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Three coplanar moving Griffith cracks in an infinite elastic strip

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THE DYNAMIC anti-plane problem of determining stress and displacement due to three coplanar Griffith cracks moving steadily at a subsonic speed in an infinite elastic strip has been considered. Employing Fourier integral transform, the problem when the lateral boundaries are subjected to shearing stress, has been reduced to solving a set of four integral equations. These integral equations have been solved using finite Hilbert transform technique and Cook's result [9] to obtain the exact form of crack opening displacement and stress intensity factors. Numerical results for stress intensity factors have been presented in the form of graphs.

1. Introduction

IN FRACTURE MECHANICS, the problem of diffraction of elastic waves by cracks of finite dimension in a strip of elastic material has been investigated by several investigators. Sih and Chen [1] investigated the problem of propagation of a crack of finite length in a strip under plane extension. Closed-form solutions for a finite length crack moving in a strip under anti-plane shear stress was obtained by SINGH *et al.* [2]. Using finite Hilbert transform technique developed by SRIVASTAVA and LOWENGRUB [3], LOWENGRUB and SRIVASTAVA [4] solved the statical problem of distribution of stress and displacement in an infinitely long elastic strip containing two coplanar Griffith cracks. Several dynamic problems of determining stress and displacement due to two coplanar moving Griffith cracks have been solved by DAS and GHOSH [5–7].

As regards the crack problem, research has been restricted mainly to the case of a single crack or a pair of cracks because of severe mathematical complexity encountered in solving the problems of three or more cracks. Recently, DHAWAN and DHALIWAL [8] solved the statical problem of determining the stress distribution in an infinite transversely isotropic medium containing three coplanar Griffith cracks.

To the best knowledge of the author, the problem of stress distribution around three coplanar moving Griffith cracks in an infinite elastic strip has not been investigated so far. In this paper, the problem of propagation of three coplanar Griffith cracks in a fixed direction with constant velocity V in an infinitely long elastic strip of finite width has been considered. Employing Fourier integral transform, the problem when the lateral boundaries are subjected to shearing stress, has been reduced to solving a set of four integral equations using finite Hilbert transform technique [3] and COOK'S result [9] to derive the exact form of stress intensity factors and the crack opening displacement. Numerical results for the stress intensity factors are presented graphically to show their variations with crack speed, crack lengths and the separation distance between the cracks.

2. Statement of the problem

Consider an infinitely long elastic strip occupying the region $-h \le Y \le h$, weakened by three coplanar Griffith cracks moving steadily at a constant velocity V in the X-direction referred to a fixed coordinate system (X, Y, Z) as shown in the Fig. 1.

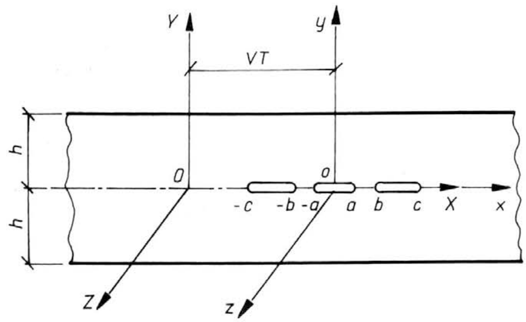

FIG. 1. Geometry and coordinate system.

In dynamic problem of anti-plane shear, the non-vanishing component of displacement W in the Z-direction satisfies the equation of motion

$$W_{,XX} + W_{,YY} = \frac{1}{C_2^2} W_{,TT}, \tag{2.1}$$

where $C_2 = (\mu/\rho)^{1/2}$ is the shear wave velocity, ρ is the material density and $W_{,X}$ represents partial derivatives of W with respect to X.

For cracks moving at constant velocity V in the X-direction it is convenient to introduce the Galilean transformation

$$x = X - VT, \quad y = Y, \quad z = Z, \quad t = T, \tag{2.2}$$

where (x, y, z) represents the moving coordinate system as shown in the Fig. 1.

Let the positions of the coplanar Griffith cracks referred to the coordinates (x, y, z) be $-a < x < a$, $-c < x < -b$ and $b < x < c$ on $y = 0$, and let the uniform shearing stress p be applied to the lateral boundaries $y = \pm h$ of the strip. The equivalent problem involves the application of shear stress $-p$ to the crack faces at $y = 0$. Accordingly, the boundary conditions of the proposed problem are

$$\sigma_{yz}(x, 0) = -p, \quad |x| < a, \quad b < |x| < c, \tag{2.3}$$

$$\sigma_{yz}(x, \pm h) = 0, \quad -\infty < x < \infty, \tag{2.4}$$

$$W(x, 0) = 0, \quad a < |x| < b, \quad |x| > c. \tag{2.5}$$

In the moving coordinate system, the equation of motion becomes independent of time and takes the form

$$s^2 W_{,xx} + W_{,yy} = 0, \tag{2.6}$$

with

$$s = \sqrt{1 - V^2/C_2^2}. \tag{2.7}$$

Due to the symmetry about x, z-plane we need to consider the region $0 < y < h$ only. Introducing the Fourier transforms

$$\overline{W}_C(\xi, y) = \int_0^\infty W(x, y)\cos(\xi x)\,dx,$$
(2.8)
$$W(x, y) = \frac{2}{\pi}\int_0^\infty \overline{W}_C(\xi, y)\cos(\xi x)\,d\xi,$$

in Eq. (2.6), the solution of Eq. (2.6) is obtained as

(2.9)
$$W(x, y) = \frac{2}{\pi}\int_0^\infty [C_1(\xi)e^{-\xi ys} + C_3(\xi)e^{\xi ys}]\cos(\xi x)\,d\xi,$$

with

(2.10)
$$\sigma_{yz}(x, y) = -\frac{2\mu s}{\pi}\int_0^\infty \xi[C_1(\xi)e^{-\xi ys} - C_3(\xi)e^{\xi ys}]\cos(\xi x)\,d\xi.$$

Using the expression for $\sigma_{yz}(x, y)$ given by Eq. (2.10) in Eq. (2.4), it has been found that

$$C_1(\xi) = \frac{C(\xi)}{1 + e^{-2\xi hs}},$$
$$C_3(\xi) = \frac{C(\xi)e^{-2\xi hs}}{1 + e^{-2\xi hs}},$$

where the unknown function $C(\xi)$ is to be determined. From conditions (2.3) and (2.5) it is found that $C(\xi)$ satisfies the following quadruple integral equations:

(2.11)
$$\int_0^\infty \xi C(\xi hs)\,\mathrm{th}(\xi hs)\cos(\xi x)\,d\xi = \frac{\pi p}{2\mu s}, \qquad x \in I_1, I_3,$$

and

(2.12)
$$\int_0^\infty C(\xi)\cos(\xi x)d\xi = 0, \qquad x \in I_2, I_4,$$

where

$$I_1 = (0, a), \quad I_2 = (a, b), \quad I_3 = (b, c), \quad I_4 = (c, \infty).$$

3. Method of solution

In order to solve the quadruple integral equations (2.11) and (2.12), let us take

(3.1)
$$C(\xi) = \frac{1}{\xi}\int_0^a h(u)\sin(\xi u)\,du + \frac{1}{\xi}\int_b^c g(v^2)\,\mathrm{ch}(ev)\sin(\xi v)\,dv,$$

where $h(u)$ and $g(v^2)$ are the unknown functions to be determined from the boundary conditions of the problem considered. Substituting the value of $C(\xi)$ given by Eq. (3.1) into Eq. (2.12) and using the well-known result

$$\int_0^\infty \frac{\sin(x\xi)\cos(y\xi)}{\xi}\,d\xi = \begin{cases} \dfrac{\pi}{2}, & x > y > 0, \\ \dfrac{\pi}{4}, & x = y > 0, \\ 0, & y > x > 0 \end{cases}$$

it is found that this choice of $C(\xi)$ leads to the condition

$$\int_b^c g(v^2)\,\mathrm{ch}(ev)\,dv = 0. \tag{3.2}$$

Rewritting Eq. $(2.11)_1$ in the form

$$\frac{d}{dx}\int_0^\infty C(\xi)\,\mathrm{th}(\xi hs)\sin(\xi x)\,d\xi = \frac{\pi p}{2\mu s}, \qquad x \in I_1 \tag{3.3}$$

and inserting the value of $C(\xi)$ from Eq. (3.1) in (3.3) it is found that $h(u)$ is the solution of the following singular integral equation:

$$\int_0^a h(u)\log\left|\frac{\mathrm{sh}(ex)+\mathrm{sh}(eu)}{\mathrm{sh}(ex)-\mathrm{sh}(eu)}\right| du = \pi f(x), \qquad x \in I_1 \tag{3.4}$$

with

$$f(x) = \int_0^x \left[\frac{p}{\mu s} - \frac{1}{\pi}\int_b^c \frac{e g(v^2)\,\mathrm{ch}(ex')\,\mathrm{sh}(2ev)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex')}\,dv\right] dx',$$

where the following result [10] has been used:

$$\int_0^\infty \mathrm{th}(\xi hs)\frac{\sin(\xi x)\sin(\xi u)}{\xi}\,d\xi = \frac{1}{2}\log\left|\frac{\mathrm{sh}(ex)+\mathrm{sh}(eu)}{\mathrm{sh}(ex)-\mathrm{sh}(eu)}\right|, \qquad e = \frac{\pi}{2hs}. \tag{3.5}$$

Now using the Cook's result [9], the solution of Eq. (3.4) has been obtained with the aid of the formula

$$\int_0^a \frac{\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ex)}\,e\,\mathrm{ch}(ex)\,dx}{[\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)][\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex)]} = -\frac{\pi}{2\,\mathrm{sh}(ev)}\frac{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eu)}$$

$$\text{for} \quad u \in I_1 \quad \text{and} \quad v \in I_3,$$

$$h(u) = \frac{-e\,\mathrm{sh}(2eu)}{\pi\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}\left[\frac{p}{\mu s}\int_0^a \sqrt{\frac{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)}}\,dx \right. \tag{3.6}$$

$$\left. + \int_b^c \frac{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eu)}\,g(v^2)\,\mathrm{ch}(ev)\,dv\right].$$

Substitute now the resulting value of $C(\xi)$, obtained by inserting Eqs. (3.6) into Eq. (3.1), in condition $(2.11)_2$, and make use of the following results:

$$\int_0^a \frac{e\,\mathrm{sh}^2(eu)\,\mathrm{ch}(eu)du}{[\mathrm{sh}^2(eu)-\mathrm{sh}^2(ex)][\mathrm{sh}^2(ev)-\mathrm{sh}^2(eu)]\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}$$
$$= \frac{\pi}{2[\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex)]}\left[\frac{\mathrm{sh}(ev)}{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}} - \frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}\right],$$

$$\int_0^a \frac{e\,\mathrm{sh}^2(eu)\,\mathrm{ch}(eu)du}{[\mathrm{sh}^2(eu)-\mathrm{sh}^2(ex)][\mathrm{sh}^2(ey')-\mathrm{sh}^2(eu)]\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}$$
$$= \frac{\pi}{2[\mathrm{sh}^2(ex)-\mathrm{sh}^2(ey')]}\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}, \quad \text{for} \quad x, v \in I_3 \quad \text{and} \quad y' \in I_1.$$

It can be shown that $g(v^2)$ is the solution of the following singular integral equation

$$(3.7) \qquad \int_b^c \frac{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex)} e g(v^2)\,\mathrm{ch}(ev)\,dv = \frac{\pi p}{\mu s}\left[\frac{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}{\mathrm{sh}(2ex)} + \frac{1}{\pi}\int_0^a \frac{\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ey')}}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ey')}\,dy'\right], \quad \text{for} \quad x \in I_3.$$

Using finite Hilbert transform technique [3] and the formula

$$\int_b^c \sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eb)}}\frac{\mathrm{sh}(2ex)dx}{[\mathrm{sh}^2(ex)-\mathrm{sh}^2(ey')][\mathrm{sh}^2(ex)-\mathrm{sh}^2(ev)]}$$
$$= -\frac{\pi}{e[\mathrm{sh}^2(ev)-\mathrm{sh}^2(ey')]}\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ey')}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ey')}},$$

the solution of Eq. (3.7) is found to be

$$(3.8) \quad g(v^2) = -\frac{2ep}{\mu\pi s}\frac{\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}}{\sqrt{[\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)]}}\left[\int_b^c \sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eb)}}\right.$$
$$\left.\times\frac{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ev)}\,dx - \int_0^a \sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ey')}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ey')}}\frac{\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ey')}}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ey')}\,dy'\right]$$
$$+\frac{C_1\,\mathrm{sh}(ev)}{\sqrt{[\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)]}}.$$

Next, substitution of $g(v^2)$ from Eq. (3.8) in Eq. (3.6) and finally application of the formula

$$\int_b^c \sqrt{\frac{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}}\frac{\mathrm{sh}(2ev)dv}{[\mathrm{sh}^2(ev)-\mathrm{sh}^2(eu)][\mathrm{sh}^2(ex')-\mathrm{sh}^2(ev)]}$$
$$= \frac{\pi}{e[\mathrm{sh}^2(eu)-\mathrm{sh}^2(ex')]}\left[\sqrt{\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}} - \sqrt{\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ex')}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex')}}\right], \quad \text{for} \quad u, x' \in I_1$$

yields $h(u)$ in the form

$$(3.9)\quad h(u) = -\frac{2ep}{\mu\pi s}\frac{\mathrm{ch}(eu)\,\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}}{\sqrt{[\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)]}}\Bigg[\int_0^a\sqrt{\frac{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ey')}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ey')}}$$

$$\times\frac{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ey')}}{\mathrm{sh}^2(ey')-\mathrm{sh}^2(eu)}dy' - \int_b^c\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eb)}}\frac{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)}\,dx\Bigg]$$

$$-\frac{C_1\,\mathrm{sh}(eu)\,\mathrm{ch}(eu)}{\sqrt{[\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)][\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)]}}.$$

Substitution of the value of $g(v^2)$ from Eq. (3.8) in the condition (3.2) yields

$$(3.10)\qquad C_1 = -\frac{2ep}{\pi\mu s}\Bigg[\int_b^c\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eb)}}\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}\Bigg\{\frac{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}$$

$$\times\Pi\Big\{\frac{\pi}{2},\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)},q\Big\}/F\left(\frac{\pi}{2},q\right)+1\Bigg\}\,dx$$

$$+\int_0^a\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(es)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(es)}}\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(es)}$$

$$\times\Bigg\{1-\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(es)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(es)}\Pi\Big\{\frac{\pi}{2},\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(es)},q\Big\}/F\Big(\frac{\pi}{2},q\Big)\Bigg\}\,ds,$$

where $F(\phi,q)$ and $\Pi(\phi,n,q)$ are elliptic integrals of the first and third kind, respectively, and $q=\sqrt{\dfrac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}$.

The relevant displacement and stress components in the plane of the crack can now be shown to be given by

$$(3.11)\qquad W(x,0) = \int_x^a h(u)\,du,\qquad 0\le x\le a,$$

$$= \int_x^c g(v^2)\,\mathrm{ch}(ev)\,dv,\qquad b\le x\le c,$$

and

$$[\sigma_{yz}(x,0)]_{a<x<b} = \frac{2\mu s}{\pi}\Bigg[\int_0^a\frac{eh(u)\,\mathrm{sh}(eu)\,du}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)} - \int_b^c\frac{eg(v^2)\,\mathrm{sh}(ev)\,\mathrm{ch}(ev)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex)}\,dv\Bigg]\,\mathrm{ch}(ex),$$

(3.12)

$$[\sigma_{yz}(x,0)]_{x>c} = \frac{2\mu s}{\pi}\Bigg[\int_0^a\frac{eh(u)\,\mathrm{sh}(eu)\,du}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)} + \int_b^c\frac{eg(v^2)\,\mathrm{sh}(ev)\,\mathrm{ch}(ev)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ev)}\,dv\Bigg]\,\mathrm{ch}(ex).$$

Now, insertion of the values of $h(u)$ and $g(v^2)$, as given by Eqs. (3.9) and (3.8), in the expressions (3.12) yields (after some algebraic manipulations)

$$[\sigma_{yz}(x,0)]_{a<x<b} = \frac{2pe}{\pi}\Bigg[-\sqrt{\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}$$

$$\Bigg\{\int_0^a F_2(u,x)\,du+\int_b^c F_2(v,x)dv\Bigg\}-\frac{2e[\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)]}{\pi}\Bigg\{\int_0^a F_2(u',x)\,du'\int_0^a F_4(c,u)$$

$$\times F_3(0,x,u)\,du+\int_b^c F_2(v,x)dv\int_0^a F_4(c,u)F_3(v,x,u)\,du\Bigg\}+\frac{\mu s}{ep}C_1\Bigg\{\frac{\pi}{2}$$

$$\times\frac{1-\mathrm{sh}(ex)/\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}{\sqrt{[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)]}}+e\int_0^a F_4(c,u)F_5(u,x)\,du\Bigg\}$$

$$+\frac{e[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)]}{\pi}\Bigg\{\int_b^c F_2(v',x)dv'\int_b^c F_4(a,v)F_6(v',x,v)dv+\int_0^a F_2(u,x)\,du$$

$$\times\int_b^c F_4(a,v)F_6(u,x,v)dv-\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)}\int_0^a F_1(u,x)du\int_0^a F_4(c,u')F_9(u,u')du'\Bigg\}$$

$$-\frac{\mu s}{pe}\frac{C_1}{X_1}\Bigg\{\frac{\pi}{2}\frac{\mathrm{sh}(ec)}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}+e\,\mathrm{sh}^2(ea)\int_b^c F_7(x,v)\,dv\Bigg\}\Bigg]\,\mathrm{ch}(ex),$$

$$(3.13)\qquad [\sigma_{yz}(x,0)]_{x>c} = \frac{2pe}{\pi}\Bigg[-\sqrt{\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}$$

$$\times\Bigg\{\int_0^a F_2(u,x)\,du+\int_b^c F_2(v,x)\,dv\Bigg\}-\frac{2e[\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)]}{\pi}\Bigg\{\int_0^a F_2(u',x)\,du'\int_0^a F_4(c,u)$$

$$\times F_3(0,x,u)\,du+\int_b^c F_2(v,x)\,dv\int_0^a F_4(c,u)F_3(v,x,u)\,du\Bigg\}+\frac{\mu s}{ep}C_1\Bigg\{\frac{\pi}{2}$$

$$\times\frac{1-\mathrm{sh}(ex)/\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}{\sqrt{[\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)]}}+e\int_0^a F_4(c,u)F_5(u,x)\,du\Bigg\}$$

$$-\frac{e[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)]}{\pi}\Bigg\{\int_b^c F_2(v',x)\,dv'\int_b^c F_4(a,v)F_8(v',v,x)\,dv+\int_0^a F_2(u,x)\,du$$

$$\times\int_b^c F_4(a,v)F_8(u,v,x)dv+\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)}\int_0^a F_1(u,x)\,du\int_0^a F_4(c,u')F_9(u,u')du'\Bigg\}$$

$$+\frac{\mu s C_1}{pe X_1}\Bigg\{\frac{\pi}{2}\frac{\mathrm{sh}(ec)}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}+e\,\mathrm{sh}^2(ea)\int_b^c F_7(x,v)dv\Bigg\}-\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}$$

$$\times\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ec)}}\Bigg\{\int_0^a F_2(u,x)\,du+\int_b^c F_2(v,x)\,dv\Bigg\}\Bigg]\,\mathrm{ch}(ex).$$

In the above formulae

$$F_1(u,x)=\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}}\frac{\mathrm{sh}(eu)}{\mathrm{sh}^2(ex)-\mathrm{sh}^2(eu)},$$

$$F_2(v,x)=\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}}\frac{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ex)},$$

$$F_3(v,x,u)=\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}}\tan^{-1}\left\{\frac{\mathrm{sh}(eu)}{\mathrm{sh}(ex)}\sqrt{\frac{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ea)}{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}\right\}$$

$$-\frac{\mathrm{sh}(ev)}{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}}\tan^{-1}\left\{\frac{\mathrm{sh}(eu)}{\mathrm{sh}(ev)}\sqrt{\frac{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}\right\},$$

$$F_4(\omega,u)=\frac{\mathrm{ch}(eu)\,\mathrm{sh}(eu)}{\sqrt{[\mathrm{sh}^2(e\omega)-\mathrm{sh}^2(eu)]^3[\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)]}},$$

$$F_5(u,x)=[2\,\mathrm{sh}^2(eu)-\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)]\left\{\sin^{-1}\left(\frac{\mathrm{sh}(eu)}{\mathrm{sh}(ea)}\right)-F_3(0,x,u)\right\}, \tag{3.14}$$

$$F_6(u,x,v)=\frac{\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}}$$

$$\times\log\left|\frac{\mathrm{sh}(ex)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}+\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}}{\mathrm{sh}(ex)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}-\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}}\right|-\frac{\mathrm{sh}(eu)}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}$$

$$\times\log\left|\frac{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}+\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}-\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}\right|,$$

$$F_7(x,v)=\tan^{-1}\left(\frac{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}}\frac{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}}{\sqrt{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ex)}}\right)$$

$$\times\frac{\mathrm{ch}(ev)}{\sqrt{[\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)]^3}},$$

$$F_8(u,v,x)=-\frac{2\,\mathrm{sh}(ex)}{\sqrt{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ec)}}\tan^{-1}\left\{\frac{\mathrm{sh}(ev)}{\mathrm{sh}(ex)}\sqrt{\frac{\mathrm{sh}^2(ex)-\mathrm{sh}^2(ec)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}}\right\}$$

$$+\frac{\mathrm{sh}(eu)}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}\log\left|\frac{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}+\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}-\mathrm{sh}(ev)\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}}\right|,$$

$$F_9(u,u')=\log\left|\frac{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu')}+\mathrm{sh}(eu')\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}{\mathrm{sh}(eu)\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu')}-\mathrm{sh}(eu')\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}\right|$$

and

$$X_1 = \sqrt{[\mathrm{sh}^2(eb) - \mathrm{sh}^2(ex)][\mathrm{sh}^2(ec) - \mathrm{sh}^2(ex)]}.$$

The dynamic stress intensity factors are defined by

$$\text{(3.15)}\qquad \begin{aligned} N_a &= \lim_{x\to a^+} \sqrt{2(x-a)}[\sigma_{yz}(x,0)]_{a<x<b}, \\ N_b &= \lim_{x\to b^-} \sqrt{2(b-x)}[\sigma_{yz}(x,0)]_{a<x<b}, \\ N_c &= \lim_{x\to c^+} \sqrt{2(x-c)}[\sigma_{yz}(x,0)]_{x>c}. \end{aligned}$$

Substitution of the results given by Eqs. (3.13) in expressions (3.15) yields

$$N_a = \sqrt{\frac{\mathrm{sh}(2ea)}{e}}\Bigg[-\sqrt{\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}\frac{2pe}{\pi}\Bigg\{\int_0^a F_2(u,a)\,du + \int_b^c F_2(v,a)\,dv\Bigg\} - \frac{\mu s C_1}{\sqrt{[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)]}}\Bigg],$$

$$\text{(3.16)}\qquad N_b = -\frac{\mu s C_1}{\sqrt{[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)]}}\sqrt{\frac{\mathrm{sh}(2eb)}{e}},$$

$$N_c = \sqrt{\frac{\mathrm{sh}(2ec)}{e}}\Bigg[-\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}\frac{2pe}{\pi}\Bigg\{\int_0^a F_2(u,c)du + \int_b^c F_2(v,c)\,dv\Bigg\} + \frac{\mu s C_1}{\sqrt{[\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)][\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)]}}\Bigg].$$

Again, insertion of the values of $h(u)$ and $g(v^2)$, given by Eqs. (3.8) and (3.9), in the expressions for displacements given by Eqs. (3.11) yields

$$[W(x,0)]_{0\le x\le a} = -\frac{p}{\mu\pi s}\Bigg[\frac{2[\mathrm{sh}^2(eb)-\mathrm{sh}^2(ea)]}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}}\Bigg\{\int_b^c \Pi\Big\{\lambda, \frac{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}, q\Big\} \times\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}}\frac{dv}{\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}} - \int_0^a \Pi\Big\{\lambda, \frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}, q\Big\} \times\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}}\frac{du}{\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}}\Bigg\}\Bigg] - \frac{C_1 F(\lambda,q)}{e\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}},$$

and

$$[W(x,0)]_{b\le x\le c} = \Bigg[\frac{2p}{\mu\pi s}\Bigg(\int_b^c \sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}}\sqrt{\mathrm{sh}^2(ev)-\mathrm{sh}^2(ea)}\Bigg\{F(\lambda',q) + \frac{\mathrm{sh}^2(ev)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}\Pi\Big\{\lambda', \frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ev)}, q\Big\}\Bigg\}dv + \int_0^a \sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}}$$

$$\times\sqrt{\mathrm{sh}^2(ea)-\mathrm{sh}^2(eu)}\{F(\lambda',q)-\frac{\mathrm{sh}^2(eb)-\mathrm{sh}^2(eu)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)}\Pi\{\lambda',\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eu)},q\}\}\,du\Big)$$
$$+\frac{C_1}{e}F(\lambda',q)\Big]\frac{1}{\sqrt{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ea)}},$$

where

$$\sin\lambda=\sqrt{\frac{\mathrm{sh}^2(ea)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(eb)-\mathrm{sh}^2(ex)}},\qquad \sin\lambda'=\sqrt{\frac{\mathrm{sh}^2(ec)-\mathrm{sh}^2(ex)}{\mathrm{sh}^2(ec)-\mathrm{sh}^2(eb)}}$$

and $F(\phi,q)$, $\Pi(\phi,n,q)$, and q have been defined earlier.

On putting $b=c$ and simplifying, it may be noted that the results $(3.16)_1$ and $(3.17)_1$ become those given by Eqs. (4.18) and (4.19) of SINGH *et al.* [2], and for $a=0$ the results given by Eqs. $(3.16)_2$, $(3.16)_3$ and $(3.17)_2$ coincide with those given by Eqs. (4.38), (4.39) and (4.35) of DAS and GHOSH [5].

4. Numerical results and discussions

Numerical results for stress intensity factors at the tips of the cracks for different values of crack speed, crack lengths and the separating distance between the cracks have been presented in this section. The dependence of the stress intensity factors on crack lengths and their variations with V/C_2 have been shown in Figs. 2–5. It is seen in Figs. 2–3 that stress intensity factors at the edges of the cracks increase rapidly when $V/C_2\to 1$, and variation of stress intensity factors at the edge $x=a$ is greater than that at the tips $x=b$ and $x=c$ when the length of the inner crack increases.

Variations of stress intensity factors at the edges of the cracks with a/b for different values of c/b and that with b/a for different values of c/a are plotted in Figs. 4–5, respectively. It has been found that when the distance between the inner crack and the outer pair of cracks decreases, the stress intensity factors at the tips $x=a$ and $x=b$ become greater than that at the edge $x=c$.

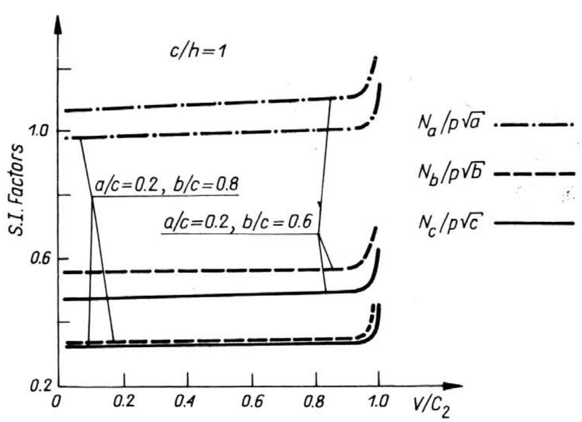

FIG. 2. Variations of stress intensity factors with V/C_2.

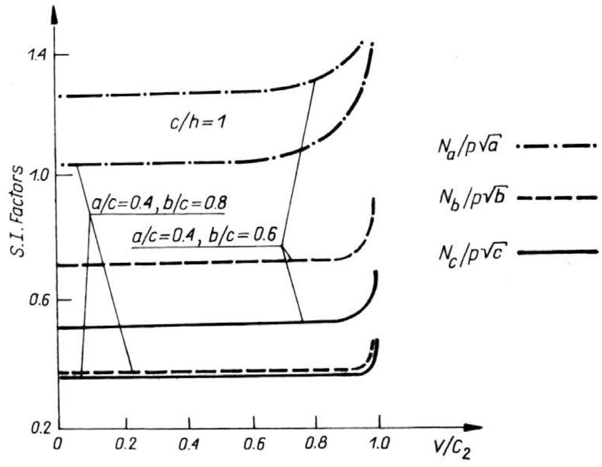

FIG. 3. Variations of stress intensity factors with V/C_2.

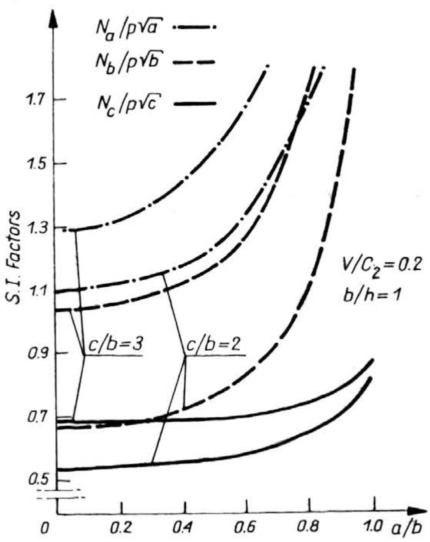

FIG. 4. Stress intensity factors Vs. a/b.

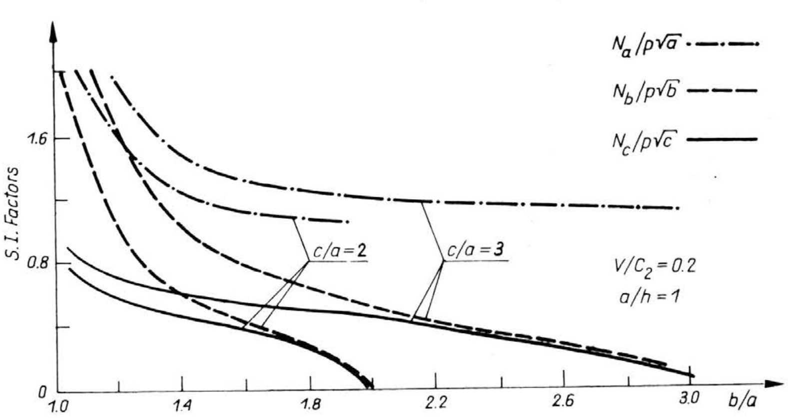

FIG. 5. Stress intensity factors Vs. b/a.

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