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On the determination of material coefficients of the theory of thermodiffusion in deformable solids for the heat and moisture transfer processes in building walls

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THE HEAT and moisture transfer processes in building walls are coupled together. The equations of the theory of thermodiffusion in deformable solids given by W. Nowacki and S. Podstrigac allow to describe the coupled thermo-humiditive processes in building walls and, moreover, they enable the determination of the stresses induced in the wall material during these processes. A numerical-measurement procedure to determine the basic material coefficients for moisture thermodiffusion in a building wall has been proposed.

1. Introduction

THE EQUATIONS of the theory of thermodiffusion in deformable solids developed by W. NOWACKI and S. PODSTRIGAC [1, 2], have been applied in [3] to the investigation of the heat and moisture transfer processes in a multilayer building wall. The equations of this theory allow to describe the coupled thermo-humiditive processes occurring in building walls and, moreover, they enable us to determine the stresses induced in the wall material during these processes.

The application of the general solutions presented in [3] in engineering practice depends on experimental determination of the numerical values of the basic material coefficients for the moisture thermodiffusion process: d, m, n, D_C, k and coefficients: μ, λ, γ_T, γ_C which are necessary for determining the stresses.

The determination of material coefficients for the Podstrigac–Nowacki theory of thermodiffusion in deformable solids presents, from the beginning of this theory, a substantial problem. The thermodynamical interpretations of the thermodiffusion coefficients occurring in this theory are given by PODSTRIGAC and PAWLINA [4]. Some attempts at determination of those thermodiffusion coefficients (including evaluation of their numerical values) are made in [5, 6]. However, the problem of a method of determination of all basic material coefficients for the theory of thermodiffusion in deformable solids is still open.

This paper offers the numerical-measurement procedure for determining the basic material coefficients of thermodiffusion theory in solids (building walls), basing on the general solution of a one-dimensional problem and the measurements of the prescribed quantities on the building wall boundary.

2. Equations of thermodiffusion theory in solids

The complete set of differential equations of the linear theory of thermodiffusion in solids by W. NOWACKI [1] and S. PODSTRIGAC [2] for slow thermodiffusion process in isotropic homogeneous elastic bodies, by disregarding the inertial forces and assuming that in the considered body volume the external heat and mass sources do not occur, has the following form:

$$
\begin{aligned}
&\mu\nabla^2\mathbf{W} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{W} = \gamma_T\,\mathrm{grad}\,\Theta + \gamma_c\,\mathrm{grad}\,c\,,\\
&k\nabla^2\Theta - c_{\varepsilon,c}\partial_t\Theta - T_0(\gamma_T\partial_T\varepsilon_{kk} - d\partial_t c) = 0\,,\\
&D_c\nabla^2 c - \partial_t c + D_\varepsilon\nabla^2\varepsilon_{kk} + D_T\nabla^2\Theta = 0\,.
\end{aligned}
\tag{2.1}
$$

One can see that in deriving the above set of equations it is necessary to assume that the considered processes of the heat conduction and the mass transfer are coupled by means of the physical relations only. The physical relations in this theory have the form

$$
\begin{aligned}
\sigma_{ij} &= 2\mu\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma_T\Theta - \gamma_c c)\delta_{ij}\,,\\
S &= \gamma_T\varepsilon_{kk} - d\Theta + mc, \qquad c = C - C_0\,,\\
M &= -\gamma_c\varepsilon_{kk} + d\Theta + nc, \qquad \Theta = T - T_0\,,
\end{aligned}
\tag{2.2}
$$

where σ_{ij} is the stress tensor, $\mathbf{W}$ — displacement vector, ε_{ij} — strain tensor, ε_{kk} — dilatation, c — increment of concentration of the diffusing medium(1) related to the concentration in the initial state C_0, Θ — increment of temperature related to the initial state temperature T_0, S — entropy per unit volume, M — chemical potential (moisture potential), μ, λ, k, γ_T, γ_c, d, m, n are the basic material coefficients: μ, λ are the Lamé elasticity coefficients, k — coefficient of thermal conductivity; the remaining material coefficients (γ_c, γ_T, d, m, n) will be determined later.

On the ground of Onsager's relations (simplified by the assumption that the considered processes of heat conduction and mass transfer are coupled by the physical relations (2.2) only), the entropy balance equation, and the mass conservation principle, when we confine our investigation to the linear theory with respect to the temperature, the concentration (humidity) and the displacement, we obtain the following equations [1]:

$$
\mathbf{q} = \frac{1}{T_0}L_{qq}\,\mathrm{grad}\,\Theta\,, \qquad \boldsymbol{\eta} = -L_{\eta\eta}\,\mathrm{grad}\,M\,,
\tag{2.3}
$$

$$
-\,\mathrm{div}\,\mathbf{q} \cong T_0\dot{S}\,, \qquad -\,\mathrm{div}\,\boldsymbol{\eta} = \dot{c}\,,
\tag{2.4}
$$

where $\mathbf{q}$ is the heat flux, $\boldsymbol{\eta}$ — mass (moisture) flux, L_{qq}, $L_{\eta\eta}$ — are Onsager's coefficients.

Combining the Eqs. (2.3) and (2.4) we obtain

$$
\dot{S} \cong \frac{1}{T_0^2}L_{qq}\,\mathrm{div}\,\mathrm{grad}\,\Theta\,,
\tag{2.5}
$$

$$
\dot{c} = L_{\eta\eta}\,\mathrm{div}\,\mathrm{grad}\,M\,.
\tag{2.6}
$$

Substituting in $(2.3)_2$ the potential M from the physical relations $(2.2)_3$ we obtain

$$
\boldsymbol{\eta} = -D_c\,\mathrm{grad}\,c - D_\varepsilon\,\mathrm{grad}\,\varepsilon_{kk} - D_T\,\mathrm{grad}\,\Theta\,,
\tag{2.7}
$$

where

$$
D_c = nL_{\eta\eta}, \qquad D_\varepsilon = -\gamma_c L_{\eta\eta}, \qquad D_T = dL_{\eta\eta}\,.
\tag{2.8}
$$

In what follows it is assumed that the influence of the stresses in elastic material on the processes of the heat conduction and the mass transfer is negligible [1] . It is an analogous assumption to that used in the theory of thermal stresses and in the uncoupled

(1) In the building walls considered here,the role of the diffusing medium plays the moisture; the concentration of moisture is called humidity.

thermoelasticity. Formally, it means that the dilatational terms in Eqs. $(2.1)_2$ and $(2.1)_3$ can be omitted.

According to the introduced assumptions, the slow one-dimensional process of moisture thermodiffusion in a building wall elastic material is described by the following equations:

from Eqs. (2.1)

$$\begin{aligned}(2\mu+\lambda)\frac{\partial^2 W}{\partial x^2} &= \gamma_T\frac{\partial\Theta}{\partial x}+\gamma_c\frac{\partial c}{\partial x},\\ k\frac{\partial^2\Theta}{\partial x^2}-c_{\varepsilon,c}\frac{\partial\Theta}{\partial t}+T_0 d\frac{\partial c}{\partial t} &= 0,\\ D_c\frac{\partial^2 c}{\partial x^2}-\frac{\partial c}{\partial t}+D_T\frac{\partial^2\Theta}{\partial x^2} &= 0,\end{aligned}\tag{2.9}$$

from Eqs. (2.2)

$$\begin{aligned}\sigma_{11} &= (2\mu+\lambda)\varepsilon_{11}-\gamma_T\Theta-\gamma_c c,\\ S &= m\Theta-dc,\\ M &= d\Theta+\eta c,\end{aligned}\tag{2.10}$$

from Eqs. (2.5) and (2.6)

$$\dot{S}=\frac{1}{T_0^2}L_{qq}\frac{\partial^2\Theta}{\partial x^2},\qquad \dot{c}=L_{\eta\eta}\frac{\partial^2 M}{\partial x^2},\tag{2.11}$$

from Eqs. (2.7)

$$\eta=-D_c\frac{\partial c}{\partial x}-D_T\frac{\partial\Theta}{\partial x}.\tag{2.12}$$

3. General solution of one-dimensional problem of moisture thermodiffusion under slow harmonic thermo-humiditive excitation

Consider the one-dimensional, linear process of heat and moisture transfer in an external building wall viewed as an isotropic homogeneous material layer, undergoing slow harmonic changes (with the angular frequency ω), of the thermo-humiditive environment parameters.

This one-dimensional problem of moisture thermodiffusion in a building wall will be solved by means of the methods of the theory of electric transmission lines, which are very suitable for solving of one-dimensional boundary problems for sinusoidally — in time — variable excitations. Application of such methods is possible owing to the electro-elasto-thermo-diffusive analogies presented in Appendix I.

We assume that in the range of the considered values of temperature, humidity and time, the system (i.e. the building wall) is linear and stationary. Thus, all the quantities describing the moisture thermodiffusion process in this building wall are changing in time sinusoidally with the same angular frequency as the excitations (i.e. the thermo-humiditive environment parameters):

$$\sigma_{11}(x,t)=\mathrm{Re}[\sigma_0(x)e^{j\omega t}],\qquad \frac{\partial W_1}{\partial t}:=V(x,t)=\mathrm{Re}[V_0(x)e^{j\omega t}],\tag{3.1}$$

(3.1) [cont.]
$$\Theta(x,t) = \mathrm{Re}[\Theta_0(x)e^{j\omega t}], \qquad \frac{\partial S}{\partial t} := \dot{S}(x,t) = \mathrm{Re}[\dot{S}(x)e^{j\omega t}],$$
$$M(x,t) = \mathrm{Re}[M_0(x)e^{j\omega t}], \qquad \frac{\partial c}{\partial t} := \dot{c}(x,t) = \mathrm{Re}[\dot{c}_0(x)e^{j\omega t}],$$

where

$$\begin{aligned} \sigma_0(x) &:= \sigma_{0m}(x)e^{j\varphi_\sigma(x)}, & V_0(x) &:= V_{0m}(x)e^{j\varphi_v(x)}, \\ \Theta_0(x) &:= \Theta_{0m}(x)e^{j\varphi_\Theta(x)}, & \dot{S}_0(x) &:= \dot{S}_{0m}(x)e^{j\varphi_s(x)}, \\ M_0(x) &:= M_{0m}(x)e^{j\varphi_M(x)}, & \dot{c}_0(x) &:= \dot{c}_{0m}(x)e^{j\varphi_c(x)}. \end{aligned} \tag{3.2}$$

The quantities denoted by index 0 are the complex amplitudes; they are vectors in the Gaussian complex plane, and in Eqs. (3.2) they are expressed in terms of real amplitudes (the quantities with the index "$0m$") and of the appropriate phase shift angles $\varphi_{(\cdot)}(x)$. For example, $M_0(x)$ — is the complex amplitude of moisture potential, $\varphi_M(x)$ — is the phase shift angle of moisture potential.

The one-dimensional process of moisture thermodiffusion running in a material layer in the x direction, at slow harmonic state changes, can be described by analogy to Eqs. (I.13)–(I.15) (see Appendix I) by means of the following equations:

$$\frac{dV_0(x)}{dx} = Z_1\sigma_0(x) + Z_{12}\Theta_0(x) + Z_{13}M_0(x), \qquad \frac{d\sigma_0(x)}{dx} = 0, \tag{3.3}$$

$$\frac{d}{dx}\begin{bmatrix} \dot{S}_0(x) \\ \dot{c}_0(x) \\ \Theta_0(x) \\ M_0(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 & Z_2 & -Z_{23} \\ 0 & 0 & -Z_{23} & Z_3 \\ Y_2 & 0 & 0 & 0 \\ 0 & Y_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{S}_0(x) \\ \dot{c}_0(x) \\ \Theta_0(x) \\ M_0(x) \end{bmatrix}, \tag{3.4}$$

where

$$Z_1 = j\omega\frac{1}{2\mu+\lambda}, \qquad Z_2 = j\omega\frac{mn+d^2}{n}, \qquad Z_3 = j\omega\frac{1}{n}, \tag{3.5}$$

$$Z_{12} = j\omega\frac{\gamma_T n - \gamma_c d}{n(2\mu+\lambda)}, \quad Z_{13} = j\omega\frac{\gamma_c}{n(2\mu+\lambda)}, \quad Z_{23} = j\omega\frac{d}{n}, \tag{3.6}$$

$$Y_2 = T_0\frac{1}{k}, \qquad Y_3 = \frac{1}{L_{\eta\eta}} = n\frac{1}{D_c}. \tag{3.7}$$

The quantities (3.5) and (3.6), by analogy to the electric impedances (I.16) (Appendix I), may be called the thermo-humiditive impedances of a material layer.

Equation (3.4) is the so-called homogeneous state equation; it can be written in the compact form as

$$\frac{d\mathbf{S}(x)}{dx} = \mathbf{A}\mathbf{S}(x), \tag{3.8}$$

where

$$\mathbf{S}(x) = [\dot{S}_0(x), \dot{c}_0(x), \Theta_0(x), M_0(x)]^T \tag{3.9}$$

is the so-called state vector, **A** is the matrix of the system.

The solution of the state equation (3.8) is given by the state vector $\mathbf{S}(x)$ presented by the following transmission equation:

$$\mathbf{S}(x) = e^{\mathbf{A}x}\mathbf{S}(0), \tag{3.10}$$

where $e^{\mathbf{A}x}$ is the so-called transmission (or transition) matrix, $\mathbf{S}(0) = \mathbf{S}(x = 0)$ is the state vector at the input of the system (external surface of the building wall).

Thus, the solution of the state equation (3.8) reduces to the determination of the transmission matrix $e^{\mathbf{A}x}$. First, we find the eigenvalues of the matrix $\mathbf{A}$ by solving its characteristic equation:

$$\det(\mathbf{A} - \lambda \mathbf{1}) = 0 \tag{3.11}$$

we obtain from Eq. (3.11)

$$\lambda^4 - \lambda^2(P_2 + P_3) + P_2P_3 - Q_2Q_3 = 0\,, \tag{3.12}$$

where

$$\begin{aligned} P_2 &= Z_2Y_2, \quad & Q_2 &= Y_2Z_{23}\,, \\ P_3 &= Z_3Y_3, \quad & Q_3 &= Y_3Z_{23}\,. \end{aligned} \tag{3.13}$$

The roots of Eq. (3.12)

$$\lambda_{1,2} = \pm\gamma_1, \qquad \lambda_{3,4} = \pm\gamma_2 \tag{3.14}$$

are the eigenvalues of the matrix $\mathbf{A}$, and

$$\gamma_1 = (\overline{P} + K)^{1/2}, \qquad \gamma_2 = (\overline{P} - K)^{1/2}\,, \tag{3.15}$$

where

$$\begin{aligned} \overline{P} &= \frac{P_2 + P_3}{2}, \qquad K = [(\Delta P)^2 + Q^2]^{1/2}\,, \\ \Delta P &= \frac{P_2 - P_3}{2}, \qquad Q^2 = Q_2Q_3\,. \end{aligned} \tag{3.16}$$

The matrix $e^{\mathbf{A}x}$ can be determined by means of the Cayley – Hamilton theorem [3] and as the result we obtain

$$\begin{aligned} e^{\mathbf{A}x} = {} & \left[\frac{\operatorname{sh}\gamma_1 x}{\gamma_1(\gamma_1^2 - \gamma_2^2)} - \frac{\operatorname{sh}\gamma_2 x}{\gamma_2(\gamma_1^2 - \gamma_2^2)}\right]\mathbf{A}^3 + \frac{\operatorname{ch}\gamma_1 x - \operatorname{ch}\gamma_2 x}{\gamma_1^2 - \gamma_2^2}\mathbf{A}^2 \\ & + \left[\frac{\gamma_1^2 \operatorname{sh}\gamma_2 x}{\gamma_2(\gamma_1^2 - \gamma_2^2)} - \frac{\gamma_2^2 \operatorname{sh}\gamma_1 x}{\gamma_1(\gamma_1^2 - \gamma_2^2)}\right]\mathbf{A} + \frac{\gamma_1^2 \operatorname{ch}\gamma_2 x - \gamma_2^2 \operatorname{ch}\gamma_1 x}{\gamma_1^2 - \gamma_2^2}\mathbf{1}\,. \end{aligned} \tag{3.17}$$

Knowing the transmission matrix $e^{\mathbf{A}x}$ we can determine, on the basis of Eq. (3.10), the components of the state vector $\mathbf{S}(x)$, which is the solution of Eq. (3.8), namely:

$$\begin{aligned} \dot{S}_0(x) = {} & \frac{(K + \Delta P)\dot{c}_0(0) - Q_3\dot{S}_0(0)}{2K}\operatorname{ch}\gamma_1 x + \frac{(K + \Delta P)A_2^0 - Q_3A_3^0}{2K\gamma_1}\operatorname{sh}\gamma_1 x \\ & + \frac{(K - \Delta P)\dot{c}_0(0) + Q_3\dot{S}_0(0)}{2K}\operatorname{ch}\gamma_2 x + \frac{(K - \Delta P)A_2^0 + Q_3A_3^0}{2K\gamma_2}\operatorname{sh}\gamma_2 x\,, \end{aligned} \tag{3.18}$$

$$\begin{aligned} \dot{c}_0(x) = {} & \frac{(K - \Delta P)\dot{S}_0(0) - Q_2\dot{c}_0(0)}{2K}\operatorname{ch}\gamma_1 x + \frac{(K - \Delta P)A_3^0 - Q_2A_2^0}{2K\gamma_1}\operatorname{sh}\gamma_1 x \\ & + \frac{(K + \Delta P)\dot{S}_0(0) - Q_2\dot{c}_0(0)}{2K}\operatorname{ch}\gamma_2 x + \frac{(K + \Delta P)Y_2\dot{c}_0(0) - Q_2Y_3\dot{S}_0(0)}{2K\gamma_2}\operatorname{sh}\gamma_2 x\,, \end{aligned} \tag{3.19}$$

$$(3.20)\quad \Theta_0(x) = \frac{(K+\Delta P)Q_0(0) - Q_2 M_0(0)}{2K}\operatorname{ch}\gamma_1 x + \frac{(K+\Delta P)Y_2\dot{c}_0(0) - Q_2 Y_3 \dot{S}_0(0)}{2K\gamma_1}\operatorname{sh}\gamma_1 x + \frac{(K-\Delta P)\Theta_0(0) + Q_2 M_0(0)}{2K}\operatorname{ch}\gamma_2 x + \frac{(K-\Delta P)Y_2\dot{c}_0(0) + Q_2 Y_3\dot{S}_0(0)}{2K\gamma_2}\operatorname{sh}\gamma_2 x\,,$$

$$(3.21)\quad M_0(x) = \frac{(K-\Delta P)M_0(0) - Q_3\Theta_0(0)}{2K}\operatorname{ch}\gamma_1 x + \frac{(K-\Delta P)Y_3\dot{S}_0(0) - Q_3 Y_2\dot{c}_0(0)}{2K\gamma_1}\operatorname{sh}\gamma_1 x + \frac{(K+\Delta P)M_0(0) + Q_3\Theta_0(0)}{2K}\operatorname{ch}\gamma_2 x + \frac{(K+\Delta P)Y_3\dot{S}(0) + Q_3 Y_2\dot{c}_0(0)}{2K\gamma_2}\operatorname{sh}\gamma_2 x\,,$$

where

$$(3.22)\quad \begin{aligned} A_2^0 &= Z_2\Theta_0(0) - Z_{23}M_0(0)\,,\\ A_3^0 &= Z_3 M_0(0) - Z_{23}\Theta_0(0)\,. \end{aligned}$$

Equations (3.18)–(3.21), and equations for $V_0(x)$ and $\sigma_0(x)$ obtained by simple integration of Eqs. (3.3), constitute the general solution for one-dimensional process of coupled thermo-diffusion of moisture in a material layer of an external building wall, running at slow harmonic changes of thermo-humiditive environment parameters.

Equation (3.21) describes the moisture potential field in the wall, Eq. (3.19) presents the field of the humidity rate, Eq. (3.20) concerns the temperature field, and Eq. (3.18) describes the field of the entropy rate. They are the equations for the complex amplitudes of a.m. quantities, the instantaneous values of which one can obtain on the basis of Eq. (3.1).

3.1. Determination of material coefficients n, d, D_c by means of a numerical-measurement procedure

The differential equations of thermodiffusion (2.1) and the physical relations (2.2) contained 11 material coefficients μ, λ, k, γ_T, γ_c, d, m, n, D_c, D_T, D_ε. There are 9 independent material coefficients; the last two coefficients D_T, D_ε can be calculated from Eqs. (2.8) and expressed in terms of the independent ones.

The material coefficients n, d, D_c will be determined on the basis of the general solution given by Eqs. (3.18)–(3.21), and by some measurements of several prescribed quantities on the boundary of the building wall, in which the moisture thermodiffusion process occur. Note that in the physical relations (2.10) for one-dimensional thermodiffusion in an elastic material (where the effect of stresses on the heat and moisture transfer processes are assumed to be negligible), which are used to construct the general solution (3.18)–(3.21), occur all these material constants of thermodiffusion which are present in the 3-dimensional physical relations (2.2). Thus, the generality of our considerations is ensured.

The schematic diagram of the measurement system proposed for determining the material coefficients n, d, D_c is shown in Fig. 1. The thermodiffusion process goes inside a building wall of thickness g in the x direction, under the influence of the temperature and humidity gradients changing sinusoidally in time. The wall is placed in a chamber possessing thermo–humiditive insulation, and divides this chamber into two parts 1 and 2. The temperature and the humidity in those two parts of the chamber are denoted Θ_1, c_1 and θ_2, c_2, respectively.

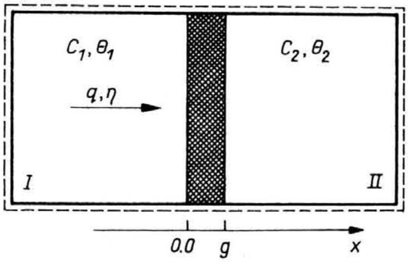

FIG. 1. Schematic diagram of a measurement system to determine the material coefficients for thermodiffusion of moisture in a building wall material layer.

In order to determine the numerical values of the material coefficients n, d, D_c, the following procedure is proposed;

1. In Part 1 of the system shown in Fig. 1 slow harmonic changes of e.g. temperature Θ_1 and humidity c_1 should be used to measure(2)

$M_0(0)$ complex amplitude of the moisture potential for $x = 0$;

$\dot{S}_0(0)$, $\dot{S}_0(g)$ complex amplitudes of the entropy rate for $x = 0$ and $x = g$, respectively (by means of the heat flux and the temperature measurements);

$\dot{c}_0(0)$, $\dot{c}_0(g)$ complex amplitudes of the humidity rate for $x = 0$ and $x = g$, respectively;

$\Theta_0 c$, $\Theta_0 g$ the complex amplitudes of the temperature for $x = 0$ and $x = g$, respectively.

2. The following nonlinear set of equations should be solved numerically:

$$
\begin{aligned}
(3.24)\qquad n = j\omega Q_3 M_0(0)\frac{\operatorname{sh}\gamma_1 g}{\gamma_1}\Big/\{[(K+\Delta P)\dot{c}_0(0) - Q_3\dot{S}_0(0)]\operatorname{ch}\gamma_1 g \\
+[(K-\Delta P)\dot{c}_0(0) + Q_3\dot{S}_0(0)]\operatorname{ch}\gamma_2 g + [(K+\Delta P)A_2^0 + Q_3 Z_{23}\Theta_0(0)]\frac{\operatorname{sh}\gamma_1 g}{\gamma_1} \\
+[(K-\Delta P)A_2^0 + Q_3 A_3^0]\frac{\operatorname{sh}\gamma_2 g}{\gamma_2} - 2K\dot{S}_0(g)\},
\end{aligned}
$$

$$
\begin{aligned}
(3.25)\qquad d = \frac{n}{j\omega Y_2\dot{c}_0(0)\operatorname{ch}\gamma_1 g}\{[(K-\Delta P)A_3^0 - Q_2 A_2^0]\frac{\operatorname{sh}\gamma_1 g}{\gamma_1} \\
+[(K-\Delta P)A_3^0 - Q_2 A_2^0]\frac{\operatorname{sh}\gamma_2 g}{\gamma_2} + (K-\Delta P)\dot{S}_0(0)\operatorname{ch}\gamma_1 g \\
+(K+\Delta P)\dot{S}_0(0) - Q_2\dot{c}_0(0)]\operatorname{ch}\gamma_2 g - 2K\dot{c}_0(g)\},
\end{aligned}
$$

(2) The measurements should enable us to express the proposed quantities as functions of time; next it will be possible to determine the real amplitudes and the shift phase angles — i.e. the complex amplitudes of these quantities.

$$D_c = \frac{nQ_2\dot{S}_0(0)\,\mathrm{sh}\,\gamma_1 g}{\gamma_1} / \{[(K + \Delta P)\Theta_0(0) - Q_2M_0(0)]\,\mathrm{ch}\,\gamma_1 g \tag{3.26}$$
$$+[(K - \Delta P)\Theta_0(0) + Q_2M_0(0)]\,\mathrm{ch}\,\gamma_2 g + (K + \Delta P)Y_2\dot{c}_0(0)\frac{\mathrm{sh}\,\gamma_1 g}{\gamma_1}$$
$$+[(K - \Delta P)Y_2\dot{c}_0(0) + Q_2Y_3\dot{S}_0(0)]\frac{\mathrm{sh}\,\gamma_2 g}{\gamma_2} - 2K\Theta_0(g)\},$$

which are obtained by a simple transformation of Eqs. (3.18)–(3.21) to the form:

$$\begin{aligned} n &= f_1(n, d, D_c), \\ d &= f_2(n, d, D_c), \\ D_c &= f_3(n, d, D_c). \end{aligned} \tag{3.27}$$

The set of equations in the form (3.27) is suitable for numerical solution by means of e.g. the successive approximation method.

The convergence of the iterative process will depend here partly upon the functions $f_1, \ldots, f_3$ for a given moisture thermodiffusion process going on in a given elastic material, and partly on the choice of the initial approximation [10, 11].

Substituting $x_1 = n$, $x_2 = d$, $x_3 = D_c$, and introducing the vectors

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{F}(\mathbf{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}, \tag{3.28}$$

one can present the set (3.27) in the form

$$\mathbf{x} = \mathbf{F}(\mathbf{x}). \tag{3.29}$$

After appropriate selection of the initial vector $\mathbf{x}_0$, we construct the sequence $\{\mathbf{x}^n\}$ of vectors $\mathbf{x}^n$ determined according to the formula

$$\mathbf{x}^{n+1} = \mathbf{F}(\mathbf{x}), \tag{3.30}$$

where n is the iteration index.

If $\{\mathbf{x}^n\}$ tends to the limit $\mathbf{x}^*$ when $n \to \infty$, then, of course, $\mathbf{x}^*$ is the solution of Eq. (3.28).

Let us suppose that, if $\mathbf{x} = [x_1, x_2, x_3]^T$ and $\mathbf{x}' = [x_1', x_2', x_3']^T$ belong to the region D containing all the vectors $\mathbf{x}^n$, then such positive numbers A_{ij} exist that

$$|f_i(x_1, x_2, x_3) - f_i(x_1', x_2', x_3')| \le \sum_j A_{ij}|x_j - x_j'| \quad \text{for} \quad i = 1, 2, 3. \tag{3.31}$$

We define the following norms for matrices

$$\begin{aligned} \|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{x}')\| &= \max_i |f_i(x_1, x_2, x_3) - f_i(x_1', x_2', x_3')|, \\ \|\mathbf{x} - \mathbf{x}'\| &= \max_i |x_i - x_i'|, \\ \|\mathbf{A}\| &= \max_i \Big(\sum_j |A_{ij}|\Big) = K. \end{aligned} \tag{3.32}$$

From the assumptions made it follows that

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{x}')\| \le K\|\mathbf{x} - \mathbf{x}'\|. \tag{3.33}$$

Thus, if $K < 1$ then the sequence $\{\mathbf{x}^n\}$ possesses the limit. In addition, since the functions f_1, f_2, f_3 are differentiable we have

$$f_i(x_1, x_2, x_3) - f_i(x'_1, x'_2, x'_3) = \sum_j \frac{\partial f_i}{\partial x_i}(\xi_1, \xi_2, \xi_3)(x_j - x'_j) \tag{3.34}$$

thus, the derivatives $\partial f_i / \partial x_j$ are bounded from above by the numbers A_{ij}.

3.2. Determination of the remaining material coefficients

The material coefficients μ, λ, k, γ_c, γ_T, m can be determined independently. The coefficients μ, λ are the Lamé elastic constant, k is the coefficient of thermal conductivity from Fourier's law. The methods of determination of these coefficients (μ, λ, k) are well-known.

From the theory of thermal stresses in solids it is known, that

$$\gamma_T = (3\lambda + 2\mu)\alpha_T \,, \tag{3.35}$$

where α_T is the coefficient of linear thermal expansion of the body, determined experimentally.

By analogy, for the isothermal processes of moisture diffusion in deformable solids we have [1]:

$$\gamma_c = (3\lambda + 2\mu)\alpha_c \,, \tag{3.36}$$

where α_c is the coefficient of linear diffusive expansion of the body. This coefficient can be determined in a similar way as α_T, i.e. by measurement of the elongation Δl of a standard beam of the length l, made of the building wall material, the elongation of which is due to the humidity increment Δc,

$$\Delta l = \Delta c \alpha_c l \,. \tag{3.37}$$

From the thermodynamical considerations presented in [1], we have

$$m = \frac{c_{\varepsilon,c}}{T_0}, \qquad c_{\varepsilon,c} = T\left(\frac{\partial S}{\partial T}\right)_{\varepsilon,c}, \tag{3.38}$$

where $c_{\varepsilon,c}$ is the specific heat at constant strain and humidity; T_0 is the absolute temperature of natural state. The specific heat $c_{\varepsilon,c}$ is a measure of the amount of heat stored in unit volume of the solid due to a variation by one degree of its temperature at constant strain and humidity. This quantity $(c_{\varepsilon,c})$ may be determined by means of the calorimetric method, similar to that used in determining the specific heat at constant strain (c_ε) in the theory of thermal stresses [15].

4. Final remarks

Although the processes of the heat conduction and the moisture transfer in building walls are coupled, up till now, in the thermo-humiditive design of building walls, one usually does not take into account the influence of the slow process of moisture transfer on the temperature field inside the wall [12]. The temperature field and the moisture potential field in the wall are determined separately, from two partial differential equations with variable coefficients of the Fourier's type.

The linear theory of thermodiffusion in deformable solids, developed by W. NOWACKI [1] and S. PODSTRIGAC [2], enables us to describe the coupled thermo-humiditive processes occurring in building walls; here the role of the diffusing medium plays the moisture; the humidity defines its concentration, the chemical potential of diffusing medium is replaced by the moisture potential. This theory gives also a possibility to calculate the stresses and strains which appear during the heat and moisture transfer in the wall material.

In this paper, a method has been proposed concerning the determination of material coefficients of the theory of thermodiffusion in deformable solids, for a building wall material. This method is based on the general solution of the one-dimensional problem of moisture thermodiffusion in a material layer, obtained in [3], and on the measurements of several quantities at the wall boundary.

Appendix I. Electro-elasto-thermo-diffusive analogies and electric transmission lines

One-dimensional, slow process of linear thermodiffusion of moisture in an elastic material layer by neglecting the influence of stress state on the heat and moisture transfer processes, is described by the Eqs. (2.9) and (2.10).

Figure 2 presents the system of three electric transmission lines coupled magnetically. It will be seen in the following that line *1* corresponds to the elastic deformation of the solid, line *2* — to the heat conduction, and line *3* — to the moisture transfer.

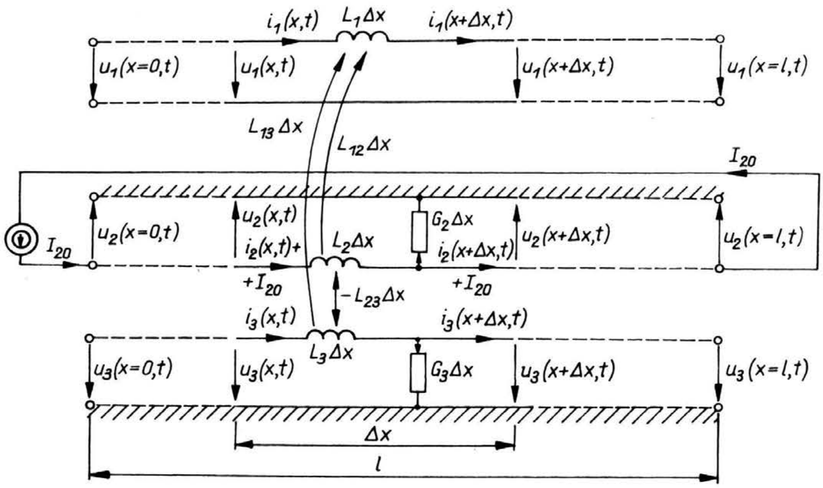

FIG. 2. System of three electric transmission lines coupled magnetically — the electrical analogue of a slow, one-dimensional process of thermodiffusion in elastic solids.

In Fig. 2 u_1, u_2, u_3 are the line voltages, i_1, i_2, i_3 — the currents, L_1, L_2, L_3 — the self-inductances per unit length, L_{12}, L_{13}, L_{23} — the mutual inductances per unit length, G_2, G_3 — the self-conductances per unit length.

For the electric system from Fig. 2, we have on the basis of the second Kirchhoff's law [8], by neglecting the influence of the magnetic field of line *1* on the lines *2* and *3* (what corresponds to neglecting the influence of the stress state on the heat and moisture transfer), the following equations

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{\partial \Psi_1}{\partial x} + L_1 i_1 + L_{12} i_2 + L_{13} i_3\right) = 0 \\
&\frac{\partial}{\partial t}\left(\frac{\partial \Psi_2}{\partial x} + L_2 i_2 - L_{23} i_3\right) = 0, \\
&\frac{\partial}{\partial t}\left(\frac{\partial \Psi_3}{\partial x} - L_{23} i_2 + L_3 i_3\right) = 0,
\end{aligned}
\tag{A.1}
$$

where Ψ is magnetic flux associated with the corresponding transmission line ($k = 1, 2, 3$).

Between the line voltage and the associated magnetic flux, the following relationship holds true [8]:

$$
u = \frac{\partial \Psi}{\partial t}. \tag{A.2}
$$

From Eqs. (A.1) we obtain

$$
\begin{aligned}
i_1 = J_1 A &= \frac{1}{L_1}\frac{\partial(-\Psi_1)}{\partial x} - \frac{L_3 L_{12} + L_{13} L_{23}}{L_1 L_3} i_2 - \frac{L_{13}}{L_1 L_3}\frac{\partial(-\Psi_3)}{\partial x}, \\
\frac{\partial(-\Psi_2)}{\partial x} &= \left(L_2 - \frac{L_{23}^2}{L_3}\right) i_2 - \frac{L_{23}}{L_3}\frac{\partial(-\Psi_3)}{\partial x}, \\
i_3 = J_3 A &= \frac{L_{23}}{L_3} i_2 + \frac{1}{L_3}\frac{\partial(-\Psi_3)}{\partial x},
\end{aligned}
\tag{A.3}
$$

where J_k ($k = 1, 2, 3$) is the current density in line k, A is the cross-sectional area of each transmission line.

It is easy to see that Eqs. (A.3) are analogous to the physical relations of thermodiffusion (2.10).

On the ground of the first Kirchhoff's law for the lines *2* and *3* from Fig. 2, we obtain the equations

$$
u_2 = -\frac{1}{G_2}\frac{\partial i_2}{\partial x}, \qquad u_3 = -\frac{1}{G_3}\frac{\partial i_3}{\partial x}. \tag{A.4}
$$

Differentiating Eqs. (A.4) with respect x we have

$$
\frac{\partial(-u_2)}{\partial x} = \frac{1}{G_2}\frac{\partial^2 i_2}{\partial x^2}, \qquad \frac{\partial(-u_3)}{\partial x} = \frac{1}{G_3}\frac{\partial^2 i_3}{\partial x^2}. \tag{A.5}
$$

Equations (A.5) are analogous to Eqs. (2.11).

Substituting i_3 from Eq. $(\text{A.3})_3$ into (A.4), we obtain

$$
u_3 = -\frac{1}{L_2 G_3}\frac{\partial^2(-\Psi_3)}{\partial x^2} - \frac{L_{23}}{L_3 G_3}\frac{\partial i_2}{\partial x}. \tag{A.6}
$$

Equation (A.6) is analogous to Eq. (2.12).

From the complete set of differential Eqs. (2.9) describing one-dimensional, slow process of linear thermodiffusion of moisture in an elastic material layer, we obtain the following analogous equations describing the system of coupled electric transmission lines from Fig. 2

$$
\text{(A.7)}\qquad
\begin{gathered}
\frac{1}{L_1}\frac{\partial^2(-\Psi_1)}{\partial x^2} = \frac{L_3L_{12}+L_{13}L_{23}}{L_1L_3}\frac{\partial i_2}{\partial x} - \frac{L_{13}}{L_1L_3}\frac{\partial^2(-\Psi_3)}{\partial x^2}\,,\\
\frac{I_{20}}{G}\frac{\partial^2 i_2}{\partial x^2} - \frac{I_{20}(L_2L_3-L_{23}^2)}{L_3}\frac{\partial i_2}{\partial t} + \frac{I_{20}L_{23}}{L_3}\frac{\partial(-u_3)}{\partial x} = 0\,,\\
\frac{1}{L_3G_3}\frac{\partial^2}{\partial x^2}\frac{\partial(-\Psi_3)}{\partial x} - \frac{\partial}{\partial t}\frac{\partial(-\Psi_3)}{\partial x} + \frac{L_{23}}{L_3G_3}\frac{\partial^2 i_2}{\partial x^2} = 0\,.
\end{gathered}
$$

Comparing Eqs. (A.3) to (2.10), (A.6) to (2.12) and (A.7) to (2.9) we establish the following correspondences:

$$
\text{(A.8)}\qquad
\begin{aligned}
&\sigma_{11} \leftrightarrow J_1, \qquad && \frac{\partial W_1}{\partial x} = \varepsilon_{11} \leftrightarrow \frac{\partial(-\Psi_1)}{\partial x}\,,\\
&\Theta \leftrightarrow J_2, && S \leftrightarrow \frac{\partial(-\Psi_2)}{\partial x}\,,\\
&M \leftrightarrow J_3, && c \leftrightarrow \frac{\partial(-\Psi_3)}{\partial x}\,.
\end{aligned}
$$

Correspondences $(A.8)_1$ indicate that the electro-elasto-thermodiffusive analogies presented above constitute an extension of the so-called Firestone system of electro-mechanical analogies [13, 14].

Let us assume in the system from Fig. 2

$$
\text{(A.9)}\qquad
\begin{aligned}
u_k(x,t) &= \mathrm{Re}[U_k(x)e^{j\omega t}]\,,\\
i_k(x,t) &= \mathrm{Re}[I_k(x)e^{j\omega t}]\,, \qquad k = 1,2,3,
\end{aligned}
$$

where $u_k(x,t)$, $i_k(x,t)$ are the instantaneous values of voltages and currents, respectively, $U_k(x)$, $I_k(x)$ are the complex amplitudes, $j = \sqrt{-1}$ is the imaginary unit, and ω is the angular frequency.

The system of electric transmission lines Fig. 2 is described by the following equations:

$$
\text{(A.10)}\qquad \frac{\partial(-u_1)}{\partial x} = L_1\frac{\partial i_1}{\partial t} + L_{12}\frac{\partial i_2}{\partial t} + L_{13}\frac{\partial i_3}{\partial t}, \qquad \frac{\partial i_1}{\partial x} = 0\,,
$$

$$
\text{(A.11)}\qquad \frac{\partial(-u_2)}{\partial x} = L_2\frac{\partial i_2}{\partial t} - L_{23}\frac{\partial i_3}{\partial t}, \qquad \frac{\partial i_2}{\partial x} = -G_2u_2\,,
$$

$$
\text{(A.12)}\qquad \frac{\partial(-u_3)}{\partial x} = -L_{23}\frac{\partial i_2}{\partial t} + L_3\frac{\partial i_3}{\partial t}, \qquad \frac{\partial i_3}{\partial x} = -G_3u_3\,.
$$

If we introduce (A.9) into Eqs. (A.10)–(A.12), we obtain the following set of equations for complex amplitudes

$$
\text{(A.13)}\qquad \frac{d(-U_1)}{dx} = Z_1^{(e)}I_1 + Z_{12}^{(e)}I_2 + Z_{13}^{(e)}I_3, \qquad \frac{dI_1}{dx} = 0\,,
$$

$$
\text{(A.14)}\qquad \frac{d(-U_2)}{dx} = Z_2^{(e)}I_2 - Z_{23}^{(e)}I_3, \qquad \frac{dI_2}{dx} = -G_2U_2\,,
$$

$$
\text{(A.15)}\qquad \frac{d(-U_3)}{dx} = -Z_{23}^{(e)}I_2 + Z_3^{(e)}I_3, \qquad \frac{dI_3}{dx} = -G_3U_3\,.
$$

Here

(A.16) $$Z_1^{(e)} = j\omega L_1, \quad Z_{12}^{(e)} = j\omega L_{12}, \quad Z_{13}^{(e)} = j\omega L_{13},$$

(A.17) $$Z_2^{(e)} = j\omega L_2, \quad Z_{23}^{(e)} = j\omega L_{23}, \quad Z_3^{(e)} = j\omega L_3.$$

are the complex electric impedances.

References

1. W. Nowacki, *Thermodiffusion in solids*, [in] Thermoelasticity [in Polish], Ossolineum, Wrocław 1972.
2. J. S. Podstrigac, *Differential equations for thermodiffusion problems in deformable isotopic solids* [in Russian], DAN USSR, 2, 1961.
3. W. Dudziak and R. Uklejewski, *Heat and moisture transfer in a two-layer building wall at slow harmonic changes of thermo-humiditive environment parameters based on the theory of thermodiffusion in solids*, Engng. Trans., **39**, 1, 1991.
4. J. S. Podstrigac and W. S. Pawlina, *Problems of thermodynamics in solids structures* [in Russian], Ukr. Phys. J., **6**, 5, 1961.
5. W. Dudziak, *On the influence of dilatational terms in equations of thermodiffusion in solids on the temperature and concentration* [in Polish], IFTE Reports, 65, 1978.
6. J. Weres, *Modelling of thermoelastodiffusion processes in interstitial solid solutions by means of FEM in space and time*, Ph.D. Thesis, Poznań University of Technol., 1979.
7. L. Pipes, *Direct computation of transmission matrices of electrical transmission lines. Part I, II*, J. Frankl. Inst., 281, 1966.
8. T. Cholewicki, *Electrical lines and nonuniform ladder networks*, [in Polish], PWN, Warszawa 1974.
9. K. Ogata, *State space methods in control theory* [in Polish, transl. from English], WNT, Warszawa 1974.
10. J. Legras, *Practical methods of numerical analysis* [in Polish, transl. from French], WNT, Warszawa 1974.
11. A. Ralston, *Introduction to numerical analysis* [in Polish, transl. from English], PWN, Warszawa 1975.
12. W. N. Bogosłowski, *Thermo-humiditive processes in buildings*, [in Polish, transl. from Russian], Arkady, Warszawa 1985.
13. W. Dudziak, and R. Uklejewski, *On the application of the electrical transmission line theory in thermodiffusion problems in solids*, Bull. Acad. Polon. Sci., Série Sci. Techn., **34**, 3–4, 1986.
14. J. Wehr, *Electro-mechano-acoustical analogies for the systems with lumped and distributed parameters in the light of dimensional analysis* [in Polish], Arch. Akust., 2, pp. 211–226, 1967.
15. W. Zielenkiewicz and E. Margas, *Theoretical foundations of dynamical calorimetry* [in Polish], Ossolineum, Wrocław 1990.
16. K. Kiessl, *Kapillaren und dampfformiger Feuchtetransport in mehrschichtigen Bauteilen*, Diss. Univ., Essen 1983.
17. R. Kohonen, *A method to analyse the transient hygrothermal behaviour of building materials and components*, Techn. Res. Center of Finland, Publ., 21, Espoo 1984.

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