STATISTICS OF WAITING TIMES BETWEEN SUDDEN CLIMATE CHANGES AS A TOOL FOR IDENTIFYING POSSIBLE CAUSES

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Sudden climate shifts between two different climatic states have been identified in ice core records covering the last ice age. There has been a long-standing argument among climate scientists on whether these shifts are due to some hitherto unknown external periodic influence maybe from the Sun, or if they are due to internal chaotic fluctuations. In the latter case these fluctuations would most likely occur erratically, with a "memory" reflecting the short time scales of the weather fluctuations.

The erratic appearance of the oxygen isotope record ($\delta^{18}O$) shown and explained in the figure below makes it natural to think of the record as a random response of a stochastic noise driven system. However, the saw tooth character of the sample curve makes it not quite obvious how such a system can be identified from the sample. While the oxygen isotope record is a proxy for the temperature (first pointed out by Dansgaard) there are other climate proxies embedded in the ice cores. One of these is the dust sedimentation recognized by the variation of the calcium ion concentration down through the ice cover as shown in the bottom panel of the figure [log (Ca)]. The DO-events are clearly recognized in this dust record, and no obvious saw tooth behavior is seen.

In fact, a quite good suggestion of a stochastic model is given in [1]. The model gives responses with statistical properties close to those of the dust record. It is shown that the marginal distribution (after suitable first and second moment normalization of the record) as well as the DO-event behavior are well modeled by the stochastic differential equations $\mathrm{d}Y = -(\mathrm{d}U/\mathrm{d}Y)\,\mathrm{d}t + \sigma_1\,\mathrm{d}X + \sigma_2\,\mathrm{d}L$, $\mathrm{d}X = -X\,\mathrm{d}t + \sqrt{1+X^2}\,\mathrm{d}B$, where U is a bistable potential inferred from the data, $\mathrm{d}B$ is standard Brownian noise, $\mathrm{d}L$ is Levy noise with stability index $\alpha = 1.75$, and $\sigma_1/\sigma_2 = 3$. The stationary marginal distribution of the X-process is the Cauchy distribution with density proportional to $1/(1+x^2)$.

It is a challenge to find a similar model for the oxygen isotope record. The problem is to capture the gradual cooling that takes place after each DO-event (the saw tooth behavior). Due to strong negative correlation between the $\delta^{18}O$-record and the log(Ca)-record (even though they are from two different locations) it seems reasonable to look for an extension of the established stochastic differential equation model for the log(Ca)-record.

However, before going on in this direction some preliminary much simpler statistical investigations of the time point series of the DO-events are highly relevant. Even though the analysis is simple and within the toolbox of the elementary statistics textbook, some points of the analysis are interesting. In particular the problem that appear because of the small sample size deserves attention. The simplest possible well fitting model of memoryless random point generation turns out not to be distinguishable from a model built on random deviations from a periodic forcing of the DO-events. It seems that the principle of simplicity of description (known as Occam's razor) is the only way to favor the assumption of generation by pure randomness.

This will be the topic of the presentation even though the work has already been published thanks to a quite fast publication policy of the Journal of Climate [2].

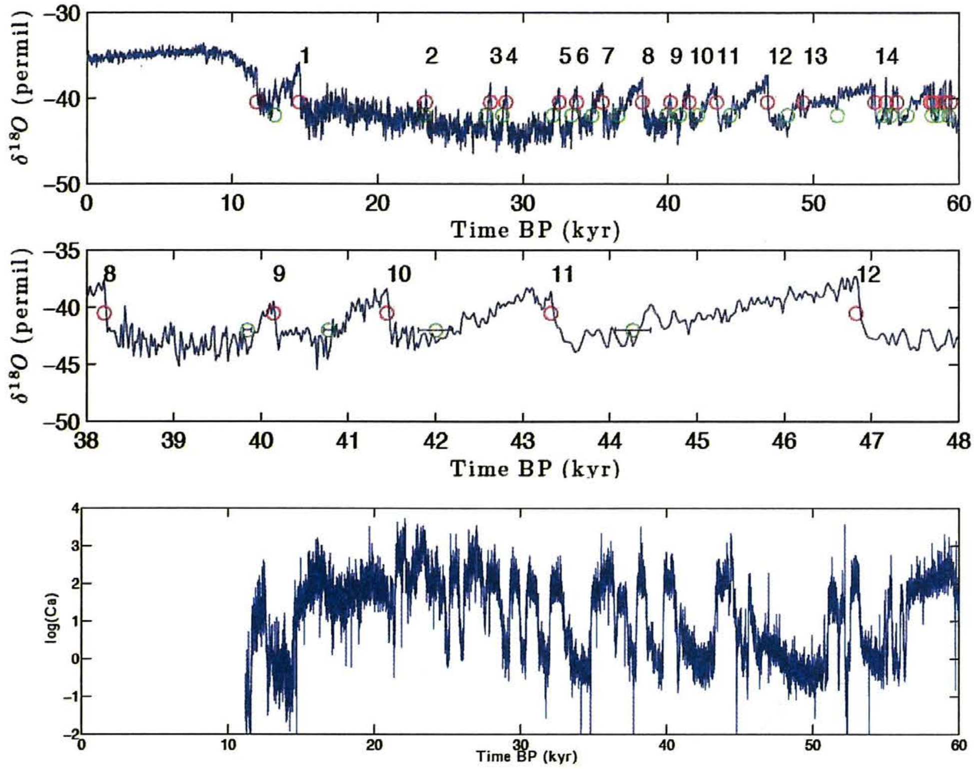

Figure 1: The upper panel shows the $\delta^{18}O$ oxygen isotope record measured down through the ice cover at a given location in central Greenland (the NGRIP ice core project). The upper circles mark the transitions from the stadial cold climate to the interstadial warm climate (off-on points). These are the so-called Dansgaard-Oeschger (DO) events. The lower circles mark the transitions back from the interstadial to the stadial climate (on-off points). The middle panel zooms into the period 38-48 kyr BP, where it is seen that the determination of the on-off transitions are much more uncertain than determining the sharp off-on transitions. This is indicated by errorbars. The bottom panel shows the logarithm of the calcium ion concentration as a function of time in the GRIP ice core. The temporal resolution is about 1 year, much better than for the $\delta^{18}O$ record that suffers from the effect of vapor diffusion. (For clarification it should be noted that the time scales in the top and the bottom panels differ by some factor implying that the DO events are not at the same nominal time on the two scales. Thus the numerically largest correlation is obtained with some linearly increasing time shift).

References

[1] P. D. Ditlevsen. Observation of alpha-stable noise and a bistable climate potential in an ice-core record. *Geophys. Res. Lett.*, 26:1441–1444, 1999.

[2] P. D. Ditlevsen and O. Ditlevsen. On the stochastic nature of the rapid shifts during the last ice age. *Journal of Climate*, 22:446–457, 2009.