

A6. $\exists \{\mathcal{F}_s^t\}_{0\le s\le t\le\infty}$ — a family of sub-σ-fields of $\mathcal{F}$, such that

$$\mathcal{F}_{s_1}^{t_1} \subset \mathcal{F}_{s_2}^{t_2} \qquad \text{if} \qquad (s_1; t_1) \subset (s_2; t_2),$$

$\exists \beta : [0; \infty) \to R$, — decreasing, and such that

$$\lim_{t\to\infty} t^6 \beta(t) = 0,$$

$\forall x \in R_n \quad \forall T, t > 0 \quad \forall B \in \mathcal{F}_{t+T}^{\infty}$

$$\sigma\big(F^0(x, t, \omega), F^1(x, t, \omega)\big) \subset \mathcal{F}_t^t$$

and

$$|\mathcal{P}(B|\, F_0^t) - \mathcal{P}(B)| < \beta(T).$$

Assumption A6, the, so called, mixing condition is just that condition by which we intend to replace the assumption of finiteness of correlation length. Note that any stochastic process which is weakly correlated in the sense of definition of sec. 2 satisfies the strong mixing condition cited above. Obviously, the oposite statement is false.

Theorem (cf. Khasminskij, [5])

Suppose that the assumptions A1–A6 are satisfied. Then, for any fixed $\tau_0 > 0$ *the sequence of processes* $X^\epsilon(\tau) := Y(\varepsilon^2 t)$ *(where* $\tau = \varepsilon^2 t$*) converges weakly on the interval* $[0;\tau_0]$ *to the solution* X^0 *of the following Itô equation:*

$$dX(t) = (K + \phi^1)(X(t))dt + A^{1/2}(X(t))\, dW(t),$$
$$X(0) = Y_0.$$

The probabilistic characteristics of the limit process are thus as follows.

$$\mathbf{E}[\Delta X^0(t) | X^0(t) = x] = (K(x) + \phi^1(x))\, \Delta\tau + o(\Delta\tau),$$
$$\mathbf{E}[\Delta X^0(t)\, \Delta X^0(t) | X^0(t) = x] = A(x)\Delta\tau + o(\Delta\tau).$$

Remark. Since it is the weak convergence which is considered above one can equally well put $F^i = F^i(y,t,\omega,\varepsilon)$ into eqn.(2) (instead of $F^i = F^i(y,t,\omega)$) as long as the probabilistic characteristics ϕ, K, A remain independent of ε.

4. Deterministic Equation with Multiplicative Noise.

Let us consider a special case where

$$F^1 \equiv 0, \quad F^0(y,t,\omega,\varepsilon) \equiv G(t) f(t,\omega,\varepsilon), \quad G(t) := H(t/\varepsilon^2),$$

function H is square integrable, and $f(\cdot,\cdot,\varepsilon)$ is any family of stochastic processes satisfying the following assumption.

A7. The family $f(\cdot,\cdot,\varepsilon)$ consists of measurable, zero mean stochastic processes, such that the assumptions A2–A6 hold true for function $F(y,t,\omega,\varepsilon) = f(t,\omega,\varepsilon)$ along with the assertion of the remark, that is, the limit

$$A = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} ds \int_t^{t+T} \mathbf{E}[f^T(s,\omega,\varepsilon) f(r,\omega,\varepsilon)]\, dr$$

does not depend on ε.

LEMMA

Let $f_\varepsilon(\tau,\omega)$ be a family of zero mean w.c.r.f.-s with continuous sample-paths and with correlation lengths ε^2 and such that the intensity function

$$a(\tau) = \lim_{\varepsilon\to 0} \frac{1}{\varepsilon^2} \int_{-\varepsilon^2}^{\varepsilon^2} \mathbf{E}[f_\varepsilon(\tau,\omega) f_\varepsilon(\tau+s,\omega)]\, ds$$

exists and is bounded on the set $\mathcal{D} = [0;\tau_f]$. Then there exists a family of w.c.r.f.-s with continuous sample-paths and with correlation lengths 1, satisfying the assumption A7 *and such that*

$$f(\tau/\varepsilon^2,\omega,\varepsilon) = f_\varepsilon(\tau,\omega) + o(\varepsilon^2)$$

The proof of this lemma consists of two steps. Firstly, it is easy to check that all assumptions A2–A6 hold true for

$$\tilde{f}(\tau/\varepsilon^2,\omega,\varepsilon) = f_\varepsilon(\tau,\omega)$$

as these processes are of zero mean, are independent on the space variable and, as w.c.r.f.-s with correlation length 1, are strongly mixed. To accomplish the proof it is enough to notice that the family $\tilde{f}$ can be modified to a form f by addition of terms small with respect to ε^2, as ε tends to 0, in such a way that the additional condition of assumption A7 is satisfied — that the limit A for f is independent of ε. This possibility follows from the existence of intensity a for initial family f_ε.

The formal application of the thesis of Hasminskij's theorem recalled in sec. 2, in the present case, yields for the limit process

$$X^0(\tau_l) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \int_D H(\tau) f(\tau,\omega)\, d\tau$$

the following expressions:

$$K = \phi^1 \equiv 0 \qquad \text{and} \qquad A = \int_D H(\tau) H^T(\tau) a(\tau)\, d\tau$$

determining the probabilistic characteristics of the limit distribution.

Let us sketch the proof of the above limit result. To justify these formally derived formulae it is necessary to prove two facts.

Firstly, it is easy to notice that the multiplication of processes f by a space independent, square integrable function of time has no influence on the properties of function F which are required for assumptions A2–A6.

Secondly, to arrive at the thesis of Hasminskij's theorem without assumption A1 the specific features of the case considered must be employed. In the present case, as the function F is space independent, assumption

A1 reduces essentially to the uniform boundedness with probability one. This property is used three times in the proof of Hasminskij's theorem. In the proof of Lemma 2.2 (p.452 in [5]) and in the proof of lemma 2.5 (p.454 [5]) it was necessary to obtain some estimates on approximations of solution process resulting from discretization of function F with respect to the spatial variable.

In our case the thesis of Lemma 2.2 is obvious and the estimate necessary for lemma 2.5 follows from spatial independence of F. Similarly, in the proof of Lemma 2.7 (pp. 456, 457 [5]) an estimate based upon the assumption A1 is employed. This lemma was necessary to gain an estimate on the second moment of increments of solutions to prove the weak compactness of measures induced by the sequence of these solutions. Again, in our case the necessary estimate follows directly from the existence of the limit A and from independence of function F of the space variable. Thus, the rest of the proof applies and the use of the thesis is justified.

Now, it is noticed that the envelope (slowly varying) function, $H(\cdot)$, which takes multidimensional values can be treated as a doubly indexed scalar function $h_i(\sigma, \cdot)$. Thus, the above justified result can be rewritten in the form of the following

COROLLARY

For the sequence of stochastic processes f_ϵ, *satisfying the assumption* A7 *and for the family of envelopes* $h_i(s,t)$ *square integrable with respect to* t *the family of random fields*

$$X_i^\epsilon(\sigma,\tau,\omega) = \frac{1}{\varepsilon}\int_0^\tau h_i(\sigma,s) f_\epsilon(s,\omega)\, ds$$

converges weakly to the Gaussian centered random field

$$\mathcal{X} = \mathcal{X}(\sigma, \tau) = [X_1^0(\sigma, \tau), \dots, X_r^0(\sigma, \tau)]^T$$

whose correlation function is as follows.

$$\mathrm{E}[\mathcal{X}_i^0(\sigma_1, \tau_1) X_j^0(\sigma_2, \tau_2)] = \int_0^{\tau_1 \wedge \tau_2} h_i(\sigma_1, s) h_j(\sigma_2, s) a(s)\, ds.$$

It is evident that, due to the lemma concerning w.c.r.f.-s satisfying assumption A7, the above corollary can be applied to f being weakly correlated in the sense of sec. 2. In this case, as it shoud be, the corollary matches exactly the limit theorem of section 2. However, it should be underlined, that the assumptions of corollary are satisfied by processes f which are not necessarily of finite correlation length.

5. Closing Remarks.

The above comparison was concerned with a special situation. In an one-dimensional, particular case a generalization of the fundamental limit theorem 2.7 from [3] was obtained. The corollary of sec. 4 generalizes theorem 2.7 in that a domain, $\mathcal{D}$, may be variable and an external random field, f, needs not be a weakly correlated random field in the sense of the definition of sec. 2 — it needs not have a finite correlation length, in particular (of course, it is a restriction of 2.7 to one-dimensional domains $\mathcal{D}$).

Despite of its peculiarity this result strongly indicates that also other tools dveloped within the asymptotic theory of WCRFT do not necessarily require the finiteness of correlation radius.

It seems to be also a prompting task to explore the possibility of extension of WCRFT to the case where a space dependent kernel would be employed. A more elaborate study would be necessary to this aim, of course.

A WCRFT is valuable in that it delivers also some results concerned with higher order approximations. The problem of generalization of these results to the case of random fields whose correlation does not vanish outside of finite correlation length is also a chalanging one. However, it seems to be difficult to employ to such problems a methodology similar to the one used in this study — in the proof of Hasminskij's theorem the theory of diffusion Markov processes was used.

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