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Modelling and computation of delamination for composite laminates

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A NEW COMPUTATIONAL delamination method using Damage Mechanics of composite laminates is proposed. A laminate is modelled as a stacking sequence of homogeneous layers and interlaminar interfaces. Both components are subject to damage. Deterioration such as fiber rupture, matrix and interface degradation are introduced at an intermediate level, which is called the meso-level. Damage is assumed to be uniform throughout the ply thickness. This makes it possible to avoid the main computational difficulties such as mesh dependency. For delamination analysis around a hole, an efficient numerical treatment is proposed to solve nonlinear (constitutive law) three-dimensional (edge effects) problems at a reasonable cost. Simulations are given.

1. Introduction

DELAMINATION REFERS to debonding of adjacent layers due to interlaminar stresses acting on weak interfaces. Due to its practical interest delamination gives rise to many studies. The analysis of this phenomenon is commonly split into the study of initiation and analysis of an existing delaminated area. Present computations of both initiation and propagation have assumed perfect connection between elastic layers. Up to now industrial initiation analysis involves such empirical criteria as point-stress or average stress. More predicting tools based on edge effects analysis [1–6] or singularity computation [7–10] are used to identify different stacking sequences with a more or less pronounced tendency to delamination. Most of the studies of propagation of existing delamination areas involve extensions of Fracture Mechanics [11–15], a theory which is currently used for metallic materials.

For one-directional fibre-reinforced elementary ply, damage may be the main mechanical phenomenon, since damage very quickly reaches high levels in such media (which is not the case for metallic material). We propose to use Damage Mechanics of Composite Materials [16–18] to take all the various types of degradation into account. This allows us to predict delamination initiation and propagation much more precisely, and to include these two aspects in a single model. For laminate damage modelling we introduce two mechanical constituents: the elementary layer and the interface. The layer modelling is founded on theoretical developments and numerous experimental tests, it has been achieved in collaboration with Aerospatiale, especially for T300-914 laminates. The layer model includes anisotropic unilateral damage and elasto-plastic behaviour. It is to be noticed that the state of damage is assumed to be uniform throughout the ply thickness. This enables us to avoid numerical difficulties such as mesh dependency [19–20]. To avoid them completely, delayed damage modelling methods are also used.

Interface is a two-dimensional element which ensures transmission of displacements and traction from one ply to another. It depends on the angle between fibre directions of the adjacent layers. In the first instance it is assumed to be elastic and damageable.

A computational tool for delamination initiation and growth prediction around a hole has been developed [21–22]. Because of the edge effects and nonlinear behaviour, the problem is three-dimensional and nonlinear. Nevertheless, this analysis can be restricted to the edge vicinity where three-dimensional effects are located. We use the LArge Time INcrement approach [23], and a semi-analytical technique which allow for solving only two-dimensional problems, thanks to the particular geometry of the edge. The latter method combines a gradient conjugate method and the Fourier expansions using the Fast-Fourier-Transform. An axially symmetric pre-conditioned operator is used to solve nonaxially symmetric problems. The LArge Time INcrement method consists in a single global iterating procedure of the whole loading history which considerably reduces, first, the number of transfers between local and global levels and, second, the number of global solutions and hence, the numerical cost of calculation. First results are presented which show the efficiency of this approach.

2. Meso-modelling of long fiber laminates

We first recall some aspects of the meso-modelling of composite laminates introduced by P. LADEVEZE and others [16, 19–20]. The damage concept introduced by KACHANOV [24] is defined as a progressive deterioration of loaded materials due to initiation and growth of micro-cracks. Damage Mechanics of composite laminates consists in modelling of these phenomena at a structural analysis scale. At this level a laminate can be reduced to a stacking of elementary constituents:

a homogeneous single layer in the thickness;

an interface which is a two-dimensional medium connecting two adjacent layers and which depends on their relative orientations.

These elements being modelled and identified, the mechanical behaviour of any laminate is then easly deduced.

3. Single layer modelling

It should be noticed that we limit ourselves to single layers with only one reinforced direction. The behaviour is linear elastic and brittle in the fiber direction N_1, nonlinear elastic and brittle in compression, so that the modulus in fiber direction is [26]:

$$E_1^c = E_1^0\left[1 - \frac{\alpha}{E_1^c}\langle -\sigma_{11}\rangle_+\right],$$

where α is a material parameter and $\langle\ \rangle_+$ denotes the positive part. Apart from the fiber fracture, the main damage occurring in layers is due to matrix micro-cracking, which is parallel to N_1, and to degradations of the fiber-matrix bond. This more or less qualitative information is transferred at the single-layers level by means of a homogenization process. The transverse rigidity in compression being supposed equal to E_2^0, one obtains the following energy for the damaged material:

$$E_D = \frac{1}{2}\left[\frac{\langle\sigma_{11}\rangle_+^2}{E_1^0} + \frac{\varphi(\langle -\sigma_{11}\rangle_+)}{E_1^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_{11}\sigma_{22} + \frac{\langle -\sigma_{22}\rangle_+^2}{E_2^0} + \frac{\langle\sigma_{22}\rangle_+^2}{(1-d')E_2^0} + \frac{\sigma_{12}^2}{(1-d)G_{12}^0}\right].$$

d and d' are two scalar damage variables constant throughout the ply thickness, the main problem is to describe their evolution. The thermodynamic variables associated with d and d' are

$$Y_d = -\rho \frac{\delta[\Psi]}{\delta d}\bigg|_{\tilde{\sigma}} = \frac{\delta[E_D]}{\delta d}\bigg|_{\sigma} = \frac{1}{2}\frac{[\sigma_{12}^2]}{G_{12}^0(1-d)^2},$$

$$Y_d' = -\rho \frac{\delta[\Psi]}{\delta d'}\bigg|_{\tilde{\sigma}} = \frac{\delta[E_D]}{\delta d'}\bigg|_{\sigma} = \frac{1}{2}\frac{[\langle\sigma_{22}\rangle_+^2]}{E_2^0(1-d')^2},$$

where Ψ is free energy and [ ] denotes mean valueaveraged over the thickness. To describe the inelastic phenomena due to damage one uses a plasticity model with isotropic hardening. $\tilde{\sigma}$ is the effective stress which governs the inelastic evolution when there is damage

$$\tilde{\sigma}_{11} = \sigma_{11}, \qquad \tilde{\sigma}_{12} = \frac{\sigma_{12}}{1-d}, \qquad \tilde{\sigma}_{22} = \frac{\langle\sigma_{22}\rangle_+}{1-d'} - \langle-\sigma_{22}\rangle_+ .$$

Microscopic deterioration evolutions affect both damage variables and from experimental results it follows that the governing quantities of damage evolution are

$$\mathbf{Y} = \sup_{\tau\leq t}[Y_d + bY_{d'}]^{1/2}, \qquad \mathbf{Y}' = \sup_{\tau\leq t} = [Y_d']^{1/2},$$

where b is a material constant. Experimentally, one obtains:

$$d = \left\langle\frac{\mathbf{Y}-Y_0}{Y_c}\right\rangle_+ \quad \text{if} \quad d<1; \quad d=1 \quad \text{otherwise},$$

$$d' = bd \quad \text{if} \quad d'<1 \quad \text{and} \quad \mathbf{Y}' < Y_c'; \quad d'=1 \quad \text{otherwise}.$$

Identification of material parameters has been done for several laminates. Results for T300 - 914 and IM6 - 914 are given in [20] and [27]. This modelling technique has been verified on numerous experimental tests. Figure 1 shown below gives an example of the damage material curves as a function of Y in the case of a T300/914.

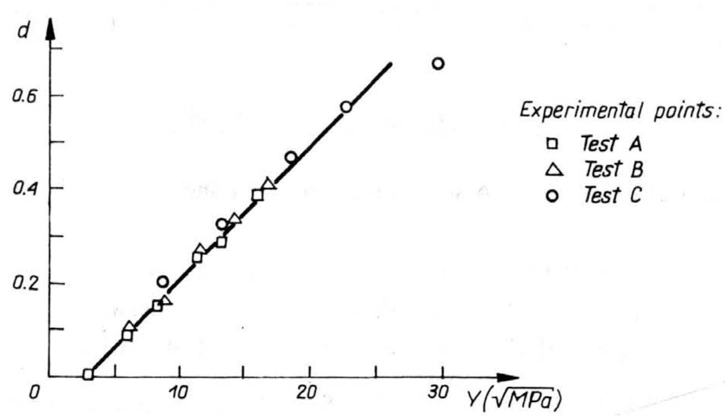


FIG. 1.

The failure mode of the single layer is derived from damage modelling through a multi-criterion approach of the same kind as that of HASHIN [25].

4. Interface modelling

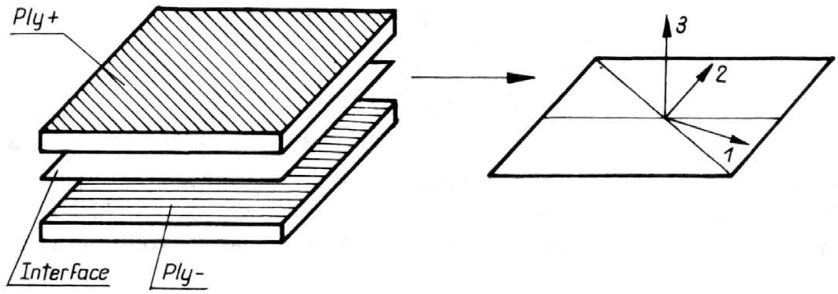


FIG. 2.

Interface is a zero thickness medium which ensures transmission of stress and displacement from one ply to another (Fig. 2). (N_1, N_2) axes are bisectors of the fiber directions. Displacement discontinuities are denoted by $[U]$. In a first step, interface is assumed to be elastic and damageable. This modelling will be completed through comparisons between analytical computations and experimental tests, what creates an additional difficulty. The energy per unit area is written as follows:

$$E_D = \frac{1}{2}\left[\frac{\langle -\sigma_{33}\rangle_+^2}{k^0} + \frac{\langle \sigma_{33}\rangle_+^2}{k^0(1-d)} + \frac{\sigma_{13}^2}{k_2^0(1-d_1)} + \frac{\sigma_{23}^2}{k_2^0(1-d_2)}\right],$$

k^0, k_1^0, k_2^0 are interface elastic characteristics. The thermodynamic variables associated with d, d_1 and d_2 are

$$Y_d = \frac{1}{2}\frac{\langle \sigma_{33}\rangle_+^2}{k^0(1-d)^2}, \quad Y_{d_1}\frac{\sigma_{13}^2}{k_2^0(1-d_1)}, \quad Y_{d_2} = \frac{\sigma_{23}^2}{k_2^0(1-d_2)^2}$$

and

$$Y = \sup_t [Y_d + \gamma_1 Y_{d_1} + \gamma_2 Y_{d_2}]^{\frac{1}{2}},$$

γ_1, γ_2 are material parameters. A standard model is defined by the choice of a function W,

$$\begin{aligned} d &= W(Y) \quad \text{if} \quad d < 1, \quad d = 1 \text{ otherwise};\\ d_1 &= \gamma_1 W(Y) \quad \text{if} \quad d_1 < 1, \quad d_1 = 1 \text{ otherwise};\\ d_2 &= \gamma_2 W(Y) \quad \text{if} \quad d_2 < 1, \quad d_2 = 1 \text{ otherwise}. \end{aligned}$$

The first choice for the material function W is:

$$W(Y) = \frac{Y(t')}{Y_c} \quad \text{if} \quad d < 1, \quad \text{otherwise} \quad d = 1,$$

Y_c is a critical energy which equals Y_d for a unit value of d.

5. Relation between two approaches of delamination: fracture mechanics and damage mechanics

The comparison has been done on a D.C.B. specimen under the assumption of an damageable interface connecting two elastic layers under a pure Mode I loading.

The computation uses Reissner's plate theory, under the assumption of plane strain state in the (N_1, N)-plane (Fig. 3).

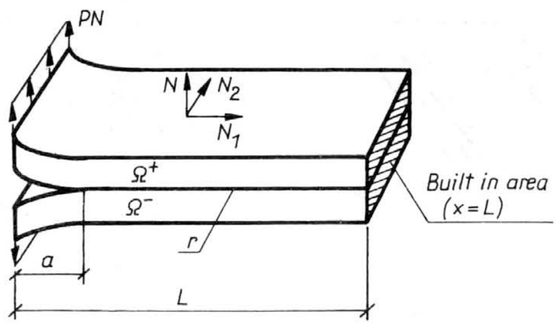


FIG. 3.

Because of interface deterioration, there exists a maximum value of the applied load P denoted by P_c. This limit point is reached for $d = 1$ which means for the delamination propagation. This allows us to calculate the critical energy release rate G_{I_c}. For a crack long enough one obtains that $G_{I_c} \# Y_c/2$. A similar result has been obtained for the Mode II loading. Within this framework, Fracture Mechanics appears to be a simplified tool for delamination study in the case of elastic layers. It is to be noted that this analysis based on the model of elastic layers allows for an approximative identification of the interface behaviour only. A complete identification must involve a comparison between the computations and the tests, what represents a further difficulty.

6. Some computational difficulties arising in damage evaluation [19–20]

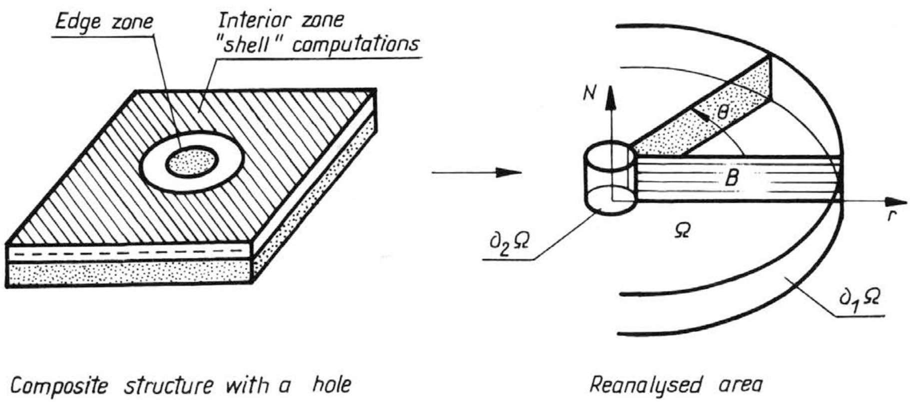
It is now a well known property that classical damage theory leads to a severe mesh-dependency, the size of the damage area being (in general) connected with the size of the finite element applied. Different solutions are proposed to avoid this fundamental difficulty. For composite one can use meso-modelling-technique as the one previously used solutions are proposed for the single layer. To avoid numerical difficulties, a delayed damage model has also been introduced. Such model differs from the previous one for high damage rates only. For example, in the case of the ply it is defined as follows

$$\dot{d} = k\left\langle \frac{Y - \mathbf{Y}(d)}{Y_c} \right\rangle_+^n \quad \text{if} \quad d < 1; \quad d = 1 \quad \text{otherwise} \quad (\mathbf{Y}(d) = Y_0 + Y_c d)$$

$$\dot{d}' = k'\left\langle \frac{Y' - \mathbf{Y}'(d')}{Y_c} \right\rangle_+^{n,} \quad \text{if} \quad d' < 1; \quad d' = 1 \quad \text{otherwise} \quad (\mathbf{Y}'(d') = Y_{0'} + Y_c d').$$

7. Damage mechanics for delamination initiation analysis

Implementation of the above mechanical approach in a computer program to study the initiation and propagation of damage in vicinity of an initially circular hole is presented below (Fig. 4).



The problem to solve is: Find (σ, ε) such that

$$\forall U^* \in \mathbf{U} = \{U/U \text{ regular and } U|_{\partial_1 \Omega} = U_d(t)\}\,, \qquad \forall t \in [0, T]\,,$$

$$\int\limits_{\Omega} \mathrm{Tr}(\sigma\varepsilon(U^*))\, d\Omega + \sum_{i}^{n-1} \int\limits_{\Gamma} \sigma N[U^*] d\Gamma = \int\limits_{\partial_2 \Omega} F_d(t) U^*\, dS$$

and (σ, ε) satisfies the constitutive relations in Ω and in Γ. The analysis is restricted to the vicinity of edges in Ω, where three-dimensional effects are significant. A link with a solution obtained by a "shell" approximation is made on $\partial_1 \Omega$ area through prescribed displacement U_d; load F_d is prescribed in the region $\partial_2 \Omega$.

8. Computational technique

To solve this nonlinear three-dimensional evolution problem at a reasonable cost, (i) the "LArge Time INcrement method", and (ii) a semi-analytical method (which requires the solution of two-dimensional problems only) are used. The "LArge Time INcrement method" developed by P. Ladeveze *et al.*, breaks with the step by step scheme of all previous computation techniques (Newton–Raphson for example). It proceeds by a single global iterational procedure over the whole loading history $[0, T]$ between the global linear steps (satisfying the equilibrium conditions) and the local nonlinear steps (satisfying the constitutive law) which considerably reduces the number of global solutions. The global step represents then a linear but three-dimensional evolution problem.

This three-dimensional problem is reduced to two-dimensional problems in a finite strip by using (i) — a conjugate gradient method, and (ii) — the Fast-Fourier-Transform. An axially symmetric operator is used to solve non-axially symmetric problems, so the problem is solved iteratively by means of a series of intermediate problems

$$K_0 X(t) = F^{(i)}(t)\,,$$

where

$$K_0 = \frac{1}{2\pi} \int_0^{2\pi} K_e \, d\theta$$

(K_e is the elasticity operator). To solve this problem, the generalized force $F^{(i)}(t)$ at the i-th step is written as a sum of products of time and space functions. By means of the Fast-Fourier-Transform, displacement is expanded into a Fourier series, and then the two-dimensional elastic problems associated with K_0 and with each component of displacements are solved in a finite strip. An example of computation of a structure loaded in pure Mode I is given below. The displacement is prescribed in the form $U(r = 0) = \lambda U_0$.

Figure 5a shows the exterior force power divided by λ as a function of λ. This curve allows us to predict through the global instability condition the maximum applied load for a structure. To know where delamination occurs one has to check at what Gauss point the interface damage equals one. The other figure shows a typical evolution of peeling stress at this Gauss point.

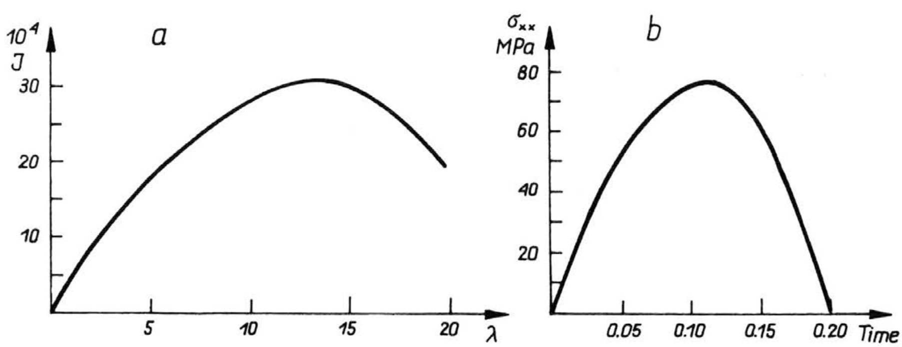


FIG. 5.

This approach may prove to be too expensive for a parametered study or for a composite with a great number of layers, so a simplified method has been developed in the case of a quasi-straight edge under the assumption of elastic layers. First comparison between the analytical results and experiments are very satisfying [28].

9. Conclusion

Delamination consists in a degradation of layers and of bonds between layers. An approach based upon the Damage Mechanics is proposed. A connection between Damage Mechanics and Fracture Mechanics has been established. Fracture Mechanics appears to be a simplified tool for studying the growth of an existing delamination under the assumption of elastic layers. Damage Mechanics enables us to predict precisely the instant of delamination initiation and propagation on the basis of a single model. This approach leads to a nonlinear three-dimensional evolution problem. A method of solution of such a problem at a sensible numerical expense is presented for an initially circular hole.

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