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Variational bounds on the effective moduli of viscoelastic periodic composites

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VARIATIONAL THEOREMS are established and applied to the derivation of the bounds for the effective viscoelastic dynamical moduli of micro-heterogeneous, periodic composites. For the case of two-phase isotropic materials the bounds are calculated and graphical illustration is presented.

1. Introduction

THE FIELD of composite materials includes the methods of derivation of the effective (overall macroscopic) properties of inhomogeneous media.

The effective properties are the properties of the equivalent homogeneous body, the behaviour of which is the same as the macroscopic behaviour of the microperiodic composite.

The most purely mathematical method is the homogenization method [1, 2]. There are many papers concerned with the problem of theoretical determination of the effective properties of different inhomogeneous media [3, 4, 5, 9]. To obtain the formulae for the effective properties by the homogenization method we must have full information about the structure of periodic composite. If, for example, all the available information on the structure of composite is limited to the properties of the constituents and their volume fractions only, one is not able to calculate explicitly the effective properties but rather to find their bounds.

The techniques of derivation of the bounds for macroscopic properties have an extensive history dating back to XIX century (Mossotti 1836, Voigt 1887). Some of the recent advances in bounding the effective properties are due to the developments of new variational principles [6, 7].

The aim of this paper is to find the bounds on the effective dynamical modulus of viscoelastic material with microperiodic structure. It is assumed that viscoelastic body undergoes the steady-state harmonic vibrations. Due to this fact, complex fields are introduced and the properties of the body are described by linear equations with complex coefficients. Complex effective tensor governs the response of the composite to oscillating field in the quasi-static limit. In this case the wave length and attenuation length are sufficiently large compared with the characteristic length of microstructure.

For the above problem the saddle-point variational principle of GIBIANSKY and CHERKAEV [7] is formulated. This principle is converted via the Legendre transforms into a Dirichlet-type variational principle from which the bounds on effective constants are obtained. As an example of application, the mixture of two-phase isotropic constituents is considered and the bounds are calculated (figures are shown).

2. Formulation of the problem of viscoelasticity

We consider the steady-state vibrations of a linear viscoelastic, heterogeneous body [8]. Using complex functions we define:

complex displacement of the body

$$\tilde{\mathbf{u}}e^{i\omega t} = (\mathbf{u}_1 + i\mathbf{u}_2)(\cos\omega t + i\sin\omega t) \tag{2.1}$$

(only the real part of (2.1) describes the behaviour of harmonically vibrating viscoelastic body with frequency ω);

complex strain

$$\tilde{\mathbf{e}} = \frac{1}{2}(\nabla\tilde{\mathbf{u}} + (\nabla\tilde{\mathbf{u}})^T); \tag{2.2}$$

complex stress

$$\tilde{\boldsymbol{\sigma}} = \tilde{\mathbf{L}}\tilde{\mathbf{e}}; \tag{2.3}$$

and equilibrium equations

$$\operatorname{div}\tilde{\boldsymbol{\sigma}} = 0, \tag{2.4}$$

where $\tilde{\mathbf{L}}$ is the complex tensor of 4-th order called the dynamical modulus, which depends on ω.

Tensor $\tilde{\mathbf{L}}$ has the form

$$\tilde{\mathbf{L}} = \mathbf{L}_1 + i\mathbf{L}_2, \tag{2.5}$$

where $\mathbf{L}_1(x)$, $\mathbf{L}_2(x)$ are positive definite and periodic functions of position (Q — cell of periodicity).

$\tilde{\mathbf{L}}$ has the properties of symmetry

$$\tilde{L}_{ijkl} = \tilde{L}_{klij} = \tilde{L}_{jikl}.$$

Separating real and imaginary parts in (2.2) — (2.4) we obtain equivalent relations for real functions

$$\begin{gathered}\mathbf{e}_1 = \frac{1}{2}(\nabla\mathbf{u}_1 + (\nabla\mathbf{u}_1^T)), \quad \mathbf{e}_2 = \frac{1}{2}(\nabla\mathbf{u}_2 + (\nabla\mathbf{u}_2)^T),\\ \boldsymbol{\sigma}_1 = \mathbf{L}_1\mathbf{e}_1 - \mathbf{L}_2\mathbf{e}_2, \qquad \boldsymbol{\sigma}_2 = \mathbf{L}_1\mathbf{e}_2 + \mathbf{L}_2\mathbf{e}_1,\\ \operatorname{div}\boldsymbol{\sigma}_1 = 0, \quad \operatorname{div}\boldsymbol{\sigma}_2 = 0.\end{gathered} \tag{2.6}$$

Let us split the local fields, depending on position, $\mathbf{e}_1(x)$, $\mathbf{e}_2(x)$, $\boldsymbol{\sigma}_1(x)$, $\sigma_2(x)$ into their average and fluctuating components:

$$\begin{aligned}\mathbf{e}_i(x) &= \mathbf{e}_i^* + \mathbf{E}_i^*(x),\\ \boldsymbol{\sigma}_i(x) &= \boldsymbol{\sigma}_i^* + \boldsymbol{\Sigma}_i^*(x), \quad i = 1,2,\end{aligned} \tag{2.7}$$

where $\mathbf{e}_i^*$, $\boldsymbol{\sigma}_i^*$ — constant tensors, $\mathbf{E}_i^*$, $\boldsymbol{\Sigma}_i^*$ — rotation and divergence-free fields, respectively, which means that

$$\mathbf{e}_i^*, \boldsymbol{\sigma}_i^* \in \mathcal{U}, \quad \mathbf{E}_i^* \in \mathcal{E}, \quad \boldsymbol{\Sigma}_i^* \in \mathcal{J}, \quad i = 1,2$$

and

$$\mathcal{H} = \mathcal{U} \oplus \mathcal{E} \oplus \mathcal{J}$$

is a Hilbert space of tensorial fields over $L^2(Q)$, $\mathcal{U}$ — the space of uniform tensor fields, $\mathcal{E}$ — the subspace of curl-free fields, $\mathcal{J}$ — the subspace of div-free fields.

The fluctuating parts of (2.7) satisfy

$$\int_Q \mathbf{u}^* \cdot \mathbf{E}_i^*(x)dx = 0, \quad \int_Q \mathbf{u}^* \cdot \mathbf{\Sigma}_i^*(x)dx = 0, \quad \forall \mathbf{u}^* \in \mathcal{U} \tag{2.8}$$

$$\int_Q \mathbf{E}_i^*(x) \cdot \mathbf{\Sigma}_i^*(x) = 0, \quad i = 1,2,$$

which means that $\mathbf{u}^*$, $\mathbf{E}_i^*(x)$, $\mathbf{\Sigma}_i^*(x)$ are mutually orthogonal. Using the above definitions we can write the local constitutive law in the following block form:

$$\begin{bmatrix} \sigma_1^* + \mathbf{\Sigma}_1^* \\ \sigma_2^* + \mathbf{\Sigma}_2^* \end{bmatrix} = \begin{bmatrix} -\mathbf{L}_1 & -\mathbf{L}_2 \\ -\mathbf{L}_2 & \mathbf{L}_1 \end{bmatrix} \begin{bmatrix} -(\mathbf{e}_1^* + \mathbf{E}_1^*) \\ \mathbf{e}_2^* + \mathbf{E}_2^* \end{bmatrix}. \tag{2.9}$$

Similarly, the effective tensor $\mathbf{L}^*$ is defined by

$$\begin{bmatrix} \sigma_1^* \\ \sigma_2^* \end{bmatrix} = \begin{bmatrix} -\mathbf{L}_1^* & -\mathbf{L}_2^* \\ -\mathbf{L}_2^* & \mathbf{L}_1^* \end{bmatrix} \begin{bmatrix} -e_1^* \\ e_2^* \end{bmatrix}. \tag{2.10}$$

3. Variational principle for viscoelasticity

Following GIBIANSKY and CHERKAEV [7], we formulate the saddle-point variational principle

$$W^*(\mathbf{e}_1^*, \mathbf{e}_2^*) = \max_{\mathbf{E}_1 \in \mathcal{E}} \min_{\mathbf{E}_2 \in \mathcal{E}} \int_Q dx W(\mathbf{e}_1^* + \mathbf{E}_1(x), \mathbf{e}_2 + \mathbf{E}_2(x)) \tag{3.1}$$

associated with the problem (2.9), where

$$W^*(P_1, P_2) = \frac{1}{2} \begin{bmatrix} P_1^T \, P_2^T \end{bmatrix} \mathbf{L}^* \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \tag{3.2}$$

$$W(P_1, P_2) = \frac{1}{2} \begin{bmatrix} P_1^T P_2^T \end{bmatrix} \mathbf{L} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}. \tag{3.3}$$

This principle can be converted via the Legendre transforms into a Dirichlet type variational principle which is more useful to bound $\mathbf{L}^*$.

First we introduce the Legendre transforms of W^* and W,

$$\widetilde{W}^*(Q_1, P_2) = \max_{P_1 \in \mathcal{T}} [W^*(P_1, P_2) - (Q_1, P_1)], \tag{3.4}$$

$$\widetilde{W}(Q_1, P_2) = \max_{P_1 \in \mathcal{T}} [W(P_1, P_2) - (Q_1, P_1)]. \tag{3.5}$$

Substitution of (3.1) and (3.2) into these expressions gives the explicit formulae

$$\widetilde{W}^*(Q_1, P_2) = \frac{1}{2} \begin{bmatrix} Q_1^T \, P_2^T \end{bmatrix} \widetilde{\mathbf{L}}^* \begin{bmatrix} Q_1 \\ P_2 \end{bmatrix}, \tag{3.6}$$

$$\widetilde{W}(Q_1, P_2) = \frac{1}{2} \begin{bmatrix} Q_1^T \, P_2^T \end{bmatrix} \widetilde{\mathbf{L}} \begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} \tag{3.7}$$

for these transforms, where

$$\widetilde{\mathbf{L}}^* = \begin{bmatrix} \mathbf{L}_1^{*-1} & \mathbf{L}_1^{*-1}\mathbf{L}_2^* \\ \mathbf{L}_2^*\mathbf{L}_1^{*-1} & \mathbf{L}_1^* + \mathbf{L}_2^*\mathbf{L}_1^{*-1}\mathbf{L}_2^* \end{bmatrix}, \tag{3.8}$$

and

$$\tilde{\mathbf{L}} = \begin{bmatrix} \mathbf{L}_1^{-1} & \mathbf{L}_1^{-1}\mathbf{L}_2 \\ \mathbf{L}_2\mathbf{L}_1^{-1} & \mathbf{L}_1 + \mathbf{L}_2\mathbf{L}_1^{-1}\mathbf{L}_2 \end{bmatrix}, \tag{3.9}$$

Since $W^*(P_1, P_2)$ and $W(P_1, P_2)$ are concave functions of P_1 at fixed P_2, they can be recovered from $\widetilde{W}^*$ and $\widetilde{W}$ using the inverse Legendre transforms

$$W^*(P_1, P_2) = \min_{Q_1 \in \mathcal{T}} [\widetilde{W}^*(Q_1, P_2) + (Q_1, P_1)], \tag{3.10}$$

$$W(P_1, P_2) = \min_{Q_1 \in \mathcal{T}} [\widetilde{W}(Q_1, P_2) + (Q_1, P_1)]. \tag{3.11}$$

By substituting (3.11) into (3.1) and the result into (3.4), we obtain the variational inequality

$$\begin{aligned} \widetilde{W}^*(\boldsymbol{\sigma}_1^*, \mathbf{e}_2^*) &= \max_{\mathbf{e}_1 \in \mathcal{U}} [W^*(\mathbf{e}_1, \mathbf{e}_2^*) - (\boldsymbol{\sigma}_1^*, \mathbf{e}_1)] \\ &= \min_{\mathbf{E}_2 \in \mathcal{E}} \max_{\mathbf{e}_1 + \mathbf{E}_1 \in \mathcal{U} \oplus \mathcal{E}} \int_Q dx [W(\mathbf{e}_1 + \mathbf{E}_1(x), \mathbf{e}_2^* + \mathbf{E}_2(x)) - (\boldsymbol{\sigma}_1^*, \mathbf{e}_1)] \\ &= \min_{\mathbf{E}_2 \in \mathcal{E}} \max_{\mathbf{e}_1 + \mathbf{E}_1 \in \mathcal{U} \oplus \mathcal{E}} \min_{\boldsymbol{\sigma}_1 \in \mathcal{H}} \int_Q dx [\widetilde{W}(\boldsymbol{\sigma}_1(x), \mathbf{e}_2^* + \mathbf{E}_2(x)) + (\boldsymbol{\sigma}_1(x), \\ &\qquad \mathbf{e}_1 + \mathbf{E}_1(x)) - (\boldsymbol{\sigma}_1^*, \mathbf{e}_1)] \\ &\le \min_{\mathbf{E}_2 \in \mathcal{E}} \max_{\mathbf{e}_1 + \mathbf{E}_1 \in \mathcal{U} \oplus \mathcal{E}} \min_{\boldsymbol{\Sigma}_1 \in \mathcal{J}} \int_Q dx [\widetilde{W}(\boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \\ &\qquad \mathbf{e}_2^* + \mathbf{E}_2(x)) + (\boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \mathbf{e}_1 + \mathbf{E}_1(x)) - (\boldsymbol{\sigma}_1^*, \mathbf{e}_1)] \\ &= \min_{\boldsymbol{\Sigma}_1 \in \mathcal{J}} \min_{\mathbf{E}_2 \in \mathcal{E}} \int_Q dx [\widetilde{W}(\boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \mathbf{e}_2^* + \mathbf{E}_2(x))], \end{aligned} \tag{3.12}$$

where inequality arises bacause we have restricted the minimum over $\boldsymbol{\sigma}_1 \in \mathcal{H}$ to a smaller class of fields, namely to those of the form

$$\boldsymbol{\sigma}_1(x) = \boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \quad \text{where} \quad \boldsymbol{\Sigma}_1(x) \in \mathcal{J}. \tag{3.13}$$

Now the solutions $\boldsymbol{\sigma}_1^*$, $\mathbf{e}_1^* \in \mathcal{U}$, $\boldsymbol{\Sigma}_1^*(x) \in \mathcal{J}$ and $\mathbf{E}_1^* \in \mathcal{E}$ to the direct problem (2.9) satisfy

$$\widetilde{W}^*(\boldsymbol{\sigma}_1^*, \mathbf{e}_2^*) = \int_Q dx [\widetilde{W}(\boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \mathbf{e}_2^* + \mathbf{E}_2(x))]. \tag{3.14}$$

Thus the variational inequality (3.12) is strong; it implies the Dirichlet–type variational principle,

$$\widetilde{W}^*(\boldsymbol{\sigma}_1^*, \mathbf{e}_2^*) = \min_{\boldsymbol{\Sigma}_1 \in \mathcal{J}} \min_{\mathbf{E}_2 \in \mathcal{E}} \int_Q dx [\widetilde{W}(\boldsymbol{\sigma}_1^* + \boldsymbol{\Sigma}_1(x), \mathbf{e}_2^* + \mathbf{E}_2(x))] \tag{3.15}$$

of Gibiansky and Cherkaev.

As an example of application of (3.15) we take the trial fields $\boldsymbol{\Sigma}_1 = \mathbf{E}_2 = 0$ and set

$$\boldsymbol{\sigma}_1^* = -L_0 \mathbf{e}_2^*, \tag{3.16}$$

where L_0 is any given real selfadjoint operator. Then (3.15) and the expressions (3.6) and

(3.7) for $\widetilde{W}^*$ and $\widetilde{W}$ imply that the operator inequality

$$(3.17)\qquad L_0L_1^{*-1}L_0 - L_0L_1^{*-1}L_2^* - L_2^*L_1^{*-1}L_0 + L_1^* + L_2^*L_1^{*-1}L_2^* \le \int\limits_Q dx[L_0L_1^{-1}L_0 - L_0L_1^{-1}L_2 - L_2L_1^{-1}L_0 + L_1 + L_2L_1^{-1}L_2]$$

is satisfied for all L_0. This has the equivalent form

$$(3.18)\qquad (L_0 - L_2^*)L_1^{*-1}(L_0 - L_2^*) + L_1^* \le \int\limits_Q dx[(L_0 - L_2)L_1^{-1}(L_0 - L_2) + L_1].$$

If we introduce the complex operators

$$(3.19)\qquad (\widetilde{L}^* - iL_0)^{-1} = [(L_0 - L_2^*)L_1^{*-1}(L_0 - L_2^*) + L_1^*]^{-1} + i[L_1^*(L_0 - L_2^*)^{-1}L_1^* + (L_0 - L_2)]^{-1},$$

$$(3.20)\qquad (\widetilde{L} - iL_0)^{-1} = [(L_0 - L_2)L_1^{-1}(L_0 - L_2) + L_1]^{-1} + i[L_1(L_0 - L_2)^{-1}L_1 + (L_0 - L_2)]^{-1},$$

where $\widetilde{L}^* = L_1^* + iL_2^*$ and $\widetilde{L} = L_1 + iL_2$, then (3.18) can be rewritten as

$$(3.21)\qquad [\mathrm{Re}(\widetilde{L}^* - iL_0)^{-1}]^{-1} \le \int\limits_Q dx[\mathrm{Re}(\widetilde{L}(x) - iL_0)^{-1}]^{-1}.$$

Now the complex direct problem

$$(3.22)\qquad \widetilde{\boldsymbol{\sigma}}^* + \widetilde{\boldsymbol{\Sigma}}^* = \widetilde{\mathbf{L}}(\widetilde{\mathbf{e}}^* + \widetilde{\mathbf{E}}^*)$$

is isomorphic with the rotated complex direct problem

$$(3.23)\qquad \widetilde{\boldsymbol{\sigma}}'^* + \widetilde{\boldsymbol{\Sigma}}'^* = \widetilde{\mathbf{L}}'(\widetilde{\mathbf{e}}'^* + \widetilde{\mathbf{E}}'^*)$$

obtained by a rotation by θ in the complex plane, where $\theta \in [0, 2\pi]$ is any fixed angle

$$(3.24)\qquad \widetilde{\boldsymbol{\sigma}}' = e^{i\theta}\widetilde{\boldsymbol{\sigma}},$$

and

$$(3.25)\qquad \widetilde{\boldsymbol{\sigma}}'^* = e^{i\theta}\widetilde{\boldsymbol{\sigma}}^* \in \widetilde{\mathcal{U}}, \quad \widetilde{\mathbf{e}}' = \widetilde{\mathbf{e}} \in \widetilde{\mathcal{U}},$$
$$\widetilde{\boldsymbol{\Sigma}}'^* = e^{i\theta}\widetilde{\boldsymbol{\Sigma}} \in \widetilde{\mathcal{J}}, \quad \widetilde{\mathbf{E}}'^* = \widetilde{\mathbf{E}} \in \widetilde{\mathcal{E}}.$$

Hence (3.21) implies the more general bounds

$$(3.26)\qquad [\mathrm{Re}(e^{i\theta}\widetilde{L}^* - iL_0)^{-1}]^{-1} \le \int\limits_Q dx[\mathrm{Re}(e^{i\theta}\widetilde{L}(x) - iL_0)^{-1}]^{-1},$$

which hold for any real selfadjoint operator L_0 and any angle $\theta \in [0, 2\pi]$ such that

$$(3.27)\qquad \mathrm{Re}(e^{i\theta}\widetilde{L}(x)) \ge 0 \quad \text{for all} \quad x \in Q.$$

4. Example

We consider a viscoelastic periodic composite (macroscopically isotropic) built from two isotropic constituents. We don't know the geometry of the cell of periodicity, but we know the properties of constituents and their volume fractions.

$$L_1(x) = [3\kappa_R^1, 2\mu_R^1]\chi_1(x) + [3\kappa_R^2, 2\mu_R^2]\chi_2(x),$$
$$L_2(x) = [3\kappa_I^1, 2\mu_I^1]\chi_1(x) + [3\kappa_I^2, 2\mu_I^2]\chi_2(x), \tag{4.1}$$

where $[\kappa, \mu]$ denotes Hill's convention for isotropic elastic moduli, χ_i — characteristic functions of i-th constituent ($i = 1, 2$),

$$\int\limits_Q \chi_i(x)dx = v_i, \quad \chi_2(x) = 1 - \chi_1(x).$$

We assume that $v_i = 1/2$ for $i = 1, 2$. Under above assumptions the inequality (3.26) for effective complex "shear" modulus takes the form

$$\frac{1}{\mathrm{Re}(e^{i\theta}\widetilde{\mu}^* - i\mu_0)^{-1}} \le \int\limits_Q \frac{dx}{\mathrm{Re}(e^{i\theta}\mu(x) - i\mu_0)^{-1}}, \tag{4.2}$$

where

$$\widetilde{\mu}^* = \mu_R^* + i\mu_I^*, \quad \mu_0 \in (0, \infty), \quad \theta \in [0, 2\pi].$$

After calculations we get

$$\frac{(\mu_R^* - \mu_0\sin\theta)^2 + (\mu_I^* - \mu_0\cos\theta)^2}{\mu_R^*\cos\theta - \mu_I^*\sin\theta} \le \frac{\frac{1}{2}[(\mu_R^1)^2 + (\mu_I^1)^2 + \mu_0^2] - 2\mu_0(\mu_R^1\sin\theta + \mu_I^1\cos\theta)}{\mu_R^1\cos\theta - \mu_I^1\sin\theta} + \frac{\frac{1}{2}[(\mu_R^2)^2 + (\mu_I^2)^2 - 2\mu_0(\mu_R^2\sin\theta + \mu_I^2\cos\theta)}{\mu_R^2\cos\theta - \mu_I^2\sin\theta}. \tag{4.3}$$

This inequality confines $\widetilde{\mu}^*$ to lie within a circle in the complex plane. As θ and μ_0 are varied, $\widetilde{\mu}^*$ must lie within the region of the complex plane formed by the intersection of these circles.

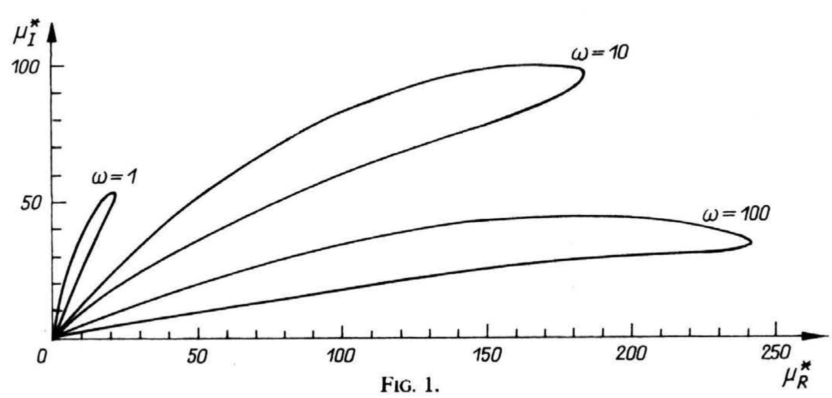

FIG. 1.

The graphical illustration (Fig. 1) of the regions for 3 different frequencies ω = 1, 10, 100 is done under the following data:

$\omega = 1$	$\omega = 10$	$\omega = 100$
$\mu_R^1 = 0.05,$	$\mu_R^1 = 2.5,$	$\mu_R^1 = 5,$
$\mu_R^2 = 19.2,$	$\mu_R^2 = 400,$	$\mu_R^2 = 500,$
$\mu_I^1 = 0.5,$	$\mu_I^1 = 2.5,$	$\mu_I^1 = 0.5,$
$\mu_I^2 = 96,$	$\mu_I^2 = 200,$	$\mu_I^2 = 25.$

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