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Optimal allocation of resources in risk reduction

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Existing buildings and facilities in accident-prone sites are sources of risks, from the viewpoints of possible loss of "life and limb", of deterioration of the "quality of life", and of direct and indirect economic losses. Risk reduction is (or should be) a major concern to owners and public authorities, and the available economic resources (usually limited) should be allocated in the most rational (or *optimal*) way. Several problems arise in this process, like the necessity of taking into account multiple objectives of the optimisation, the "system" behaviour that implies reciprocal influence of several facilities, the fact that the amount of resources is certainly limited but may be initially undefined so that its choice may become part of the optimisation itself, etc. These lectures present the methodology developed over several years (1991-99) to tackle some of these problems, with specific reference to preventive interventions for seismic risk reduction, and some examples of applications to buildings and road networks. The proposed methodology might be extended to other facilities and lifeline networks, and to other hazards (fires, floods, landslides, etc.). Open problems will be pointed out.

INTRODUCTION: How to formulate an optimal allocation problem?

These lectures originate from a series of researches on the techniques that can be used to formulate and solve the problem of optimal resource allocation in a campaign for seismic risk reduction of constructed facilities and lifeline networks, taking account of several objective functions. Indeed, this type of optimisation, which can be a decisive help in the formulation of a rational strategy for seismic risk reduction, had previously received little attention.

Let us start by defining *risk* as the product (better, the convolution integral) of three factors, namely:

- *hazard* – probability of occurrence of a dangerous event (the *action*): to each *risk*, a different *hazard* corresponds. These lectures will deal exclusively with *seismic risk*, and consequently with the *seismic action* (i.e. the *hazard* will be described by the probability of occurrence of an *earthquake* of each relevant *intensity*);
- *exposure* (or *exposition*) – probability that the *action* finds something that can be damaged;
- *vulnerability* – (conditional) probability that the object or facility is damaged when hit by the dangerous *action*. These lectures will deal exclusively with the *seismic vulnerability*.

According to these definitions, *risk* is the *absolute* probability of *failure* (or the integral of the probabilities of exceeding each degree of *damage*), while *vulnerability* is defined by the corresponding *conditional* probabilities.

To reduce risk, one must consider and should act on each of the three factors *(hazard, exposure, vulnerability)*. However, with reference to existing built facilities, it is difficult, if not impossible, to modify *hazard* and *exposure*: hence, *risk reduction* is usually identified with preventive interventions aimed at reducing the *vulnerability*.

In planning a risk-reduction campaign, the most common approach is to fix a quantitative target for the considered objective (e.g. that the probability of failure P_f must be below a certain threshold) and then to calculate the amount of resources necessary to attain that target.

It appears more rational to formulate an optimization problem, that in turn can be put in at least three alternative ways. Namely (indicating by C_{tot} the total cost of interventions, deterministic once they have been chosen, and by G_{tot} the *gain* or reduction of *expected losses*, always uncertain in whatever way it is defined and calculated):

I. just minimize the expected losses ($C_{tot} - G_{tot}$);
II. maximize the ratio between total gain and total expenditure (G_{tot}/C_{tot});
III. reverse the problem, and assume that the economic resources available for maintenance and interventions are limited (as indeed they usually are): therefore, to optimize will mean to maximize the total gain for a given maximum total expenditure (G_{tot}/C_{tot}).

The approach that we followed (and is illustrated in these lectures) is the third one, as it appeared more realistic: a problem of *optimal allocation of resources* was thus set up. In simple words, it is searched how one can make the most out of the limited resources available, without a pre-determined quantitative target.

As a prerequisite for implementing such an *optimal allocation of resources*, possible interventions on each facility must be designed, and their costs and expected *gains* (or *loss reduction*) evaluated. Since any intervention requires a finite amount of resources, the relation between the resources employed and the expected losses is a discontinuous (stepwise) relation, as qualitatively shown in Fig. 1.

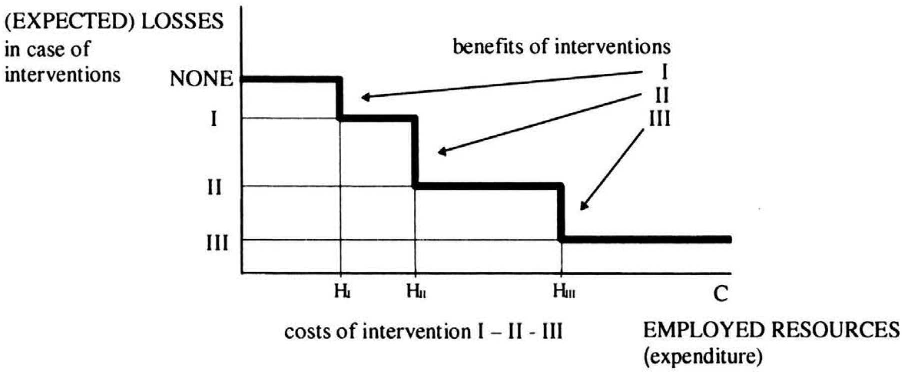

FIGURE 1. Effects of interventions on expected losses for each facility.

Provided that such relations were available, the procedures that have been developed and shall be described allow to estimate and optimize the benefit to be expected from any amount C of resources employed in upgrading. However, this requires that all data are available or can be estimated, and this is often the greatest challenge for the actual implementation of the procedures.

In particular, three points must be underlined:

- Apart from the difficulties in collecting data and calculating the relevant quantities, these are all uncertain; therefore, the treatment is inherently *probabilistic*, and all quantities must be seen as *expected* and not *deterministic*.
- The actual total loss is seldom just the sum of the losses on each building or facility, especially if the losses are widespread, as in the case of a severe earthquake or a flood, because of the general disruption of economy and/or social life in a region. Moreover, some facilities, like road networks and other lifelines, act as *systems*, and this has a direct effect on the losses. In the examples that will illustrate the procedures, the losses on *buildings* will be considered simply additive, while *system effects* will play an essential role in the study of *lifeline networks*.

- Several types of losses (and benefits) must be taken into account, and they are often incommensurable (e.g. economic losses, casualties, damage to the cultural heritage, etc.); the optimization can be performed separately for each objective, but also "multiple-objective" optimization techniques are needed. Also this point will be illustrated by examples.

On the contrary, the problem of actual quantitative derivation of the relevant quantities (in most cases an open problem) will not be tackled; instead, reasonable estimates will be introduced when necessary, since the primary aim of these lectures is to present methodologies rather than results.

PART I: Buildings

Generalities

Two strong earthquakes hit Italy in the last decades, procuring widespread damage and many victims: the Friuli earthquake of 1976 and the Irpinia earthquake of 1980. The epicentral area of both quakes was a rural but densely populated area, and most of the damages and victims were caused by the collapse of old masonry buildings; the disruption of the transportation networks increased much the difficulties, in particular in the latter case, also because of the collapse of the largest local hospital. A third, much weaker earthquake hit the central Italian region of Umbria in 1997: it called for great attention because of the damage to some artistic shrines, like the San Francesco Basilica in Assisi.

These events spurred much research and led to widespread surveys on the *vulnerability* of existing buildings.

Apart from still open problems concerning the elaboration of significant statistics from the survey data, two evident difficulties now face the exploitation of the collected information to formulate a rational strategy for reduction of earthquake losses, namely: the limited amount of resources that may be available for any preventive upgrading programme, and the multiplicity of the quantities whose reduction should be pursued in any such programme, like direct and indirect economic losses, casualties and deaths, damage to artistic and cultural heritage, environmental damages, deterioration of the *quality of life*.

1. Seismic vulnerability and upgrading

The prerequisite for the optimal allocation of the available resources is of course the availability of sufficient statistical data on seismic vulnerability and hazard. Much work has indeed be made on both these aspects in recent years; in particular, significant statistics have been and are being collected on the *seismic vulnerability* of buildings and constructed facilities.

Several alternative ways of describing such *vulnerability* exist, which can be divided into three categories.

In general, the seismic vulnerability of a structure is fully described by a set of *fragility curves*, that relate the probability of reaching a certain *degree* (or *level*) *of damage* (or a well defined *limit state*) with the *intensity* (i.e. the local dangerousness) of the earthquake.

However, due to the lack of sufficient data and the difficulties of using directly the fragility curves, seismic vulnerability is often measured in an approximate way by a number (the *vulnerability index*) or – even more simply – by including the structure in a *vulnerability class*[1].

Each description has its appropriate field of application and can be associated to a different way of describing quantitatively the degree of *upgrading* which is necessary and of evaluating the effectiveness of preventive retrofitting measures: the two examples of optimal allocation procedures, presented in the following Sec. 3.1 and 3.2, will make use of different description of vulnerability and upgrading.

1.1. Fragility curves

Fragility curves require first the definition of:

1. the relevant *limit state(s)* or quantitative measures of the damage, and
2. the intensity of the action.

A set of *fragility curves* refers to a specific construction, and can be obtained by statistics on similar constructions or by numerical calculations. They are therefore used for important structures; for instance, in the following they will be applied to examples of reinforced concrete girder bridges: the *damage* shall be measured by an indicator of the required ductility and of the energy dissipated in the critical zones of the substructure, and the *earthquake intensity* by the peak ground acceleration. To evaluate the effectiveness of an *upgrading* intervention in this approach, a new set of fragility curves must be evaluated for the retrofitted structure, and compared with the initial one.

[1] These simplified descriptions of vulnerability do not apply to "unique structures", like the *monumental buildings* that are dealt with in another lecture by the same authors in this volume. See also: G.Augusti, M.Ciampoli (2000). Heritage Buildings and Seismic Reliability. *Progress in Structural Engineering and Materials*, Vol.2, No.2; pp.225-237.

1.2. Vulnerability index

A set of fragility curves can be replaced, in an approximate way, by a number: the *vulnerability index,* which characterizes a building without explicit reference to earthquake intensity and level of damage.

The vulnerability index can be obtained in several ways. For instance, G.N.D.T. (the Italian National Group for Earthquake Loss Reduction) elaborated a form for surveying existing masonry buildings, which has experienced many variants over the years; the 1994 version of the form is schematically reproduced in Table 1. The survey team evaluates the quality condition (a) of each item on a four-level scale; the *vulnerability index* V is then obtained by summing up the values associated to the condition of each item, multiplied by the weights indicated in column (b): the higher values of V correspond to the most vulnerable buildings (with this edition of the form, V is in the range 0-382.5).

TABLE 1. Scheme of the survey form used in 1994 by G.N.D.T.

No.	Item	Item condition (a)				Weight (b)	$(a)\times(b)$
1	Connection of walls	0	5	20	45	1	...
2	Type of walls	0	5	25	45	0.25	...
3	Total shear resistance of walls	0	5	25	45	1.5	...
4	Soil condition	0	5	25	45	0.75	...
5	Horizontal diaphragms	0	5	15	45	subjective	...
6	Plan regularity	0	5	25	45	0.5	...
7	Elevation regularity	0	5	25	45	subjective	...
8	Transv. walls: spacing/thickness	0	5	25	45	0.25	...
9	Roof	0	15	25	45	subjective	...
10	Non structural elements	0	0	25	45	0.25	...
11	General maintenance conditions	0	5	25	45	1	...
		Vulnerability index V					...

An *upgrading* intervention can be defined as affecting one or more items of the form, and be assumed to bring the concerned item(s) into the best condition, i.e. to reduce to zero the contribution of that item to the vulnerability index V, thus decreasing its value.

In the example presented in Sec. 3.2 below, following previous suggestions, three possible intervention types (denoted as L, M and H respectively) have been defined, namely:

(i) in L (*light* intervention) the horizontal connections between orthogonal walls are secured, thus the contribution of item 1 to the vulnerability index vanishes;

(ii) M (*medium* intervention) includes also the strengthening of the horizontal diaphragms and brings to zero also the contribution of item 5;

(iii) finally, H (*heavy* intervention) includes also an increase in the overall strength against horizontal actions and brings to zero the contribution of item 3.

Of course, to be of significance for prevision of damages and evaluation of the effectiveness of risk-reduction campaigns, the values of the vulnerability index must be calibrated versus actual damages. Such a calibration requires the definition of a measure of earthquake intensity (usually referred to a *macroseismic scale*) and of the *degree* of damage. Much work is in progress on the subject: however, the vulnerability-intensity-damage relationships are still very much affected by uncertainties, some due to incomplete calibration, some due to their inherently random nature. For the sake of simplicity, in the example presented in Sec. 3.2 the vulnerability index V (defined in the range 0-282, according to an earlier version of the survey form), the MSK earthquake intensity i and the degree of damage D have been assumed to be related by the deterministic curves shown in Fig. 3, which were obtained from a 1985 statistical analysis of the damages caused by recent Italian earthquakes: $D = 0$ corresponds by definition to no damage, and $D = 1$ to total collapse.

1.3. Damage Probability Matrices (DPM)

Another definition of vulnerability assumes that all relevant buildings can be subdivided into a limited (say, 3 to 5) number of *vulnerability classes,* and that each class X can be associated with a damage probability matrix (DPM). By definition, each element $P_{ii}(\mathrm{X})$ of the DPM pertaining to the vulnerability class X, is the probability that a building of that class undergoes a damage of level j, if subjected to an earthquake of intensity i. The damage of the buildings and the intensity of the earthquakes must therefore be described according to discrete scales.

DPMs can be obtained from statistical analyses of the damages due to one or more earthquakes, when many buildings of a similar nature are affected in areas of different intensities. For instance, the DPMs shown in Table 2, which are used in the example presented in Sec. 3.1, originated from the statistics of the damages to masonry buildings caused by the 1980 Irpinia earthquake; they are based on an eight-level scale of damage (ranging from

TABLE 2. Damage probability matrices, elaborated from data on damages subsequent to the 1980 Irpinia earthquake[2].

Class X		A (worst)			B (medium)			C (best)		
MSK int. i		6	7	8	6	7	8	6	7	8
Damage level j	1	0.15	0.07	0.01	0.33	0.20	0.04	0.64	0.52	0.06
	2	0.19	0.12	0.03	0.25	0.26	0.11	0.24	0.24	0.24
	3	0.25	0.16	0.05	0.25	0.26	0.20	0.08	0.15	0.20
	4	0.19	0.20	0.06	0.10	0.13	0.16	0.03	0.05	0.17
	5	0.12	0.21	0.07	0.05	0.08	0.14	0.01	0.03	0.11
	6	0.07	0.17	0.12	0.02	0.05	0.13	0.00	0.01	0.10
	7	0.03	0.05	0.32	0.00	0.02	0.12	0.00	0.00	0.09
	8	0.00	0.02	0.34	0.00	0.00	0.10	0.00	0.00	0.03

no apparent damage to complete collapse) and refer to three *vulnerability classes* A, B and C (A being the least safe, C the most). In Sec. 3.1, also a fourth ideal class D of earthquake-resistant structures, including buildings which belonged to A, B, or C and have been fully upgraded, is considered: $P_{ii}(D) = 0$ by assumption.

All significant modifications to the vulnerability of a building can be indicated by its initial and final class, e.g. AB, AC,... BD,... correspond to *upgrading* interventions, while CB, BA, CA,... would be examples of *degradation* of the structure.

2. Objectives of optimization

As already alluded to, any structural design and any programme of seismic risk reduction should take many aspects into account, like, e.g., economic losses, casualties and deaths, damages to the artistic and cultural heritage, environmental damages, deterioration of the quality of life.

Many of these quantities are incommensurable with each other, and cannot be combined into a single *objective function*, not even by means of weighting factors (how to *weigh* and compare economic costs versus human lives, or versus the destruction of a historical village?): optimizations with respect to different objectives should therefore be pursued separately from each other.[3]

[2] F. Braga, M. Dolce and D. Liberatore (1982). Southern Italy November 23, 1980 earthquake: a statistical study on damaged buildings and an ensuing review of the MSK-76 scale. *7th European Conf. on Earthquake Engineering*, Athens.

[3] The rigorous approach to a rational strategy for seismic risk reduction would thus be to formulate and solve a problem of *multi-objective optimal resource allocation.* Such a formulation will be exemplified in the final Sections of this text, but it will be seen to lead to rather cumbersome numerical procedures and therefore is not always practical.

Fortunately, the objectives of the optimizations usually do not conflict with each other (a preventive intervention aimed at reducing the expected economic losses would also reduce the expected number of victims); however, the respective optimal solutions – in general – could not coincide, as examples will show in the following.

As discussed in Sec. 1 above, much research and statistical investigations are in progress on the seismic *vulnerability* of existing buildings, so that the expected damage after an earthquake can be estimated. Also, many retrofitting techniques, aimed at *upgrading* buildings (i.e. at reducing their expected damage after an earthquake), are being developed.

However, comparatively little attention has been paid – at least to the authors' knowledge – to the several possible consequences of the damages, other than the *direct economic costs*; therefore, cost-benefit analyses of a campaign of preventive interventions seem possible only with reference to this aspect, and the question remains open on how to account for the other, non-monetary aspects (often denoted *intangibles*) that have been quoted above.

A possibility would be to correlate directly the earthquake intensity (but the same could be applied to any other environmental or man-made hazard) and each of the consequences, e.g. casualties. This approach, in principle the most correct, would require specific and independent statistics for each type of consequences, and, for instance, damage statistics, elaborated with reference to economic costs only, would be useless with regard to *intangibles*. Not much significant work is available along these lines, but some is being in progress.

On the contrary, the possibility of using the vulnerability statistics in all cases requires that *damage* be defined and measured independently from the specific consequence. Other statistical relationships should relate the damage to each relevant consequence, however, this approach could be applied only if reliable damage-consequence relationships of this type were available.

In a ideal world of perfect mathematics and complete knowledge the two approaches would not differ one from the other; in the real word, they do.

The great asset of the *vulnerability* approach lies in the unified treatment of the damage and its statistics, and in the possibility of studying the results of preventive interventions as a decrease of vulnerability, independent of the specific consequences. Its greatest liability might appear the necessity of formulating other and separate relationships between damages and consequences, thus introducing an extra step in the calculations.

However, if one considers that in any case a reliable relationship between action and consequences is necessary, but in many instances not (or not yet) available, it should be clear that such an approach allows to obtain at least approximate results through extrapolations of known relationships (e.g. assume that the expected earthquake casualties in wooden buildings are sought, and that direct statistics are not available, because the specific problem was never posed before; assume also that the structural damages of timber can be forecast, and that statistics relating damages and casualties for all buildings in the area are available; this latter statistics could be assumed valid for the wooden buildings, and introduced in the calculation of the expected casualties). If the relationship between damage and consequence is deterministic and immediate (as is implicitly assumed when no distinction is made between damage and its, say, economic cost), then the introduction of the extra relationship does not pose any problem whatsoever.

Thus, the great liability of this approach remains in the unified quantitative definition of damage, be it made in linguistic terms (e.g. slight, significant, heavy, etc.) or in fractions or percentages (usually, 0 corresponds to no damage, and 100% to complete collapse; but also intermediate values, e.g. 50% or 70%, must be defined in an unequivocal way) or, perhaps better, according to a small number of *damage levels*.

However, the vulnerability approach appears indeed essential in an optimal allocation procedure, which looks for the *best* distribution of the upgrading interventions, whose costs are assumed known, under a constraint on the total expenditure. In fact, it allows to calculate and introduce unified relationships between the costs of the interventions and the reduction of the vulnerability, to evaluate the reduction of the expected damage for each distribution of interventions, and to make use of the relationships between damages and the consequence chosen as the objective of the optimization in order to choose the *most efficient* one.

In the following, it will be seen that such alternative optimizations are possible by simplified procedures or by sophisticated mathematical tools.

3. Optimal allocation of resources among buildings

Sections 3.1 and 3.2 will present examples of optimal allocation of resources for preventive upgrading interventions on masonry buildings, with respect to either direct economic losses and number of people involved by an earthquake. As anticipated, for the sake of simplicity, the total losses will be considered simply as the sum of the losses on each building.

3.1. Allocation to vulnerability classes

The first example deals with a very simple procedure, in which no formal optimization procedure is necessary. The scope is to plan the preventive interventions to reduce seismic risk in a small town; it is assumed that the vulnerability of buildings can be described through DPMs and that Table 2 holds.

Realistic costs have been estimated (as percentages of the construction cost), both for restoring a building of each class to its original condition after a level j damage, and for each type of upgrading intervention. For the sake of simplicity, all these percentages (and the construction cost per unit building volume) have been assumed to be constant irrespective of the building volumes actually involved in the operations. These values are not explicitly reported here.

The restoration costs after an earthquake of any relevant intensity can be forecast – for each class A, B and C – by multiplying the probabilities of Table 2 by the unit restoration costs estimated for each damage level, and summing up the columns; the results (again in percentage of the construction cost) are shown in Table 3; small but non-zero costs, corresponding to minor (non structural) damages, have been assumed also for the ideal class D.

TABLE 3. Forecast restoration costs of masonry buildings (as percentage of the construction cost).

MSK intensity i		6	7	8
	A	56.4	73.3	98.3
Class X	B	41.0	48.8	75.8
	C	30.3	34.7	64.0
	D	0.00	3.30	8.30

The technical aspects of the preventive upgrading interventions are not defined, but the intervention is assumed to correspond to a change of the class of the building, and therefore of the forecast cost to be read in Table 3: columns (2)-(4) of Table 4 show the *unit gains* δr_i due to each type of intervention, forecast for each given intensity i, that is, the differences between

TABLE 4. Forecast (δr_i) and expected (δr_p) unit economic gains; cost C_l and efficiency G_c of interventions.

(1) Intervention	(2) δr_6	(3) δr_7	(4) δr_8	(5) δr_p	(6) C_l	(7) G_c
AB	15.4	24.5	22.5	14.8	23.3	0.64
AC	26.1	38.6	34.3	24.2	33.3	0.73
AD	56.4	70.0	90.0	51.2	56.6	0.90
BC	10.7	14.1	11.8	9.3	28.3	0.33
BD	41.0	45.0	67.5	36.2	43.3	0.84
CD	30.3	31.4	55.7	27.0	26.6	1.00

the forecast restoration costs without and with the interventions indicated in column (1).

Multiplying these forecast gains by the probabilities of occurrence π_i of the relevant earthquake during the design life of the building, and summing up, the *expected unit gains* δr_p can be calculated.[4] The values shown in column (5) of Table 4 have been calculated by introducing the probabilities: $\pi_6 = 0.5$, $\pi_7 = 0.2$, $\pi_8 = 0.1$ which, assuming a 100 years lifetime, correspond approximately to the seismicity of many areas in Central Italy.

Finally, column (6) of the same Table 4 shows the assumed (deterministic) costs C_l of each intervention (once more, estimated as percentages of the construction cost), and column (7) the ratio G_c between the values in columns (5) and (6), i.e. the *expected efficiency* of each type of intervention. It can be noted that, with the used numerical values (realistic, even if derived from rough estimates) most values of G_c are smaller than one, i.e. no economic advantage should be expected from preventive interventions: under this limited viewpoint, it would then appear logical not to perform any intervention. However, the danger to human lives and environment, and a number of other circumstances invalidate such a conclusion; it is therefore assumed that preventive interventions are actually performed and only their optimal choice is sought.

No formal mathematical procedure is necessary to optimize the allocation of resources in this example. It is sufficient to remark that the larger or smaller efficiency of an intervention depends on the relative values of the ratio G_c: such a comparison is easily achieved by drawing (as it has been done in Fig. 2(a) straight lines with slopes equal to the values of G_c.

Noting that, once the buildings of the considered town have been assigned to a class, the procedure does not distinguish between individual buildings but can only refer to fractions of the volume of each class, the choice of the interventions to be performed in this specific case for any amount of available resources does not present any difficulty.

In fact, Fig. 2(a) shows immediately that the most convenient interventions are, in the order, CD, AD and BD (while in Fig. 3 a more complicated case will be found). Therefore, if the amount of available resources is comparatively small, they are used to bring into class *D* the largest possible volume of buildings belonging to class C; if more money is available than necessary to upgrade all buildings of class C in the considered town, the extra resources can be employed for intervention AD; then, if also class A can be fully upgraded, further resources can be employed for intervention BD.

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[4] Note that the term *forecast* is intended as a value *conditional* on a given earthquake intensity, while *expected* includes the probability of occurrence of each earthquake intensity.

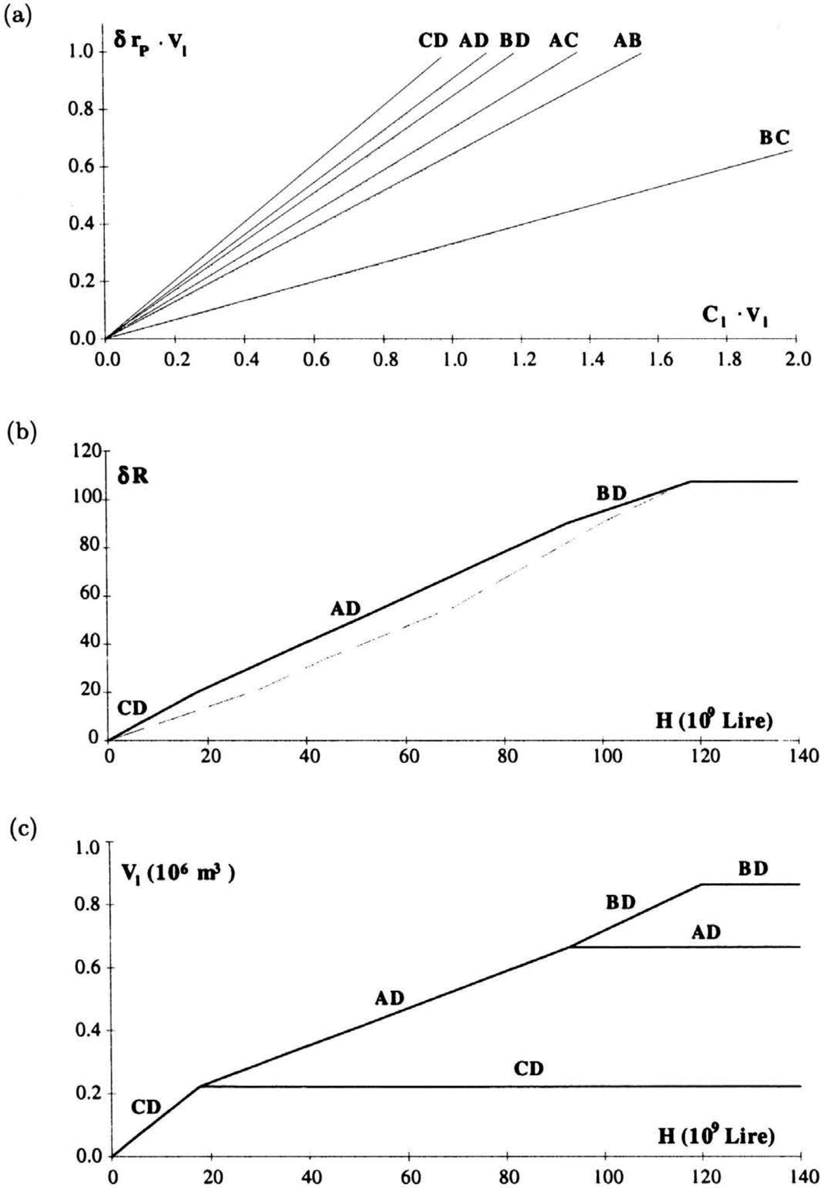

FIGURE 2. Interventions distributed among building vulnerability classes, optimized with respect to direct economic costs: (a) efficiency of interventions, (b) expected economic gain (solid line) and comparison with the economic gain expected from the solution optimized with respect to saved lives (dotted lines), (c) interested volumes.

It is thus possible to calculate the total gain

$$\delta R_p = \sum_l \delta r_p V_l,$$

where δr_p and V_l are the unit gain and volume of each intervention, and the total expenditure $H = C_c \sum_l (C_l V_l)$. Examples of plots of the total gain δR_p and of the volume V_l of each intervention versus the amount of money H available for preventive upgrading are shown in Figs. 2(b) (solid line) and 2(c); these plots have been calculated introducing the unit construction cost: $C_c = 300.000\,\text{Lire/m}^3$ (as estimated in 1991: remember that 1936 Lire = 1 Euro) and the following volumes of buildings of each vulnerability class (these values were estimated for the historic centre of Priverno, a small medieval town approximately 100 km south of Rome)

$$V_A = 441.854\,\text{m}^3, \quad V_B = 197.169\,\text{m}^3, \quad V_C = 223.543\,\text{m}^3.$$

In this way, the *intervention diagram*, optimized with respect to the direct economic losses, has been constructed in function of the available resources.

The previous optimization concerned direct economic gains and losses. As an alternative optimization objective, consider now the decrease of the number of persons endangered by an earthquake. If reliable models for the number of persons present and endangered by an earthquake are needed, the calculations for this optimization have been developed assuming that:

(i) 0.017 persons/m^3 inhabit the buildings (this value corresponds to the average density given by Italian statistics);

(ii) 60% of the inhabitants are present in the buildings at the time of the earthquake;

(iii) the ratios between the number of endangered and present persons are given by Table 5 for each level of damage.

TABLE 5. Assumed ratio η_j between endangered and present people.

Damage level j	1	2	3	4	5	6	7	8
η_j	0.00	0.00	0.00	0.04	0.12	0.24	0.40	0.80

Table 6 has been calculated in perfect analogy to Table 4. Namely, columns (2)-(4) show, for each intervention, the corresponding number δn_i of *saved* people (i.e. the reduction of endangered people) per unit volume, forecast for each earthquake intensity, while column (5) shows the expected unit number δn_p of saved people, assuming the already reported probabilities

TABLE 6. Forecast (δn_i) and expected (δn_p) number of "saved" people; cost C_l and efficiency G_v of interventions.

(1) Interv.	(2) δn_6	(3) δn_7	(4) δn_8	(5) δn_p	(6) C_l	(7) G_v
AB	0.045	0.100	0.321	0.075	23.3	0.00320
AC	0.060	0.133	0.419	0.098	33.3	0.00296
AD	0.063	0.143	0.549	0.115	56.6	0.00203
BC	0.015	0.033	0.098	0.024	28.3	0.00084
BD	0.018	0.043	0.228	0.040	43.3	0.00093
CD	0.003	0.010	0.130	0.016	26.6	0.00062
Interv. substn.				$\Delta\delta n_p$	ΔC_l	G_v
AB → AC				0.023	10.0	0.00230
AC → AD				0.017	23.3	0.00073

of occurrence. Finally, column (7) shows the efficiency of each intervention in terms of saved people, i.e. the ratio G_v between the expected unit gain of column (5) and the intervention cost of column (6); note that in the present case G_v is a ratio between two incommensurable quantities, which can be used only for comparative purposes. The last two rows of Table 6 show the differences in gains and costs between different interventions on class A,and the corresponding ratios G_v which will be necessary to construct the optimal intervention diagram in the present case.

The most convenient intervention is AB (Fig. 3(a)) and therefore this is the intervention to be performed if little resources are available.

However, if more resources are available than necessary to upgrade to class B the whole class A, it becomes next convenient not to intervene on more volumes, but to substitute intervention AC to AB; the intervention diagram is constructed as indicated in Figs. 3(b) and 3(c), taking into account that the efficiency of the substitution AB → AC is $G_v = 0.0023$ (Table 6, col. 7). If the whole class A can be upgraded to class C, the next convenient intervention is BD ($G_v = 0.00093$), then the substitution of AD to AC ($G_v = 0.00073$), and finally CD ($G_v = 0.00062$). The optimal intervention diagram is thus completed (Figs. 3(b) and 3(c)).

Thus, two optimal allocations have been performed, but their objectives are not commensurable; hence, an overall multi-objective optimum cannot be easily performed.

However, the final choice of the solution should take into account the results of both calculations. To give some indications for this purpose, Figs. 2(b) and 3(b) show also, in dotted lines, respectively (Fig. 2(b)) the

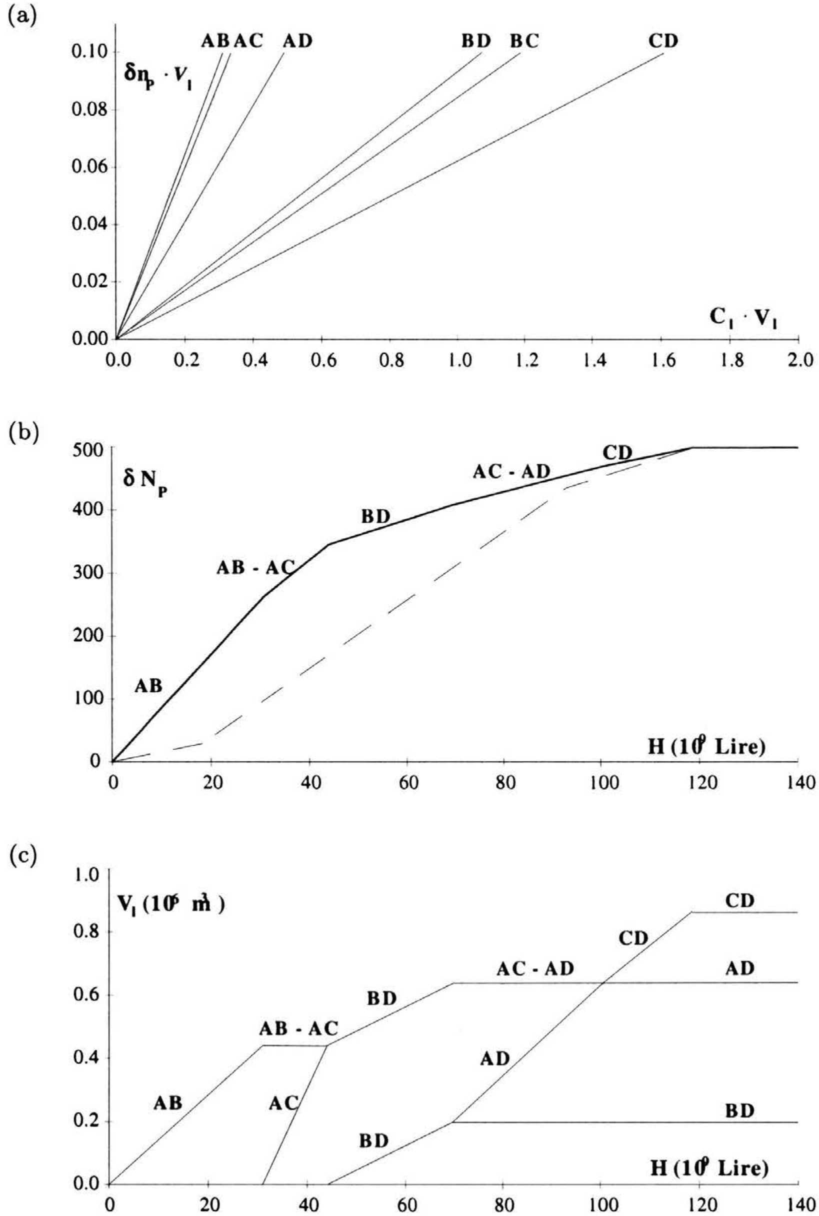

FIGURE 3. Interventions distributed among building vulnerability classes, optimized with respect to the decrease in the number of persons endangered by an earthquake (saved people): (a) efficiency of interventions, (b) expected number of saved people (solid line) and comparison with number of saved people expected from the solution optimized with respect to economic costs (dotted line,; (c) interested volumes.

total economic gain of the solution optimized in terms of saved people, and (Fig. 3(b)) the people saved by the optimal economic solution.

Although no general conclusions can be drawn, it can be noted that, in this case, the solution optimized with reference to saved people is close to optimal with respect to economic costs (Fig. 2(b)), while the reverse is not true (Fig. 3(b)).

3.2. Allocation to individual buildings by dynamic programming

In this Section, the seismic vulnerability of each building is measured by a number V (the *vulnerability index*) that, as anticipated in Sec. 1.2, is assumed to be related to the degree of damage D and the MSK earthquake intensity by the curves shown in Fig. 3. In the same Sec. 1.2, three possible types of interventions (L, M and H) have been defined; their assumed (deterministic) costs, together with the cost of construction, are shown in Table 7.

TABLE 7. Assumed construction and intervention costs per unit volume of buildings (Lire/m^3).

Construction	Intervention		
C_c	C_l	C_M	C_H
200,000	20,000	40,000	80,000

Although significant only in a statistical sense, a value of the index V is attributed to each building; therefore, the optimization will distribute the resources among individual buildings. As in Sec. 3.1, direct economic costs and number of endangered persons are taken as alternative objective functions.

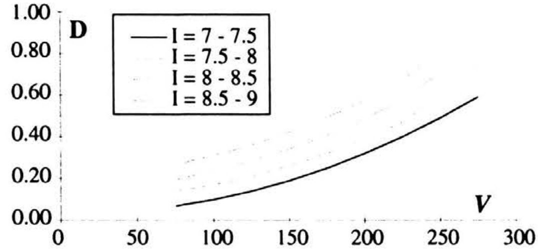

FIGURE 4. Degree of damage D vs. vulnerability index V.

The assumed relationships between the degree of damage D and respectively the monetary losses and the number of endangered persons n are shown in Figs. 5(a) and 5(b). The number of people present at the moment of the earthquake is again assumed equal to 0.6×0.017 persons/m^3.

total economic gain of the solution optimized in terms of saved people, and (Fig. 3(b)) the people saved by the optimal economic solution.

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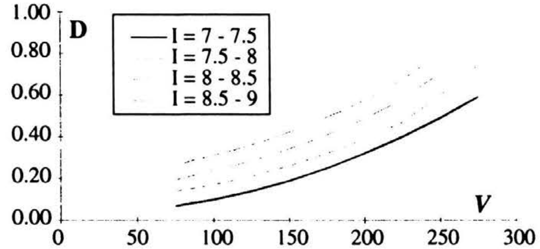

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where π_{im} is the probability of occurrence of an earthquake in the intensity interval i at the site of building m.

Under the assumptions made, the objective functions $F_{k|i}$ or F_k are the sum of as many quantities as the buildings, each in turn a function of the resources assigned to the m-th building only. But, as already noted, the nonlinearity and discontinuity of the relevant relationships do not allow the use of differential maximization procedures.

However, an optimization problem in which all relevant quantities are discrete-value variables can be tackled via a *multi-stage decisional process*, in which each stage of decision is independent of the previous ones; as stated by the *optimality principle*, such a process is optimal if, whatever the decisions taken at stage x, further decisions correspond to an optimal solution compatible with the state of the system after the decisions taken up to stage x. The optimization algorithm is then furnished by *dynamic programming*, which involves a comparatively small number of operations[6)] Without entering into the details of dynamic programming techniques, Fig. 6 shows an example of optimal allocation of 120 *resource units* (r.u.), with respect to F_c and to F_v, obtained by dynamic programming among 30 buildings located in three different areas of Umbria, a region of Central Italy, where the 100-year probabilities of earthquake occurrence shown in Table 8 had been approximately estimated; details on the volume and vulnerability of the buildings are not reported here.

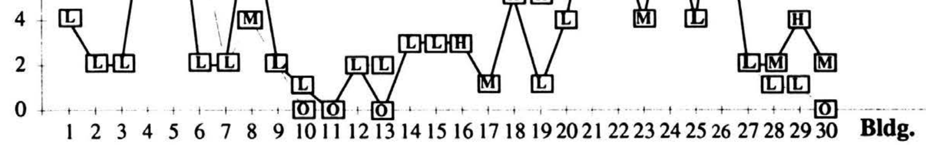

FIGURE 6. Interventions on 30 buildings allowed by a given total amount of available resources, and optimized with respect to economic damages (F_c) and to saved lives (F_v).

6) R. E. Bellman & S. E. Dreyfus, *Applied Dynamic Programming*, Princeton University Press, 1962.

TABLE 8. Assumed probabilities of earthquake occurrence in 100 years.

	I	1	2	3	4
	MSK intensity	7-7.5	7.5-8	8-8.5	> 8.5
Buildings	Site (town)	$\pi 1_m$	$\pi 2_m$	$\pi 3_m$	$\pi 4_m$
m = 1-10	Bastia Umbra	0.18	0.11	0.08	0.11
m = 11-20	Città di Castello	0.19	0.12	0.13	0.12
m = 21-30	Cascia	0.39	0.23	0.13	0.17

PART II: Lifeline networks (connectivity)

4. Optimal allocation of resources in the case of systems

4.1. General considerations; connectivity of a network

At a first sight, no significant difference appears whether the optimal allocation problems presented in Sec. 3 refer to buildings or to other facilities (e.g. bridges). However, in the case of buildings, dealt with so far, the initial vulnerability, the consequences of failures and benefits derived from an intervention on any element of the ensemble can be assumed – at least as a first approximation – to be independent from each other and then summable, which simplifies significantly the problem. On the contrary, if the facilities are the elements of a *system*, this is no more possible: the consequences of the failure of each facility, hence the effectiveness of any preventive measure, depend not only on the vulnerabilities of the single facility, but also, in an essential way, on the logical diagram of the system, the critical condition considered and the collocation *(role)* of each element; therefore the vulnerability of the *system* must be evaluated on its own account.

On the other hand, it is now a well recognized fact that the disruption of communication networks and other *lifeline* systems are among the most damaging effects of earthquakes. Indeed, as recent examples have confirmed, damages of this type can not only have immediate dramatic effects in the aftermath of an earthquake, but also consequences lasting for months and years on the economy, as well as on the conditions of life, of the whole area affected by an earthquake (or by any other disaster). And the increasing relevance of communications and services in modern life makes these effects still more important. It becomes thus essential to develop an optimal allocation methodology, not only with regard to single buildings, but also to *systems*, and in particular to *lifelines*.

A lifeline can be in general modelled as a redundant network, comprising a number of vulnerable (or *critical*) elements, that may themselves be complex redundant structural or mechanical systems. The network topology depends on the connections between the elements and on the assumed functionality condition, and is usually described by the minimal cut set or the minimal path set representations; then, from the network topology and the element vulnerabilities, it is possible to derive the reliability of the network as a whole (more details will be found in Sec. 5).

To elaborate a strategy for improving the reliability of the network, it is also necessary to estimate the costs and benefits of possible preventive measures in terms of their effects on the vulnerability of the critical elements and of the whole system.

In what follows, specific reference will be made to road networks. The main aim of such network is to ensure a connection between a *source* node S and a *destination* node D (*connectivity*); hence, the network fails when this connection is severed. Therefore, the *probability of failure* P_f of a network is defined throughout the following as the probability of *loss of connectivity*. In this Part II, optimization of networks with respect to the *probability of failure* P_f (i.e. to *connectivity*) will be dealt with; this objective has obvious limitations, because many factors are not taken into account (e.g. the capacity of traffic in the emergency that follows an earthquake), but it is certainly the one to start with. Alternative objectives will be introduced in Part III.

4.2. Critical elements (bridges); their vulnerability and upgrading

As a specific, but typical, case, all the following examples deal with road networks in which, by assumption, the only vulnerable (*critical*) elements are the bridges that form the *nodes* of the network; thus, the network can fail only because one or more bridges fail.

It is assumed that the seismic vulnerability of the bridges is described by *fragility curves*, known before and after some well defined *upgrading intervention*.

More specifically, the bridges in the examples are r.c. girder bridges: the decks are simply supported on piers of hollow rectangular section of two different types (Fig. 7).

Five structural diagrams have been considered (a to e: Fig. 8) in four different conditions, i.e. either as originally designed (O) for a peak ground acceleration $a_g = 0.10\,\text{g}$ (in accord with the Italian Regulations) or upgraded in one of three ways, which follow two different techniques, namely: jacketing of the piers with shotcrete cover and addition of reinforcement to improve the pier flexural capacity and shear strength (longitudinal reinforcement is increased by 50% in intervention I; by 100% in intervention II); elimination of

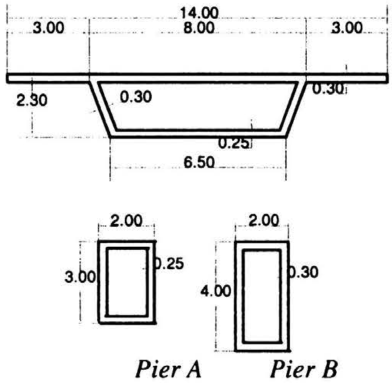

FIGURE 7. Cross sections of the deck and of the piers of the bridges depicted in Fig. 8.

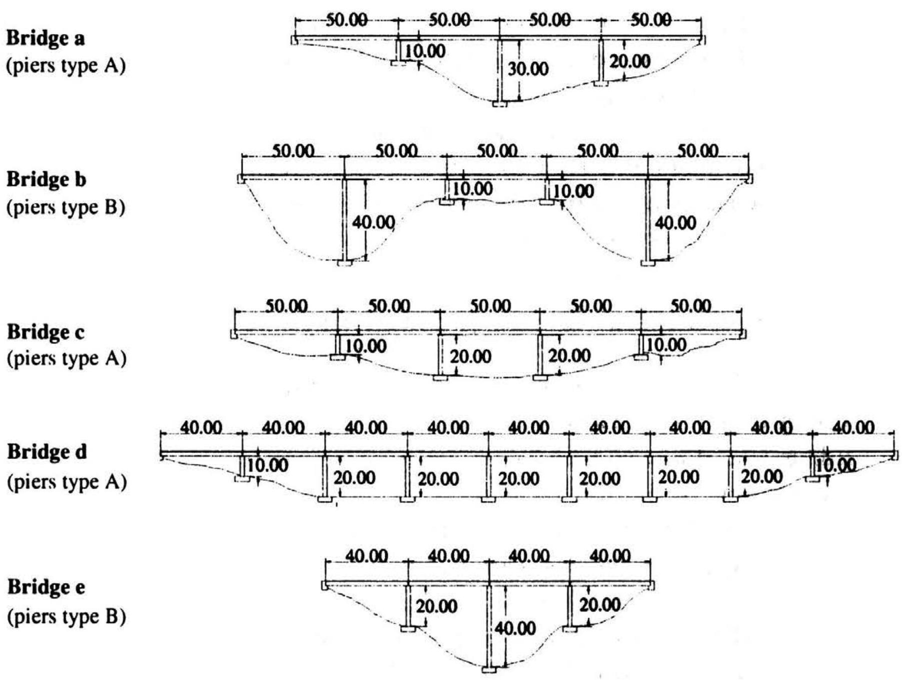

FIGURE 8. Structural diagrams of example bridges (measures in [m]).

expansion joints between the decks and introduction of isolation/dissipation devices on piers to replace the existing bearings (intervention III).

The costs of construction and intervention shown in Table 9 have been assumed in the numerical calculations: they are all normalized by reference to the construction cost of bridge D, taken equal to 100 *resource units* (r.u.).

TABLE 9. Assumed costs of construction and of upgrading of bridges; conditional probabilities of failure of original (O) and retrofitted bridges.

Bridge diagram		a	b	c	d	e
Construction cost		56	72	66	100	48
Upgrading cost	I	3	6	3	7	5
	II	4	8	4	9	6
	III	7	9	9	14	7
$P_f \mid a_g = 0.25\,g$	O	$3.15 \cdot 10^{-1}$	$2.82 \cdot 10^{-1}$	$5.60 \cdot 10^{-1}$	$6.29 \cdot 10^{-1}$	$4.43 \cdot 10^{-3}$
	I	$2.77 \cdot 10^{-1}$	$9.62 \cdot 10^{-2}$	$4.71 \cdot 10^{-1}$	$4.96 \cdot 10^{-1}$	$2.30 \cdot 10^{-3}$
	II	$1.94 \cdot 10^{-1}$	$2.71 \cdot 10^{-2}$	$3.49 \cdot 10^{-1}$	$3.59 \cdot 10^{-1}$	$3.69 \cdot 10^{-3}$
	III	$7.29 \cdot 10^{-3}$	$2.33 \cdot 10^{-3}$	$2.66 \cdot 10^{-3}$	$3.10 \cdot 10^{-3}$	$3.40 \cdot 10^{-4}$
$P_f \mid a_g = 0.35\,g$	O	1.00	1.00	1.00	1.00	$2.42 \cdot 10^{-1}$
	I	1.00	$8.72 \cdot 10^{-1}$	1.00	1.00	$1.15 \cdot 10^{-1}$
	II	1.00	$4.94 \cdot 10^{-1}$	1.00	1.00	$1.22 \cdot 10^{-1}$
	III	$3.02 \cdot 10^{-2}$	$1.54 \cdot 10^{-2}$	$1.14 \cdot 10^{-2}$	$2.50 \cdot 10^{-2}$	$7.57 \cdot 10^{-3}$

The failure condition of each bridge has been identified with the attainment of an appropriate threshold value of an indicator of the damage level in the critical sections of the piers. The fragility curves of each bridge in the four conditions have been evaluated by a Monte Carlo procedure, improved by Importance Sampling and Directional Simulation, using as inputs simulated seismic accelerograms compatible with the spectrum S2 of Structural Eurocode 8 (ENV 1998), scaled to several values of the peak ground acceleration a_g (taken as the measure of the earthquake intensity). In this way, fragility curves were plotted as functions of a_g; the probabilities of failure of the five bridges corresponding to two values of the peak ground acceleration a_g are shown in Table 9.

4.3. Upgrading the critical elements of a network: uniform upgrading vs. upgrading optimized with respect to connectivity

Five example networks diagrammatically represented in Fig. 9 have been considered. Each bridge is labelled by a serial number (1-5 or 1-10) and a letter indicating the structural diagram (defined in Fig. 8). Only the most significant results of the calculations are presented.

The first network, denoted SE, is an elementary chain of elements in series, and may correspond to bridges located along a single highway stretch. It fails if any one of the bridges fail: therefore, assuming that bridge failures under a given earthquake are stochastically independent on each other, the (conditional) probability of network survival $(1 - P_{SE})$ is equal to the product

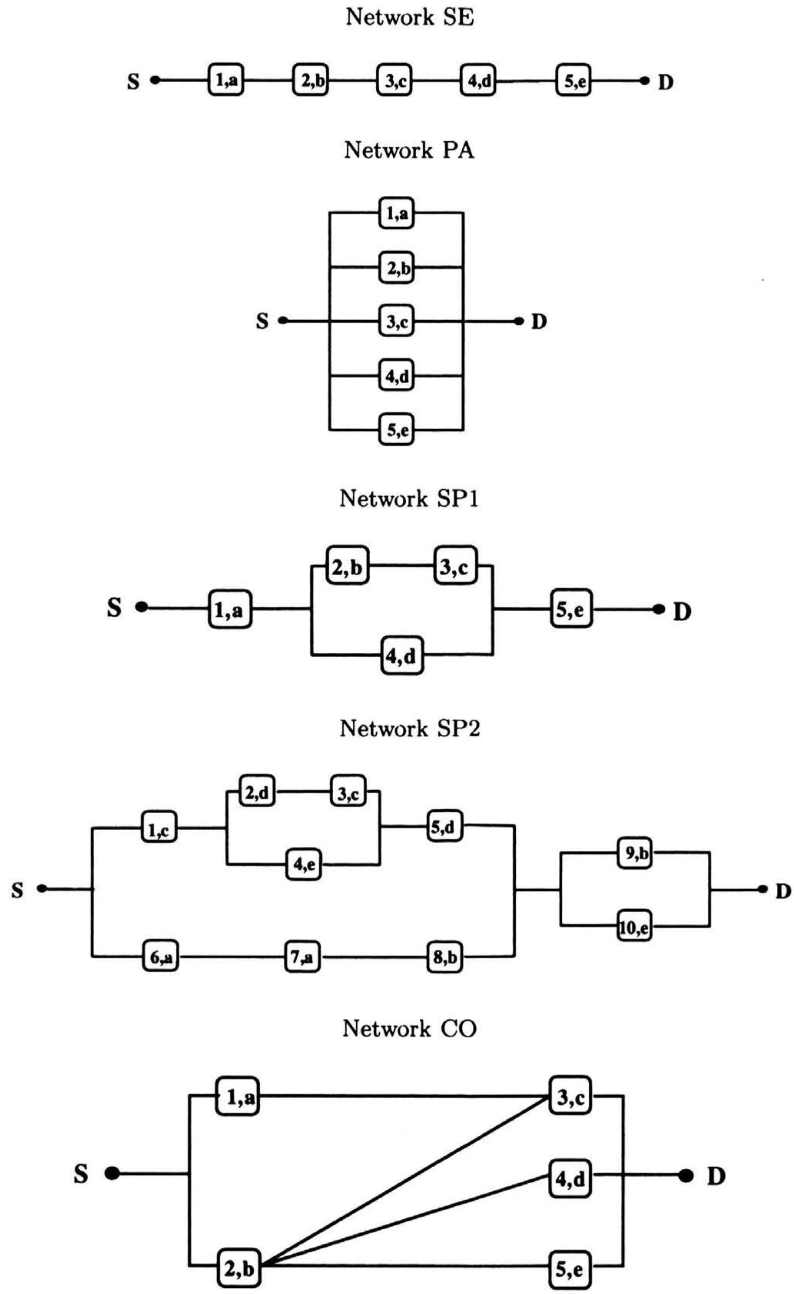

FIGURE 9. Diagrams of five example networks; locations and identification of critical elements.

of the probabilities of survival of all elements, whence:

$$P_{SE} = 1 - \Pi_i\{1 - P_i\},$$

where P_i is the probability of failure of element i subjected to a given earthquake.

The second network, denoted PA, is an elementary bundle of elements in parallel, and may represent the situation of a city cut by a river. The connection between the two banks fails if all bridges fail, whence:

$$P_{PA} = \Pi_i P_i.$$

The laws yielding the (conditional) probability of failure of the networks SP1 and SP2, that can be represented as a combination of independent subsystems in series and/or in parallel, are appropriate combinations of these rules.

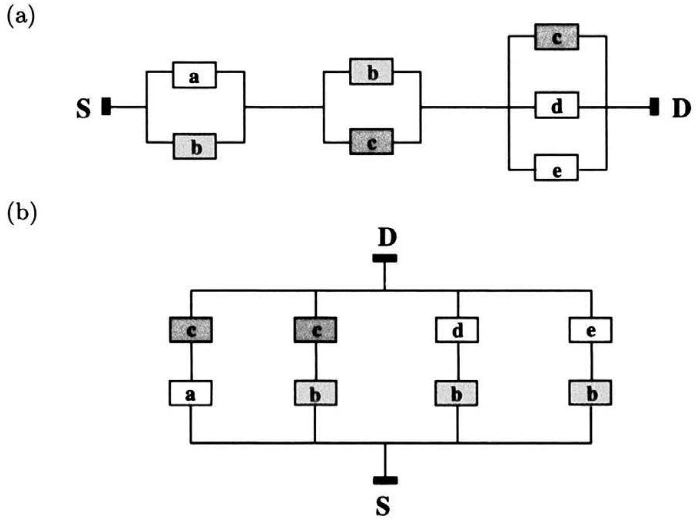

FIGURE 10. Minimal cut set (a) and minimal path set (b) representations of network CO (Fig. 8).

With regard to network CO, note that its functional logic cannot be represented by a combination of independent subsystems in series and/or in parallel. This is made evident by the minimal path set and minimal cut set representations in which some elements must be repeated in different subsystems (Fig. 10); that's why this network, according to a widely accepted definition[7], although appearing structurally simple, is *complex* from the re-

[7] S.S. Rao (1992). *Reliability-Based Design*, McGraw-Hill, Inc.

liability viewpoint. A more systematic discussion of these aspects will be presented in Sec. 5.

TABLE 10. Assumed costs of retrofitting of bridges and conditional probabilities of failure (loss of connectivity) of the networks in the original design condition (O) and after interventions of the same type (I, II or III) on all bridges.

Network		SE	PA	SP1	SP2	CO
Upgrading cost	I	26	26	26	52	26
	II	31	31	31	62	31
	III	46	46	46	92	46
$P_f \mid a_g = 0.25\,g$	O	$9.20 \cdot 10^{-1}$	$1.39 \cdot 10^{-4}$	$6.32 \cdot 10^{-1}$	$5.85 \cdot 10^{-1}$	$3.52 \cdot 10^{-1}$
	I	$8.26 \cdot 10^{-1}$	$1.43 \cdot 10^{-5}$	$4.65 \cdot 10^{-1}$	$3.87 \cdot 10^{-1}$	$7.12 \cdot 10^{-2}$
	II	$6.74 \cdot 10^{-1}$	$2.43 \cdot 10^{-6}$	$3.03 \cdot 10^{-1}$	$2.15 \cdot 10^{-1}$	$1.51 \cdot 10^{-2}$
	III	$1.56 \cdot 10^{-2}$	0	$7.64 \cdot 10^{-3}$	$1.10 \cdot 10^{-3}$	$2.32 \cdot 10^{-5}$
$P_f \mid a_g = 0.35\,g$	O	1.00	$2.42 \cdot 10^{-1}$	1.00	1.00	1.00
	I	1.00	$1.00 \cdot 10^{-1}$	1.00	1.00	$9.86 \cdot 10^{-1}$
	II	1.00	$6.03 \cdot 10^{-1}$	1.00	1.00	$7.75 \cdot 10^{-1}$
	III	$8.66 \cdot 10^{-2}$	$1.00 \cdot 10^{-9}$	$3.82 \cdot 10^{-2}$	$2.81 \cdot 10^{-3}$	$6.43 \cdot 10^{-4}$

Table 10 shows the failure probabilities of the five networks, in the original design condition (O) and after interventions of the same type on all bridges, for two values of a_g (namely 0.25 and 0.35 g, that according to Structural Eurocode 8 can be defined respectively as medium and high seismicity zones); the corresponding costs are also reported in the same table.

Instead of distributing uniformly the upgrading intervention on all bridges, it is possible, by applying dynamic programming (see Sec. 3.2), to distribute the interventions among the bridges in such a way that, for a given total amount of employed resources, the probability of network failure P_f (loss of connectivity) after an earthquake of given intensity is minimized.

The whole range of significant values of the available resources C_{ava} has been investigated, from zero up to the value that would allow the most efficient intervention (III) on all bridges, i.e. 46 r.u. for the 5-bridges networks, and 92 for the 10-bridges network SP2.

The failure probabilities of the networks, after interventions optimized in this way, are plotted in Fig. 11 for three values of a_g versus the amount of resources C_{ava} (in *resource units*, r.u.); calculations have been limited to $a_g = 0.35\,g$ for the parallel network PA, because its reliability under weaker earthquakes is already very large in the original condition.

The corresponding interventions are shown in Table 11 for the highest considered value of a_g. As it will be put in evidence in Sec. 5.2, Table 11 indicates the optimal distributions of interventions between the element of the five example networks for each given amount of resources, as well as the

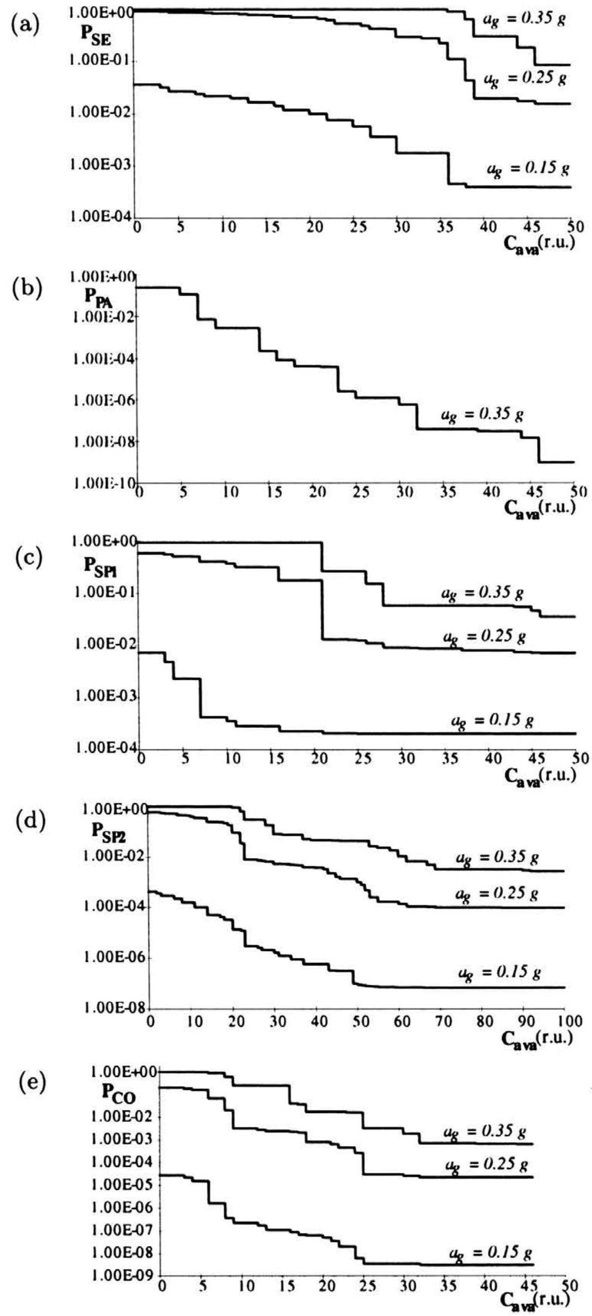

FIGURE 11. Probability of failure (loss of connectivity) versus employed resources (optimally allocated); peak ground acceleration $a_g = 0.15\,g$, $0.25\,g$ and $0.35\,g$: (a) network SE; (b) network PA; (c) network SP1; (d) network SP2; (e) network CO.

TABLE 11. Optimized interventions on each bridge of the five example networks vs. employed resources for $a_g = 0.35\,g$.

Network SE

C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III
2	-	-	-	-	-	-	-	-	-	-	-	I	III	III	III	III
3	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III
4	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	I	III

Network PA

C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	-	-	-	I	III	-	III	III	III	III	III	III	III	III	III	III
2	-	-	-	-	-	III	-	-	III	III	III	III	III	III	III	III
3	-	-	III	III	-	III	III	III	III	III	III	III	III	III	III	III
4	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III
5	-	I	-	-	III	-	I	III	-	I	III	III	-	-	I	III

Network SP1

C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III
2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	II	III
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	III	III
4	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III
5	-	-	-	-	-	-	-	-	I	III	III	III	III	III	III	III

Network SP2

C_{ava}	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	92
1	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III	III
2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	III	III
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	III	III
4	-	-	-	-	-	-	-	III	III	III	-	III	III	III	III	III	III	II	III
5	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III	III
6	-	-	-	III	III	III	III	-	-	-	III	III	III	III	III	III	III	III	III
7	-	-	-	III	III	III	III	-	-	-	III	III	III	III	III	III	III	III	III
8	-	-	-	II	III	III	III	-	-	-	III	III	III	III	III	III	III	III	III
9	-	-	-	-	-	-	III	III	III	III	III	-	III	III	III	III	III	III	III
10	-	-	-	-	-	III	-	-	II	III	-	III	-	III	III	III	III	III	III

Network CO

C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	-	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III
2	-	I	III	III	III	III	III	III	III	III	III	III	III	III	III	III
3	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III
4	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III
5	-	-	-	-	I	-	-	I	-	I	III	III	-	-	I	III

minimum amount of resources that is needed to modify the probability of network failure; the diagrams in Fig. 11 allow to evaluate the beneficial effect of a set of intervention, as well as the amount of resources that is needed to limit the probability of failure to a threshold value.

5. The general case: optimal allocation of resources in complex systems

5.1. Optimizing the upgrading of the critical elements of a complex network

Still within the definition of *network reliability* given in Sec. 4.1 (i.e. referred to *connectivity* only), let us now present a more systematic treatment.

To solve any optimization problem it is first necessary to identify the structure of the network. If seriality prevails over redundancy, the best representation is given by the minimal path sets: then, the functionality of a path can be defined by a *state function* S_i which is 0 if the path has failed, 1 if the path has not failed. Since the vulnerable elements are in series along each path i,

$$S_i = \prod_{j=1}^{n_{p_i}} S_{ij},$$

where S_{ij} is the analogous state function of the j-th among the n_{pi} vulnerable elements (bridges) encountered on the i-th path.

The algorithm which finds all the minimal paths connecting one source node S with one destination node D can be described by the following recursive procedure. The input data are the connections between the nodes of the graph, that are defined so that for each node n_i (*father*) all the nodes n_j (*children*) that are serviced by n_i are explicitly put in evidence.

Initially, let NODE be the source node S (which, by definition, has no *father*).

1. If any of the two following conditions is met, then interrupt the procedure, and go back to the current father:
   (i) NODE has been already marked.
   (ii) Any of the fathers of NODE, excluding the current one, has been already marked.
2. Mark NODE.
3. If NODE is the destination node, then generate a minimal path by collecting all the currently marked nodes and unmark all nodes of that path.

4. For each of the nodes serviced by NODE, grouped in the vector CHILD(i), apply the same procedure [the current NODE becomes the father for CHILD(i), and CHILD(i) becomes NODE]: steps 1 to 3.

The above procedure generates all and only the minimal paths from the source to the destination, even for complex structural topology, including the case of undirected and cyclic networks: as an example, the minimal paths of network CO are shown in Fig. 10(b).

While these lectures deal only with networks with one source node and one destination node, let us briefly note that if the network has more than one source (as, e.g., it may be the case when it is only a part of a larger network, and a number of peripheral source nodes simulate the effects of the excluded parts) it is possible to transform the network under study into one with a single source by introducing a fictitious node connected to each of the actual sources whilst the minimal paths do not change. On the other hand, if more than one destination node is present, that is, if many sites are to be connected to the source node, the minimal path for each destination node are found first, then the combination of the minimal paths (one per destination node) is a minimal path for the multiple destination node case. The elimination (by means of adsorption) of the redundant paths yield all and only the minimal paths for the multiple destination node network.

Once the minimal paths have been determined, the structure of the network is expressed by the overall state function S:

$$S = 1 - \prod_{i=1}^{n_{\mathrm{p}}} [1 - S_i] \,,$$

where n_{p} is the number of minimal paths in parallel, each composed by n_{pi} element in series.

The reliability of the network is the expected value of S, which, provided that the failure of different vulnerable elements are independent events, can be evaluated as:

$$R = \sum_{i=1}^{n_{\mathrm{p}}} E[S_i] - \sum_{i=1}^{n_{\mathrm{p}}-1} \sum_{j=i+1}^{n_{\mathrm{p}}} E[S_i S_j] + \sum_{i=1}^{n_{\mathrm{p}}-2} \sum_{j=i+1}^{n_{\mathrm{p}}-1} \sum_{k=j+1}^{n_{\mathrm{p}}} E[S_i S_j S_k] - \ldots \,,$$

where the first two addenda are given by:

$$E\,[S_i] = E\left[\prod_{j=1}^{n_{pi}} S_{ij}\right] = \prod_{j=1}^{n_{pi}} E\,[S_{ij}],$$

$$E\,[S_i S_j] = E\left[\prod_{h=1}^{n_{pi}} S_{ih} \prod_{k=1}^{n_{pi}} S_{jk}\right] = \prod E\,[S_{ir}],$$

with the last product extending to all different elements of the paths i and j. The other terms of the sum can be derived accordingly.

The preceding expression can be explicitly evaluated if the number n_p of paths in parallel is limited; the formulae presented in Sec. 4.3 are particular cases.

In the case of network CO (whose minimal paths are in Fig. 10(b)), the reliability is given by the relation:

$$R = 1 - P_f = 1 - (P_a \cdot P_b + P_b \cdot P_c - P_a \cdot P_b \cdot P_c + P_c \cdot P_d \cdot P_e - P_b \cdot P_c \cdot P_d \cdot P_e).$$

Here P_f is the probability of failure of the network and P_i the probability of failure of bridge $i (= \mathrm{a} - \mathrm{e})$, all conditional upon the assumed value of a_g.

The solution of this optimization problem is a set of upgrading interventions distributed among the vulnerable elements such as to maximize the objective function R (or, equivalently, to minimize P_f), which is a discontinuous function (cf. Fig. 1). Therefore, as repeatedly stated, the usual differential techniques cannot be employed to find the optimal solution, which would instead require an exhaustive search and the comparison between all possible sets of interventions. However, such search and comparison rapidly become impossible, as the number of vulnerable components and alternative paths grows.

For non-complex systems, an optimization problem in which all relevant quantities are discrete-value variables can be tackled via a multi-stage decisional process, in which each stage of decision is independent of the previous ones, i.e. by *dynamic programming*, which involves a comparatively small number of operations (cf. Sec. 3.2).

With regard to network CO, that, although appearing structurally simple, is *complex* from the reliability viewpoint, the optimal distribution of the interventions has been obtained not only by dynamic programming, but also through an *exhaustive search*. As a matter of fact, for a complex network the results of the two procedures might not coincide, because in dynamic programming the problem is analyzed by successive steps that, in this case, cannot correspond to independent minimal cut sets.

To perform a direct exhaustive search, the diagram of the network must be *exploded* by splitting each node in order to represent each vulnerable element in one of its n possible conditions (in the example: 4, namely O-I-II-III) and each S-D possible path examined. The number of nodes – hence the complexity of the problem – increases very rapidly with the number of the alternative conditions (i.e. the number of intervention types), depending on the number of vulnerable elements and of the links: the 5-node network CO of Fig. 9 *explodes* into a 58-node graph (Fig. 12).

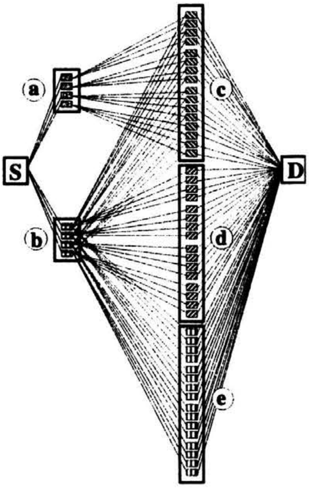

FIGURE 12. 5-node network CO transformed into an exploded graph (each node: 4 alternative conditions).

For the 5-node network, the exhaustive search is still possible with a reasonable computational effort, but it is evident that for larger networks exhaustive search can easily become intractable. It is however possible to simplify the graph, by eliminating the paths associated to vulnerable elements characterized by a reliability below an acceptable threshold.

However, the solutions obtained by dynamic programming and by exhaustive search for network CO have been found identical for all practical purposes, being different only in the range $C_{\text{ava}} = 16 - 17$ r.u. for $a_{\text{g}} = 0.25\,\text{g}$ and $a_{\text{g}} = 0.35\,\text{g}$; this result seems to indicate the possibility of applying the procedure based on dynamic programming also to complex networks.

5.2. Some remarks

Inspection of Fig. 11 and Table 11 leads to various suggestions. For instance, it is interesting to note how the distribution of the optimized interventions sometimes changes drastically when the amount of economic resources varies.

The convenience of an optimal versus a uniform distribution of resources can also be put in evidence. For instance, consider the 10-bridge network SP2; if $a_{\text{g}} = 0.25\,\text{g}$ and intervention II is performed on all bridges, 62 r.u. are employed and P_{f} is reduced from 0.58 to 0.21 (Table 10); if the same 62 r.u. are distributed in the optimal way, P_{f} becomes as low as $0.11 \cdot 10^{-3}$ (Fig. 11(d)).

In the same Fig. 11(d), it can be also noted that, when the resources are optimally allocated, the reduction of P_f with C_{ava} is very slow beyond 68 r.u.; therefore, a sensible general policy of good exploitation of resources would allocate no more than 68 r.u. to the upgrading of bridges in the considered network.

It may be also of some interest to distinguish the preferential paths automatically chosen by the optimization procedure: in the already quoted network SP2, this path is ⟨6-7-8⟩ if C_{ava} is rather small, ⟨1-4-5⟩ if it is larger.

It is important also to note that, under the set assumptions, the optimization procedure only suggests the most convenient interventions in order to increase the reliability of the connection between S and D, but does not imply an explicit control of the actual safety level of each critical element. This limit should be taken into account in applying such a strategy: for instance, if no intervention is performed on bridge d, safety requires that this bridge be closed to traffic whenever the area is hit by an earthquake with $a_g \geqslant 0.35\,g$.

Finally, it is of interest to note that a number of researches have tackled the choice of the bridge(s) on which to perform preventive upgrading interventions as a *prioritization* problem. However, as it can be seen from the example presented herein, the priority can change with the amount of available resources (and also the objective considered): therefore, it appears that taking account of the whole network at the same time can lead to more significant results.

PART III: Networks (alternative objectives)

6. Alternative objectives of the optimization

6.1. Generalities

To ensure *connectivity* can be not the only aim of a risk-reduction programme of road networks, several other objectives can also be envisaged. In the developed researches, four objectives have been considered, namely:

- the *network reliability* R, defined as the probability of maintaining the *connectivity* (i.e. the connection between the source node S and the destination node D) when an earthquake hits the area; equivalently, reference can be made to the complementary *probability of network failure* $P_f = 1 - R$; this objective has been extensively dealt with in Part II;
- the expected *traffic capacity* of the network, i.e. the maximum traffic flow that can be expected to run between the source and destination nodes after an earthquake of given intensity;

- the *out-of-service time* of the network, that is, the expected duration which the whole network remains out of service after an earthquake of given intensity;
- the *time-efficiency* of the interventions, defined as the ratio between the increase of the network reliability due to a specified set of upgrading interventions and the time necessary for their execution (the minimization of this quantity appears a key point also for the optimal planning of the repair interventions on a damaged network).

As a prerequisite for trying and employing the resources in an optimal way, the possible interventions on each facility must be designed, and their costs and gains evaluated for each objective function; relations like in Fig. 1 are thus obtained, and one is able to estimate the benefits to be expected from the amount C of resources employed in upgrading.

Starting from assumed relations of this type, the available resources can be allocated in an optimal way with respect to each of the four above stated objectives, and also in a multi-objective allocation (with assumed weights for the different objectives).

It is to be remembered that, because of the uncertainties in the physical conditions of the bridges and in their future loading history (occurrence and characteristics of the earthquakes), the treatment is inherently probabilistic: all values and results must be regarded as *expected values* and not as deterministic quantities.

Optimization for objectives other than *connectivity* will be illustrated in the following Sec. 6 and 7 with reference to the five-node network indicated as Network CO in Fig. 9 (Part II). In Sec. 8, an 8-node network will be studied.

6.2. Expected traffic capacity

Recalling that, by assumption, only the nodes of the network are vulnerable, the expected *traffic capacity* c_{ij} of each link $(n_i \rightarrow n_j)$ is defined as an *estimated capacity* of the link multiplied by the reliability of its end node in its actual (original – O – or upgraded – I, II, III) condition. The estimated capacity of the link is identified with an assumed (or measured) value, shown in Table 12 in arbitrary units. Alternatively, the capacity can be related to the users' choices by a stochastic network loading approach in which a certain measure of travel impedance or disutility associated to each link is assumed to be random.

The *traffic capacity of the network* can be identified with the *maximum flow* between the source node S and the destination node D that satisfies the link capacities and the mass balance constraints at all nodes.

TABLE 12. Estimated traffic flow capacities of each link (in arbitrary units).

LINK	CAPACITY
S → a	105
S → b	520
a → c	100
b → c	200
b → d	100
b → e	200
c → D	1200
d → D	500
e → D	1500

The search of the maximum flow is carried out by means of an *augmenting path algorithm*, a class of algorithms that can be applied to a network if the following three assumptions are satisfied:

- the network is directed (note that any undirected network can always be transformed into a directed network);
- the capacity associated with each link ($n_i \rightarrow n_j$) is a nonnegative integer;
- the network does not contain either parallel links between the same two nodes, nor an S-D path composed only by links of infinite capacity.

An augmenting path algorithm is based on the *max-flow min-cut theorem*, whose formulation requires the definition of *residual network*, *S-D cut*, and *augmenting path*.

The *residual network* with respect to a certain global flow F sent through the network is the network consisting of links ($n_i \rightarrow n_j$) with positive residual capacity r_{ij}, this latter being defined as the maximum additional flow that could be sent from node n_i to node n_j.

An S-D *cut* is a partition of the whole node set in two complementary subsets V and V^c that have no common elements, and are such that $S \in V$ and $D \in V^c$; equivalently, it can be identified with a set of links whose elimination severs the connection between S and D. The *capacity of an* S-D *cut* is the sum of the capacity of the cut links; a *minimum* S-D *cut* is the cut whose capacity is the smallest among all S-D cuts.

An *augmenting path* is a directed path from S to D in the residual network.

The *max-flow min-cut theorem* can be expressed in three equivalent formulations[8)]:

8) Refer to: K.A. RAVINDRA, T.L. MAGNANTI, and J.B. OBLIN (1993), *Network flows - Theory, Algorithms and Application*, Prentice-Hall, Inc., Englewoods Cliffs, New Jersey; T.H. CORMEN, C.E. LEISERSON, and R.L. RIVEST (1994), *Introduzione agli algoritmi*, Jackson Libri, Milan, Italy.

- the value of any flow F in the network is not larger than the capacity of any S-D cut in the network;
- the value of the maximum flow F_{max} from the source node S to the destination node D equals the capacity of the minimum S-D cut;
- the flow in the network is maximum if (and only if) no augmenting path exists in the residual network: indeed, whenever the network contains an augmenting path, it is possible to send additional flow from S to D.

In the present application, the maximum flow F_{max} in the network is evaluated by means of the basic Ford-Fulkerson algorithm, as modified by Edmond and Karps (see Note 10) to limit the computational burden and to eliminate the need of taking integer values for the link capacities. The steps of the procedure can be summarized as follows:

1. put initially $F_{max} = 0$;
2. using one of the available algorithms (see, e.g., Sec. 5.1), evaluate the minimum path between S and D on the residual network (that in the first iteration coincides with the original network);
3. along this path recognize the link(s) $(n_i \rightarrow n_j)$ of smallest capacity c_{min};
4. subtract the capacity c_{min} from the capacities of all links in the network;
5. eliminate the link(s) with capacity equal to 0; thus, the residual network with respect to the actual maximum flow $F_{max} + c_{min}$ is defined;
6. put $F_{max} = F_{max} + c_{min}$;
7. go back to step 2;
8. iterate as long as an augmenting path can be found.

When no further augmenting path can be found, the set of links with nil capacity identifies the minimum S-D cut (if more than one zero-capacity links are found on a path, the nearest to S is considered); the actual value of F_{max} is the sought maximum flow.

Therefore, in order to maximize the *expected traffic capacity* after an earthquake, the resources are optimally allocated by the following procedure:

A) fix the amount of available resources C_{ava};

B) identify in the existing network the minimum S-D cut (steps 1-8 of the above procedure);

C) by applying a dynamic programming procedure (or an exhaustive search, if the number of critical elements that must be examined is sufficiently low), find the distribution of upgrading interventions among the end nodes of the links forming the cut, such as to maximize the increase of flow in the cut;

D) verify that this increment coincides with the increment of the maximum flow in the upgraded network; otherwise, identify the new minimum cut in the upgraded network, and go back to step C) operating alternatively on the two cuts;

E) iterate until the optimal solution is derived.

The steps B) and C) of the procedure can be simplified by finding for each node n_i of the cut the minimum ratio of the expected capacities of all links that end in n_i and the expected capacities of the links that emanate from n_i, and considering at first the nodes characterized by the lowest values of this ratio.

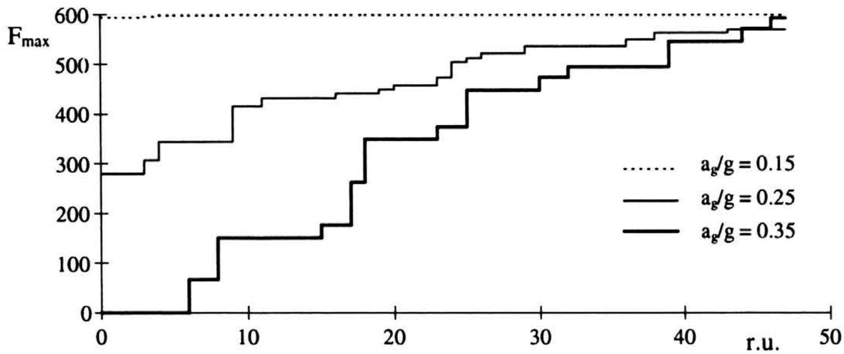

FIGURE 13. Expected traffic capacity of the example network vs. optimally allocated available resources: peak ground acceleration $a_g = 0.15\,g$, $0.25\,g$ and $0.35\,g$.

TABLE 13. Distributions of preventive upgrading interventions among bridges (a-e) for several values of the available resources (3-46 r.u.), optimized with respect to the expected traffic capacity of the network for (1) $a_g = 0.15\,g$, (2) $a_g = 0.25\,g$ and (3) $a_g = 0.35\,g$.

	C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
(1)	a	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	b	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	c	I	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	II	II	III	III	III	III	III	III	III	III	III
	e	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III	III
(2)	a	-	-	-	-	-	-	II	-	II	III	III	III	III	III	III	III
	b	-	-	-	-	-	-	-	I	I	I	I	I	I	I	I	I
	c	I	II	III	II	II	III	III	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	-	-	-	-	-	-	I	II	II	III	III
	e	-	-	-	III	III	III	III	III	III	III	III	III	III	III	III	III
(3)	a	-	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III
	b	-	I	II	II	II	III	III	III	III	III	III	III	III	III	III	III
	c	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III
	e	-	-	-	-	I	-	-	I	-	I	III	III	-	-	II	III

The results of the optimal allocation (steps A-E) are summarized in Fig. 13 and Table 13. It is interesting to note that if $a_g = 0.15\,g$, the most critical elements of the network are bridges c – d – e, in this order; if $a_g = 0.35\,g$, and if less than 18 resource units are available, it is necessary to operate on bridge b to ensure at least one connection between S and D, and therefore the chance of some traffic flow through the network. For any a_g the most important path is S-b-c-D.

6.3. Out-of-service time of the network

Another quantity of great significance in planning the interventions appears to be the *out-of-service time*, defined as the expected time necessary to restore the functionality of the network when it fails as a consequence of an earthquake.

The times required by the three types of interventions (I-II-III) on each bridge have been assumed as shown in Table 14. Conventionally, the time required by intervention II on the largest bridge (bridge d) has been put equal to one *reference interval.*

TABLE 14. Assumed intervention times and out-of-service times (nondimensional units).

BRIDGE TYPE		a	b	c	d	e
INTERVENTION TIME	I	0.33	0.67	0.33	0.83	0.50
	II	0.50	0.83	0.50	1.00	0.67
	III	0.25	0.50	0.25	0.67	0.33
OUT-OF-SERVICE TIME	O	0.91	0.82	1.00	1.00	0.64
	I	0.18	0.36	0.18	0.45	0.27
	II	0.23	0.41	0.23	0.50	0.32
	III	0.09	0.23	0.09	0.27	0.14

Also, expected *bridge out-of-service times* (i.e. the expected time required to restore a bridge hit by an earthquake) have been introduced. They are also reported in Table 14 in non-dimensional units; for simplicity, these times have been assumed, as a first rough approximation, to be the same for the three considered values of the peak ground acceleration.

Given the assumed out-of-service times of each bridge, the *out-of-service time* of the network can be evaluated in several alternative ways, and in particular as the sum of the time intervals necessary to perform either (i) all the interventions, or (ii) the interventions on the vulnerable elements of only one path connecting S with D. The optimal allocations reported here refer only to alternative (i).

To perform this optimization, the *exploded* network of Fig. 12 has been considered. Thus a *weight* can be associated to each link, corresponding to the time necessary to restore the functionality of the vulnerable element at the end of that link.

Given the amount of available resources, the minimal among the admissible S-D paths (that is, those paths that corresponds to a total expenditure less than or equal to the fixed amount of available economic resources) in the exploded network have been recognized by means of the Floyd-Warshall algorithm. The results of the optimal allocation of resources in this case are presented in Table 15.

TABLE 15. Distributions of preventive upgrading interventions among bridges (a-e) for several values of the available resources (3-46 r.u.), optimized with respect to the expected out-of-service time of the network, evaluated according to definition (i).

C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
a	I	I	I	III	III	III	III	III	III	III	III	III	III	III	III	III
b	-	-	-	-	-	-	-	I	III	III	III	III	III	III	III	III
c	-	I	I	I	I	III	III	III	III	III	III	III	III	III	III	III
d	-	-	-	-	-	-	-	-	-	-	I	I	I	I	I	III
e	-	-	-	-	-	-	-	-	-	I	-	-	III	III	III	III

The most critical elements are bridges a and c, corresponding to an optimal path S-a-c-D. If a small amount of resources is available, the interventions must be performed on bridges a and c, but if $a_g = 0.35\,\mathrm{g}$, it is in any case necessary to employ at least 16 r.u. in order to assure the S-D connection; if a larger amount is available, it is convenient to intervene also on bridge b. Further interventions yield negligible reduction of the out-of-service time.

6.4. Time-efficiency of the interventions

It can also be assumed that the most efficient set of interventions is the set that yields the largest increase of reliability in the shortest time (this is very significant in the case of repair interventions, when, because of emergency, time limits prevail); the optimization can then be referred to an objective function denoted as *time-efficiency* and assumed equal to the ratio:

$$\eta = \frac{\Delta R}{T^*},$$

between the increase ΔR of the network reliability and the time T^* necessary to implement a set of interventions.

To find the optimal allocation of resources from this viewpoint, the distribution of interventions that maximizes the reliability of the S-D connection in the shortest time is found for each S-D path, and then, the path corresponding to the largest time-efficiency is selected.

To this aim, an algorithm has been used that searches in a graph the path yielding the largest rate of the attained level of efficiency versus the time needed to attain it, that is the path that corresponds to a global optimum[9]. In Fig. 14 the time-efficiency functions η are plotted for $a_g = 0.35\,g$ and the different paths, versus the required resources. Inspection of these plots shows that S-a-c-D is the overall most efficient path; however, at least 16 r.u. are needed to ensure reasonable reliability. If a smaller amount of resources is available (but at least 6 r.u., i.e. those needed for intervention i on bridge b) the S-D connection must be entrusted upon path S-b-e-D.

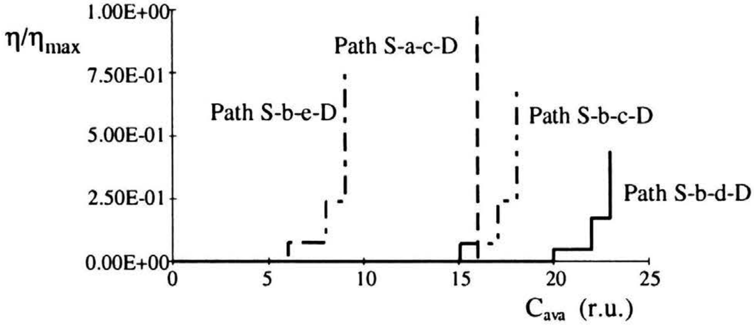

FIGURE 14. Time-efficiency functions η versus available resources ($a_g = 0.35\,g$).

In deriving these plots, the time T^* has been evaluated as the sum of the times needed to implement each intervention (case (i)). Similar results are obtained for the other limit case (ii) of T^* equal to the longest time needed for one intervention, as if all interventions were implemented at the same time. In Table 16, the two results are shown in the same cell; the differences are significant only if $a_g = 0.15\,g$, because, in a certain range of economic resources, the bridge a or b are alternatively selected; if $a_g = 0.35\,g$, the only differences appear with reference to the path S-b-e-D, because if 14 r.u. are available, an additional upgrading intervention on bridge e yields a larger increase of the efficiency than indicated by Fig. 14.

From Table 16, path S-a-c-D is confirmed as the most efficient path, as in the optimization for the out-of-service time. If the available amount of resources is larger than the amount necessary to ensure the functionality of the most efficient path, the allocation procedure is iterated on the remaining paths.

[9] Refer to: W.A. HORN, Single-machine job sequencing with tree-like precedence ordering and linear delay penalties, *SIAM, Journal of Applied Mathematics*, Vol.23, No.2, pp.189-202, 1972; N. NOJIMA and H. KAMEDA, Optimal strategy by use of tree structures for post-earthquake restoration of lifeline network systems, *Proc. Tenth World Conference on Earthquake Engineering*, Madrid, Vol.9, pp.5541-5546, 1992

TABLE 16. Distributions of preventive upgrading interventions among bridges (a-e) for several values of the available resources (3-46 r.u.), optimized with respect to time-efficiency η according to definitions (i)/(ii), for (1) $a_g = 0.15\,g$, (2) $a_g = 0.25\,g$ and (3) $a_g = 0.35\,g$.

	C_{ava}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
(1)	a	-	-	-	-	-	-/III	I/III	I/III	III	III	III	III	III	III	III	III
	b	-	-	-	-	I/-	III/-	III/-	III/II	III	III	III	III	III	III	III	III
	c	I	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	-	-	-	-	-	-	-	I	I	I	III
	e	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III
(2)	a	-	-	-	I	I	III	III	III	III	III	III	III	III	III	III	III
	b	-	-	-	-	-	-	-	II	III	III	III	III	III	III	III	III
	c	I	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	-	-	-	-	-	-	-	I	II	II	III
	e	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III
(3)	a	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III
	b	-	-	-	-	-	-	-	II	III	III	III	III	III	III	III	III
	c	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III
	d	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	III
	e	-	-	-	-	-/I	-	-	-	-	I	III	III	III	III	III	III

In this iteration the additional interventions can be planned either on the most time-efficient path among the alternative ones, or in a way such to maximize the further increase in the system reliability; however, very few differences result in the example case.

6.5. Comparison of results

Figure 15 shows, versus the available amount of resources, the variation of the failure probability P_f of the network CO corresponding to the inter-

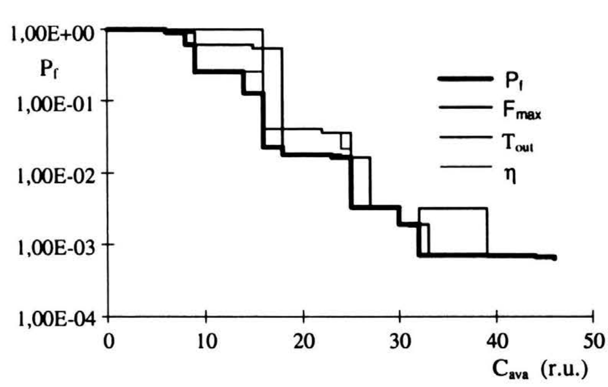

FIGURE 15. Probability of failure of example network optimized with respect to the four objective functions vs. available resources, for $a_g = 0.35\,g$.

ventions optimized with respect to the four objective function taken into consideration.

Of course, as the available amount of resources increases, the probabilities of failure tend to coincide. However, for intermediate values, there are significant differences; moreover, it has already been noted that, depending on the objective function and other data, either path S-b-c-D or paths including bridge a can be more significant. Therefore, it may be important to take account at the same time of several possible aims of the prevention strategy. This is attempted by the multi-objective allocation procedure proposed in the following Sec. 7.

7. Multi-objective optimization

Let g_h denote the h-th objective function ($h = 1, \dots, k$), d_n the n-th path of the weighted graph (in the example, Figs. 9 and 12) connecting the source node S to the destination node D, and $g_h(d_n)$ the value of the h-th objective function after the set of interventions corresponding to that path.

If the h-th objective function is summable, it is given by

$$g_h(d_n) = \sum_{(n_i \to n_j) \in d_n} f_h(n_i\, n_j),$$

where $f_h(n_i,\, n_j)$ is the value of the h-th objective function relative to that link, assumed as the weight (or length) of that link.

If the h-th objective function is not summable (for example, the reliability of the network or its expected maximum traffic capacity), a different formulation must be applied, as the value of the objective function must be referred to the whole network.

The multi-objective optimal allocation of resources consists in finding the distribution of interventions among the vulnerable elements of an S-D path (under the constraint of a fixed total amount of available resources) in such a way that all the k objective functions are taken into account. This search procedure is composed of two steps:

- determination of alternative distribution of interventions optimal in the Pareto sense under the constraint of a given maximum expenditure;
- choice of an "absolute" optimum according to a well defined decision strategy.

7.1. Search of the Pareto optimal paths

A path d^{P} is Pareto optimal if for every other path d_n, and for any h:

$$g_h(d_n) \subseteq g_h(d^{\mathrm{P}}),$$

with

$$g_h(d_n) \subset g_h(d^{\mathrm{P}}),$$

for at least one objective function. The sign $\subset$ indicates that the path d^{P} is "better" than the path d_n with respect to the h-th objective function, while the sign $\subseteq$ includes the case of equivalence between the two paths.

Usually, a set of Pareto optimal paths can be found among the admissible solutions (i.e. all the paths that require an expenditure less than or equal to the available total amount of economic resources).

For moderate-size networks, all the paths of the exploded graph (Fig. 12) can be examined, and the whole set of Pareto optimal paths individuated. For larger networks, the problem is computationally intractable, and a more efficient method of solution must be used. To this aim, an algorithm[10)] has been implemented that allows to take into account also not summable objective functions. The algorithm consists in first seeking the optimal path for an arbitrarily chosen objective function, and then in the ordered examinations of the varied paths in the exploded graph with the aim of seeking an *a priori* fixed number of Pareto optimal solutions. Selecting different objective functions for the choice of the starting path, it is possible to investigate the whole range of admissible solutions from different directions.

Let

$$g^0 = [g_1^0, \ldots, g_i^0, \ldots, g_k^0]^{\mathrm{T}},$$

denote the ideal vector that contains the optimal solution for each objective function separately, determined after finding the corresponding extreme path, $d^{0(h)}$. With reference to k summable objective functions that must be minimized, the search procedure can be described by the following steps:

1. Read the network;
2. Set $g_h^{\mathrm{P}} = \infty$ for $i = 1, 2, \ldots k$, where g_h^{P} is the value of the h-th objective function for the Pareto optimal path d_n^{P};
3. Find the extreme path $d^{0(h)}$ for each objective function separately, and the corresponding values of the objective function g_h^0 for $h = 1, 2, \ldots k$;
4. Fix the number N^a of optimal paths to be individuated;
5. Set $N = 1$;
6. Find the path $d^{(N)}$ that for $N = 1$ makes extreme the selected objective function, i.e. is such that

$$f_1(d_N) = \min_{d^{(N)}} \sum_{n_i, n_j \in d^{(N)}} f_1(n_i, n_j)$$

10) A. OSYCZKA, *Multicriterion Optimization in Engineering with FORTRAN Programs*, Ellis Horwood Limited, Chichester, 1984.

and for $N > 1$, is such that:

$$g_1\left(d^{(N)}\right) \geqslant g_1\left(d^{(N-1)}\right).$$

7. Determine the values $g_h(d^{(N)})$ of the objective functions for the path $d^{(N)}$;
8. Check if the path $d^{(N)}$ should be stored as Pareto optimal;
9. Set $N = N + 1$;
10. If $N < N^a$, go to (6); otherwise stop.

The number N^a of paths to consider is assumed a priori. Thus not all the Pareto optimal paths will be found, but only those which are contained in a selected region. If the solution individuated so far do not satisfy the decision-maker, the above procedure can be repeated for (i) a larger value of N^a or (ii) assuming another objective function to define the direction along which the investigation is carried out. Since an optimal allocation problem is dealt with in this study, only the admissible paths, i.e. those that satisfy the constraint on the amount of available economic resources, are considered.

7.2. Decision-making strategies

Once the Pareto optimal solutions have been obtained as described in Sec. 7.1, several decision-making strategies can be implemented. Two procedures have been selected here, and applied to the network CO, namely: the *Utility Value Analysis* (UVA); the *concordance and discordance analysis* formulated in the ELECTRE method. They are described below, while implementation and results are reported in Sec. 7.3.

7.2.1. Utility Value Analysis. The Utility Value Analysis[11] (whose logical diagram is shown in Fig. 16) proceeds as follows.

For each Pareto optimal solution (named *alternative* in what follows) d_n^{P} and for each objective function g_h, define *criterion-related impact* e_{hh} the value assumed by the h-th objective function for that alternative. The impacts are generally measured in various units (i.e. time unit, money, ...): in order to make them comparable, they are firstly transformed into non-dimensional *criterion utilities* u_{nh} by means of a *value* or *utility function* l_h.

In the development of the specific example, the criterion utilities u_{nh} are derived by the following relationships:

[11] M. BIELLI, M. GASTALDI, and P. CAROTENUTO (1996), Multicriteria evaluation model of public transport networks, *Advanced Methods in Transportation Analysis* (L. Bianco and P.Toth, Eds.), Springer Verlag, Berlin/Heidelberg.

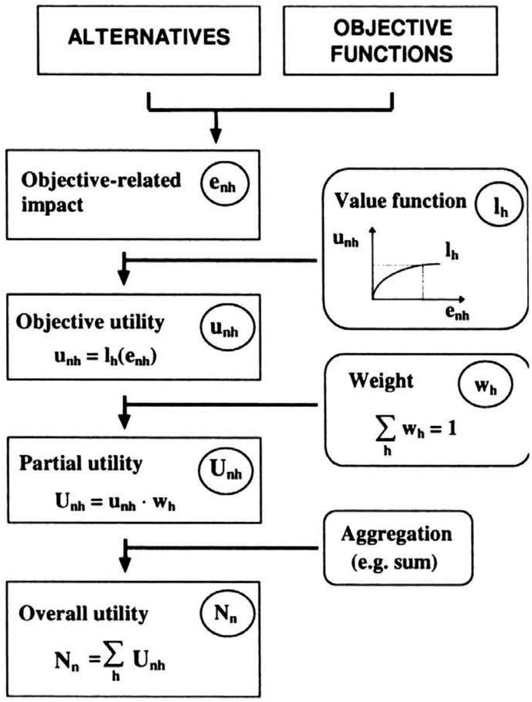

FIGURE 16. Logical diagram of Utility Value Analysis.

(a) if the utility increases when the impact decreases (this is, for example, the case of a cost or a time objective function):

$$u_{nh} = \frac{\min(e_{nh})}{e_{nh}},$$

(b) if the utility increases when the impact increases (this is, for example, the case of the network reliability):

$$u_{nh} = \frac{e_{nh}}{\max(e_{nh})}.$$

Each objective function, and therefore the corresponding utilities, can be weighted by a factor w_h that reflects the relative importance of each objective function (the sum of the weights is usually taken equal to one). The weighted criterion utilities are called *partial utilities*. The partial utilities of the alternatives are then aggregated (for example, simply summed).

The *overall utility* N_n of each alternative d_n is defined as the sum of the weighted criterion utilities of that alternative. The *absolute optimum* corresponds to the path with the highest overall utility.

7.2.2. Concordance and discordance analyses. The ELECTRE – *ELimination Et Choix Traduisant la Réalité* – methods[12)] are based on a comparison of alternatives pair by pair. The methods attempt to eliminate first a subset of less desirable alternatives from the complete set of alternatives (*elimination*), after which a complementary analysis is used to select the absolute optimal alternative, or a set of relatively good alternatives (*choice*). Let:

- $D^{\mathrm{P}} = \{d_1^{\mathrm{P}}, d_2^{\mathrm{P}}, \dots, d_{N^a}^{\mathrm{P}}\}$, the set of Pareto optimal solutions corresponding to a fixed maximum amount of economic resources;
- $G = \{g_1, g_2, \dots, g_k\}$, the set of objective functions taken into consideration;
- $W = \{w_1, w_2, \dots w_k\}$, the weights associated to the considered objective functions;
- $g_h\left(d_n^{\mathrm{P}}\right) = e_{nh}$, the value of the h-th objective function for the n-th optimal path;
- $G^+(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}) = \{h \in G \Rightarrow g_h(d_r^{\mathrm{P}} \supset g_h(d_s^{\mathrm{P}})\}$, the set of objective functions according to which the alternative d_r^{P} is preferable to the alternative d_s^{P};
- $G^=(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}) = \{h \in G \Rightarrow g_h(d_r^{\mathrm{P}} = g_h\left(d_s^{\mathrm{P}}\right)\}$, the set of objective functions according to which the alternative d_r^{P} is equivalent to the alternative d_s^{P};
- $G^-\left(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}\right) = \left\{h \in G \Rightarrow g_h\left(d_r^{\mathrm{P}}\right) \subset g_h\left(d_s^{\mathrm{P}}\right)\right\}$, the set of of objective functions according to which the alternative d_s^{P} is preferable to the alternative d_r^{P};
- $W^+(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$, $W^=(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$, $W^-(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$, the sums of the weights associated to the objective functions forming the sets $G^+(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$, $G^=(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$, $G^-(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})$.

The first step of the method consists in defining the *concordance index* for each pair of alternatives.

The concordance index

$$CI(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}),$$

is a measure of the gain of the decision-makers if the alternative d_r^{P} is preferred to the alternative d_s^{P}. It is equal to the relative (weighted) frequency of objective functions according to which the alternative d_r^{P} is not worse than

[12)] R. BENAYOUN, B. ROY, and N. SUSSMAN (1966), *Manual de Référence du Programme Electre*, Note de Synthese et Formation, No.25, Direction Scientifique SEMA, Paris (mimeographed).

the competing alternative d_s^{P}:

$$CI(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}) = \frac{W^+(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}) + W^{d_r^{\mathrm{P}}, d_s^{\mathrm{P}}})}{W^+(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}) + W^=(d_r^{\mathrm{P}},\ d_s^{\mathrm{P}}) + W^-(d_r^{\mathrm{P}}, d_s^{\mathrm{P}})}.$$

A complementary measure is given by the discordance index:

$$DI(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}),$$

that measures how much the impacts of alternative d_s^{P} are better than the impacts of alternative d_r^{P}. It can be expressed, for each pair of alternatives, in the form:

$$DI\ \left(d_r^{\mathrm{P}},\, d_s^{\mathrm{P}}\right) = \max_{h \in G^-} \frac{\left|g_h\ \left(d_s^{\mathrm{P}}\right) - g_h\ \left(d_r^{\mathrm{P}}\right)\right|}{L_h^{\max}},$$

where $L_h^{\max}$ is the largest difference between the impacts of the objective function $g_h(d_s^{\mathrm{P}})$.

The logical diagram of this analysis is shown in Fig. 17.

Once the values of concordance and discordance indices have been derived for all pairs of alternatives, it is necessary to identify a criterion for evaluating

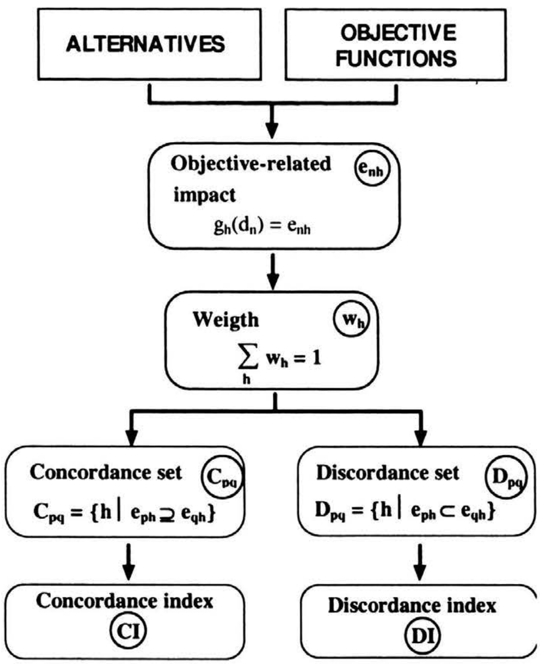

FIGURE 17. Definition of concordance and discordance index.

the (set of) absolute optimal solution(s). In general, it is possible to define threshold values of the indices, and to eliminate the alternatives characterized by a concordance smaller or a discordance larger than these critical values. In the developed example another procedure has been followed. For each alternative, the *net concordance dominant value* has been calculated as:

$$CI_{\text{net}} = CI - CI^* = \sum_{d_r^{\mathrm{P}}, d_s^{\mathrm{P}}=1,\ S \neq r}^{N^a} CI\left(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}\right) - \sum_{d_r^{\mathrm{P}}, d_s^{\mathrm{P}}=1,\ S \neq r}^{N^a} CI\left(d_s^{\mathrm{P}}, d_r^{\mathrm{P}}\right),$$

where CI is the sum of all concordance indices of alternative d_r^{P} with respect to the different alternatives, and CI^* is the degree of dominance of other alternatives with respect respect to alternative d_r^{P}. Analogously, the *net discordance dominance value* have been calculated as:

$$DI_{\text{net}} = DI - DI^* = \sum_{d_r^{\mathrm{P}}, d_s^{\mathrm{P}}=1,\ S \neq r}^{N^a} DI\left(d_r^{\mathrm{P}}, d_s^{\mathrm{P}}\right) - \sum_{d_r^{\mathrm{P}}, d_s^{\mathrm{P}}=1,\ S \neq r}^{N^a} DI\left(d_s^{\mathrm{P}}, d_r^{\mathrm{P}}\right),$$

where DI is the discordance dominant value and DI^* is the degree of discordance of other alternatives with respect to alternative d_r^{P}.

As a first indication, it can be noted that the alternative d_r^{P} is to be preferred as much as higher is the value of CI_{net}, and lower than 0 the value of DI_{net}.

In the numerical calculation, the alternative with the highest difference between the two terms have been considered.

7.3. Results and comments

The weights w_h assumed for the objective functions in the numerical examples are reported in Table 17. They have been arbitrarily chosen only to develop the example. In a real case, they must be calibrated on the basis, for example, of the marginal utilities of different objective functions; moreover, sensitivity studies on the influence of the values chosen should be performed.

TABLE 17. Weights assumed for the objective functions.

OBJECTIVE FUNCTION	WEIGHTS
Reliability	0.50
Expected traffic capacity	0.20
Out-of-service time	0.25
Time-efficiency	0.05

TABLE 18. Absolute optimal distributions of upgrading interventions, derived by UVA and ELECTRE methods for several values of the available resources (3-24 r.u.), for (1) $a_g = 0.15\,g$, (2) $a_g = 0.25\,g$ and (3) $a_g = 0.35\,g$.

UVA

	C_{ava}	3	6	9	12	15	18	21	24
(1)	a	-	-	-	-	-	-	-	-
	b	-	I	III	III	III	III	III	III
	c	I	-	-	I	II	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	-	-	-	-
(2)	a	-	-	-	-	-	-	-	-
	b	I	I	III	III	III	III	III	III
	c	-	-	-	I	II	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	-	-	-	-
(3)	a	-	-	-	-	-	-	-	-
	b	-	I	III	III	III	III	III	III
	c	I	-	-	I	-	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	I	-	-	-

ELECTRE

	C_{ava}	3	6	9	12	15	18	21	24
(1)	a	-	I	-	-	-	-	-	-
	b	-	-	I	III	III	III	III	III
	c	I	I	I	I	II	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	-	-	-	-
(2)	a	-	I	-	-	-	-	-	-
	b	-	-	-	III	III	III	III	III
	c	I	I	III	I	II	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	-	-	-	-
(3)	a	-	-	-	-	-	-	-	-
	b	-	I	III	III	III	III	III	III
	c	I	-	-	I	-	III	III	III
	d	-	-	-	-	-	-	-	-
	e	-	-	-	-	I	-	-	-

TABLE 19. Comparison between the results of multi-objective optimization (multi) and the optimizations with reference to single objective functions (probability of connectivity failure/expected traffic capacity/out-of-service time/time-efficiency of interventions) for (1) $a_g = 0.15\,g$, (2) $a_g = 0.25\,g$ and (3) $a_g = 0.35\,g$.

	BRIDGE	OBJECTIVE FUNCTION				
	TYPE	Multi	P_f	F_{max}	T_{out}	η
(1)	a	-	-	-	III	III
	b	III	III	-	-	-
	c	III	III	III	III	III
	d	-	-	II	-	-
	e	-	-	-	-	-
(2)	a	-	-	-	III	III
	b	III	III	-	-	-
	c	III	III	III	III	III
	d	-	-	-	-	-
	e	-	-	III	-	-
(3)	a	-	-	-	III	III
	b	III	III	III	-	-
	c	III	III	III	III	-
	d	-	-	-	-	-
	e	-	-	-	-	-

Table 18 shows the optimal allocations obtained with the two decision-making strategies. We observe that the results are similar (if not equal at all) with regard to the preferred path, while minor differences appear in the distribution of upgrading interventions.

Table 19 compares the results of the multi-objective optimization and the single objective optimizations, for $C_{ava} < 24$ u.r. Figure 18 shows the network reliability as a function of the peak ground acceleration and of the amount of available resources is reported. It appears quite evident that the optimal path between S and D is the path S-b-c-D; in fact the governing function is the reliability of the network, to which a relative weight equal to 0.50 has been attributed.

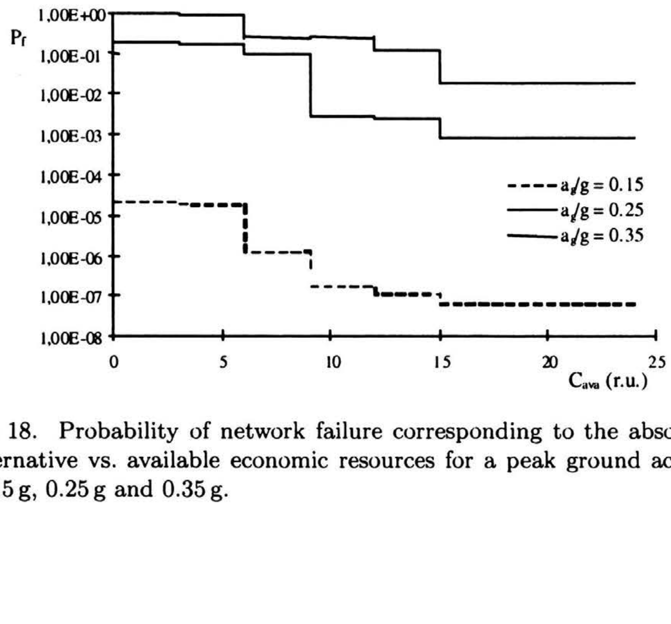

FIGURE 18. Probability of network failure corresponding to the absolute optimal alternative vs. available economic resources for a peak ground acceleration $a_g = 0.15\,g$, $0.25\,g$ and $0.35\,g$.

8. An 8-node network

As a further example, the allocation of resources in an 8-node network (Fig. 19) is now presented. In this case exhaustive search becomes prohibitive, and allocation via *dynamic programming* is absolutely necessary.

The functional logic of the network (Fig. 20) cannot be expressed by a combination of independent subsystems in series and/or in parallel; therefore, according to the definition accepted in graph theory, the network is *complex* from the reliability viewpoint.

In principle, standard dynamic programming is not applicable to such network; on the other hand, exhaustive search is impracticable with such a number of vulnerable elements. Therefore, it was necessary to extend the

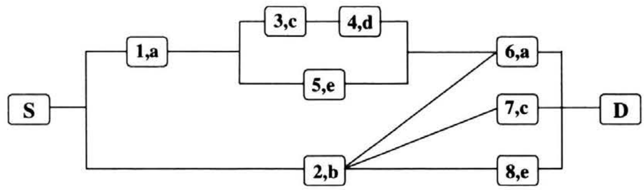

FIGURE 19. Diagrammatic representation of the 8-node complex road network; nodes and corresponding types of bridges according to Fig. 8.

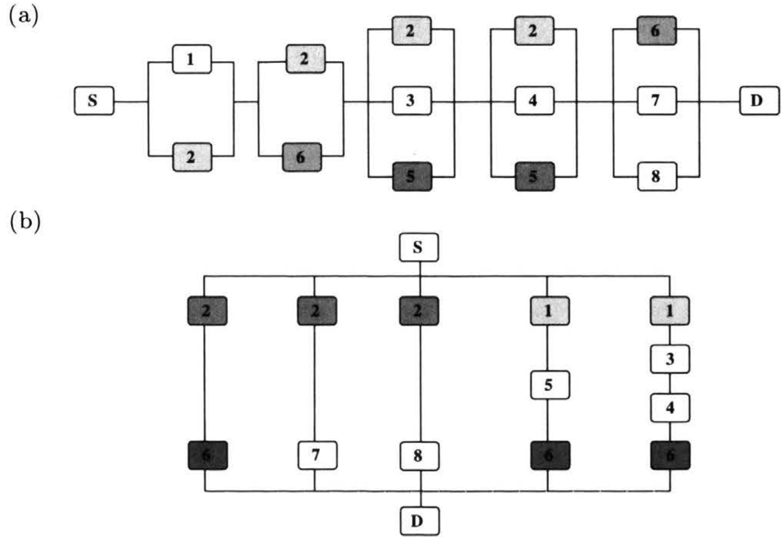

FIGURE 20. Functional logic of the road network in Fig. 19: (a) minimal cut set and (b) minimal path set representations.

dynamic programming procedure: it was then possible to apply it to all cases of single-objective optimization.

In the example, the vulnerable elements correspond to the five different typologies of prestressed concrete box girders simply supported on reinforced concrete box piers, whose structural diagrams are shown in Fig. 8 (Part I).

In the example case, the reliability of the network is:

$$\begin{aligned}R = 1 - P_{\mathrm{f}} = 1 - P_1P_2 \\ - P_2P_6 + P_1P_2P_6 - P_2P_4P_5 - P_6P_7P_8 + P_1P_2P_4P_5 \\ + P_2P_4P_5P_6 + P_2P_6P_7P_8 - P_1P_2P_4P_5P_6 - P_1P_2P_5P_6P_7P_8 \\ + P_1P_2P_3P_5P_6P_7P_8,\end{aligned}$$

P_i being the probability of failure of the i-th vulnerable element, conditional on the value of the peak ground acceleration.

The conditional probabilities of failure of the five bridge schemes, and their construction and upgrading costs are reported in Table 9, while times required by the three types of interventions (I-II-III) on each bridge and the out-of service times are reported in Table 14. The estimated traffic capacity of each link is shown in Table 20 in conventional units.

TABLE 20. Estimated traffic capacities of each link (arbitrary units).

Link	Capacity
S → 1	105
S → 2	520
1 → 3	300
1 → 5	200
3 → 4	100
2 → 6	200
2 → 7	100
2 → 8	200
6 → D	1200
7 → D	800
8 → D	1500

All single-objective optimizations have been carried out by *dynamic programming.*

The distributions of preventive upgrading interventions optimized with respect to each of the four objective functions for $a_g = 0.25\,g$ are reported in Table 21: each column shows the interventions that should be performed if the corresponding amount of resources is available, up to 69 r.u., i.e. the amount required to apply to all bridges the most effective intervention (III). (Analogous results have been obtained also for $a_g = 0.15\,g$ and $0.35\,g$).

It appears evident that if the objective function is either the reliability or the maximum traffic capacity of the network, the relevant path is S-2-6-D: thus, the most relevant element is bridge 2, on which an intervention is indicated even if a limited amount of resources is available. On the other hand, if the out-of-service time of the network or the time-efficiency of the interventions are being optimized, optimal paths include bridge 1.

Other results of the optimal allocation can be inferred from Figs. 21 and 22. Figure 21 shows that the probability of failure P_f decreases with regularity as the amount of available resources increases up to 69 r.u. However, the increase of the system reliability is rather slow already beyond 24 r.u., and becomes all but negligible beyond 32 r.u., i.e. in the range

TABLE 21. Distributions of preventive upgrading interventions between bridges (1-8) optimized with reference to: (i) the reliability; (ii) the expected traffic capacity; (iii) the out-of service time; (iv) the time-efficiency of interventions, as a function of the available economic resources (3-69 r.u.). Peak ground acceleration: $a_g = 0.25\,g$.

	C_{tot}	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69
(i)	1	I	-	-	-	-	-	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	2	-	I	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	3	-	-	-	-	-	-	-	-	-	-	-	-	-	I	II	III	II	-	II	III	III	III	III
	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III
	5	-	-	-	-	-	-	-	-	-	-	-	I	III	III	III	III	III	III	III	I	III	III	III
	6	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	7	-	-	-	I	-	-	-	-	II	-	III	-	III	III	III	III	III	III	III	III	III	III	III
	8	-	-	-	-	I	-	-	-	-	III	-	III	-	-	-	-	III	-	-	-	-	-	III
(ii)	1	I	-	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	2	-	I	-	-	I	I	I	I	II	II	III	III	III	III	III	III	III	III	III	III	III	III	III
	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	-	-	-	-	-	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	7	-	-	-	-	-	-	-	I	II	II	III	III	III	III	III	III	III	III	III	III	III	III	III
	8	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III	III
(iii)	1	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	III	I	III	III	III	III	III
	2	-	-	-	-	-	I	III	-	-	I	I	I	III	III	III	III	III	III	III	III	III	III	III
	3	I	I	I	I	I	I	I	I	I	I	I	I	I	I	III	I	I	III	III	III	III	III	III
	4	-	-	-	-	-	-	-	I	I	I	I	I	I	I	I	-	-	-	III	III	III	III	III
	5	-	-	-	-	-	-	-	-	-	-	I	III	III	III	III	III	III	III	III	III	III	III	III
	6	-	-	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	III	III	III	III
	7	-	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	III
	8	-	-	-	-	-	-	-	I	III	I	I	III	III	III	III	III	III	III	III	III	III	III	III
(iv)	1	-	-	-	-	-	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
	2	-	I	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	3	-	-	-	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
	4	-	-	-	-	-	-	-	-	-	-	I	I	I	I	I	III	III	III	III	III	III	III	III
	5	-	-	-	-	-	-	-	-	-	-	-	-	-	III	III	III	III	III	III	III	III	III	III
	6	I	-	-	-	-	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
	7	-	-	-	I	-	-	I	-	I	I	I	I	I	I	I	-	I	I	III	III	III	III	III
	8	-	-	-	-	-	-	-	II	II	II	II	II	II	II	II	II	II	II	II	II	II	II	II

in which bridges other than 1, 2, 6, 7, are upgraded. Similar conclusions can be derived from Fig. 22 in which the maximum traffic capacity is reported as a function of available resources for three values of the peak ground acceleration a_g.

It has to be noted that the optimized allocation of resources results much more efficient than a rule-of-thumb allocation: for example, the probability of failure is reduced by more than one order of magnitude if the distributions of interventions are optimized with respect to the strategy of applying the same interventions to all bridges.

Comparison of the failure probability P_f of the network corresponding to the interventions optimized with respect to each of the four objective

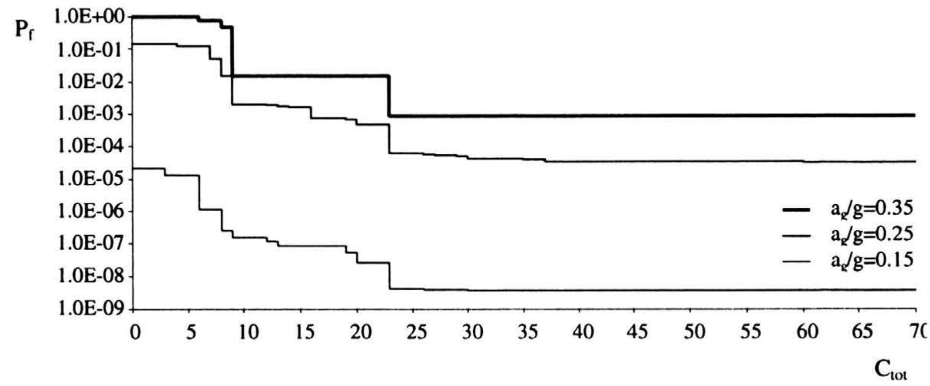

FIGURE 21. Optimized reliability of the road network vs available resources.

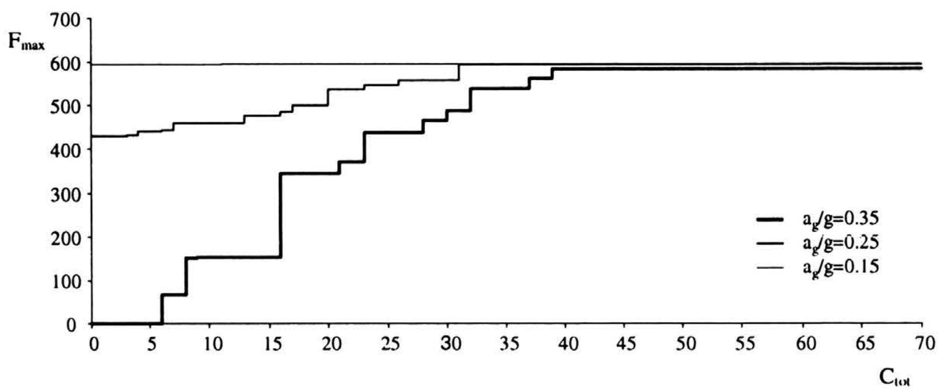

FIGURE 22. Maximum traffic flow in the road network vs available resources.

function taken into consideration, shows again that, as the available amount of resources increases, the probabilities of failure tend to coincide. However, for intermediate values, there are significant differences; moreover, it has already been noted that, depending on the objective function and other data, one or the other path may be more significant. Therefore, it can be important to take account at the same time of the different possible objectives of the prevention strategy by multi-objective optimization.

Table 22 shows the multi-objective optimal allocations obtained by the two decision-making strategies (UVA and ELECTRE), for the chosen earthquake intensity and the weights defined in Table 17; it can be noted that the results of the two strategies are similar (if not equal) with regard to the preferred path, while minor differences appear in the distribution of upgrading interventions.

TABLE 22. Absolute optimal distributions of upgrading interventions, derived by UVA and ELECTRE methods for several values of the available resources (3-24 r.u.), for $a_g = 0.25\,g$.

UVA

C_{tot}	3	6	9	12	15	18	21	24
1	-	-	-	-	-	-	-	-
2	-	I	III	III	III	III	III	III
3	I	-	-	I	II	III	III	III
4	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-
7	I	I	III	III	III	III	III	III
8	-	-	-	I	II	III	III	III

ELECTRE

C_{tot}	3	6	9	12	15	18	21	24
1	-	I	-	-	-	-	-	-
2	-	-	I	III	III	III	III	III
3	I	I	I	I	II	III	III	III
4	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-
6	-	I	-	-	-	-	-	-
7	-	-	-	III	III	III	III	III
8	I	I	III	I	II	III	III	III

By comparing the results of the multi-objective optimization and the single objective optimizations, it appears quite evident that the most significant path between S and D is the path S-2-6-D. Indeed, the governing function is the reliability of the network, to which the largest relative weight has been attributed.

9. Some final remarks

A specific optimal allocation problem has been tackled in Parts II and III: the choice of a set of preventive upgrading interventions on the bridges of a road network, such as to optimize the benefit of the available resources. The relevance of the problem arises from the usual shortage of public funds available for prevention programmes, and has been proved by the great advantages – amounting to orders of magnitude of the efficiency of the employed resources – found in the examples between "rule-of-thumb" and optimized allocation strategies.

The problem requires a *systematic probabilistic approach*, because of the many uncertainties involved. The procedures must combine several interdisciplinary competencies: of a structural engineer for the design of the interventions and the calculation of the fragility of the bridges (i.e. their probability of failure under a given earthquake), of a seismologist for the choice of the appropriate reference types and intensities of the earthquake motion, of a traffic engineer for the forecast of the network capacity.

The authors have tried and proposed procedures that cover all these aspects. The procedures have been applied to some specific examples in which realistic, but certainly not exact, numerical values have been introduced; in this way, the feasibility of the procedures has been already proved.

All optimizations of post-earthquake system response reported in these lectures have been developed with respect to a well defined assumed earthquake intensity; it would be interesting to investigate also the possibility of unconditional optimization with an assumed probability distribution of expected earthquake intensities.

Moreover, the seismic fragility of other vulnerable elements, like retaining walls, as well as the effectiveness of different intervention types should be estimated in order to include such elements in the optimization procedure.

It is also worth to note that recently the effects of continuous deterioration due to normal use, aggressive environment, and other causes are also being introduced in the optimization besides those of earthquakes.

This lecture is based on three papers [1, 2, 3], in which detailed lists of original references can be found.

References

1. G. AUGUSTI, A. BORRI, and M. CIAMPOLI, Optimal resource allocation for seismic reliability upgrading of existing structures and lifeline networks, *Reliability and Optimization of Structural Systems '94, Proceedings of* 6^{th} *IFIP WG7.5 Conference, Assisi, Italy*, Chapman & Hall for IFIP; Keynote Lecture, pp.3-24, 1994.

2. G. AUGUSTI and M. CIAMPOLI, Multi-objective optimal allocation of resources to increase the seismic reliability of highways, *Mathematical Methods of Operations Research*, Vol.47, No.1, pp.131-164, 1998;

3. G. AUGUSTI and M. CIAMPOLI, Further studies on the reduction of seismic risk in highway networks with limited resources, *Proceedings of the* 5^{th} *US Conference on Lifeline Earthquake Engineering, Seattle, Washington*, ASCE, August 1999.

Other relevant references are indicated in footnotes in these lectures.