STOCHASTIC AND CHAOTIC ANALYSIS OF SHALLOW CABLES DUE TO CHORD LENGTH ELONGATIONS

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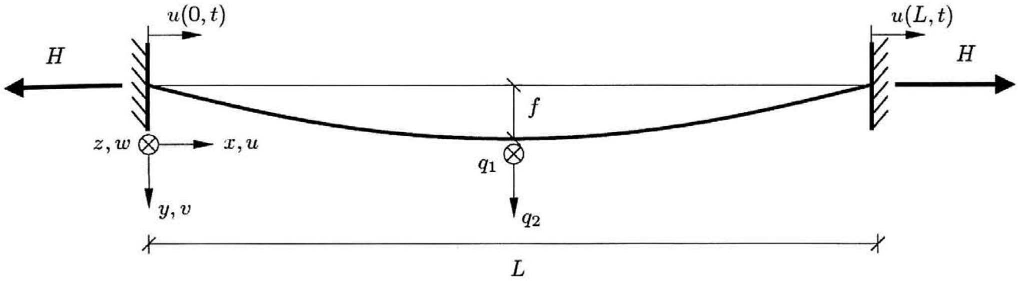

Figure 1. Cable in static equilibrium configuration.

Cable systems are of great interest in a wide range of applications in civil engineering to supply both support and stiffness to large structures. Typically, cables are used as support of cable-stayed bridges, masts and TV-towers are characterized by a sag-tochord-length ratio below say 0.01, which means that the angular eigenfrequencies for the in-plane eigenvibrations ω_2, $\omega_4,\dots$, and the out-of-plane eigenvibrations $\omega_1, \omega_3, \dots$ are pairwise close. With reference to the coordinate system defined in figure 1 the components of the support point motion in the y and z directions merely introduce additive load terms in the modal equations of motion of the cable, whereas the chord elongation $u(L,t) - u(0,t)$ along the x-axis causes additional parametric loading terms in the modal equations of motion, which may cause significant subharmonic and superharmonic responses. The chord elongation is conveniently described by the following non-dimensional parameter of the magnitude 1

$$e(t) = \frac{EA}{HL}\big(u(L,t) - u(0,t)\big) \tag{1}$$

where EA/L denotes the axial stiffness and H is the pre-stressing force. Even though the excitation only affects the in-plane motion, stable out-of-plane displacements may be brought forward by non-linear couplings in both harmonic, subharmonic and superharmonic responses.

When the chord elongation and hence $e(t)$ is harmonically varying with the e_0 and the angular frequencies ω stable stationary periodic motions exist for specific frequency ratios ω/ω_1. Figure 2a shows the trajectory of the midpoint of the cable for subharmonic response of order 2 for $\omega/\omega_1 = 2$. As seen, the in-plane modal coordinate $q_2(t)$ is rather small and harmonically moving with the same frequency as the excitation, whereas the out-of-plane coordinate $q_1(t)$ is large at subharmonic resonance with a frequency equal to half the excitation frequency. The stable trajectory is brought forward by a phase locking between the in-plane and out-of-plane components. However, in reality the chord elongation is narrow banded stochastic rather than harmonic varying, driven by the narrow-banded random response of the supported structure. In this case the subharmonic response of the cable changes dramatically, qualitatively and quantitatively, no matter

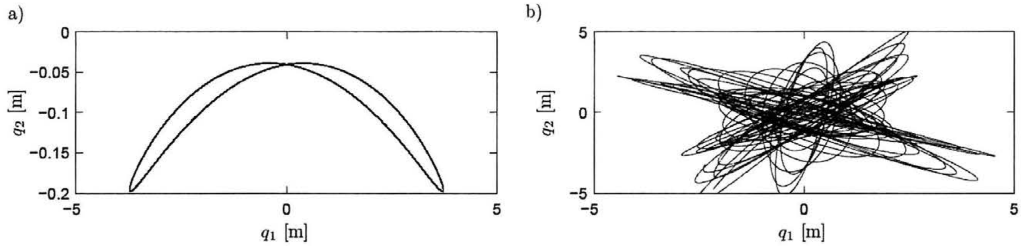

Figure 2. Subharmonic response of order 2. a) Harmonic varying chord elongation. b) Stochastic chord elongation.

how small the bandwidth of the excitation is. As shown in figure 2b, the in-plane and the out-of-plane components are coupled forming a elliptic like trajectory with slowly varying inclination and magnitude of the semi-axes.

The response is defined as chaotic with probability one, if two realizations with close initial values exposed to the same but arbitrary realization of the chord elongation process deviate exponentially with time. The exponential growth rate is measured by the Lyapunov exponent, which here is estimated numerically by the algorithm of Wolf et al. Chaotic behaviour occurs for sufficiently large standard deviation of the excitation process. In the paper stochastic chaotic response is investigated for subharmonic response of order two, and superharmonic response of the orders 3/2 and 2. It is demonstrated by means of Monte Carlo simulation that in all the indicated cases the tendency to stochastic chaotic behaviour is increased for increased standard deviation and increased bandwidth of the excitation process is increased. Further, the magnitude of the out-of-plane displacement is also dependent on the bandwidth, and ceases completely above a certain critical bandwidth parameter. Finally, it is demonstrated that stochastic excitation processes with the same auto-spectral density function, but different higher moments provide qualitatively identical stochastic ordered and chaotic responses, i.e. the dramatic influence of the stochastic excitation on the response is basically caused by the second order moments.

References

[1] A. Wolf, J.B, Swift, H.L. Swinney and J.A. Vastano (1985). Determining the Lyapunov exponents from time series. Physica, D16, 285-317.