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Nonsimilar laminar incompressible boundary layer flow over a rotating sphere

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THE HEAT and mass transfer problem for the steady laminar incompressible boundary layers for a rotating sphere under forced flow has been studied. The results indicate that the rotation has a strong effect on the skin friction but its effect on the heat transfer is comparatively small. When the temperature of the wall is greater than the temperature of the free stream, then beyond a certain critical value of the dissipation parameter, the hot wall ceases to be cooled by the stream of cooler air because the "heat cushion" provided by the frictional heat prevents cooling. The results have been compared with those obtained by other prediction methods.

Rozważono problem przepływu ciepła i masy w nieściśliwych laminarnych warstwach przyściennych dla kuli wirującej w przepływie wymuszonym. Wyniki wskazują, że ruch wirowy ma duży wpływ na tarcie powierzchniowe lecz niewielki wpływ na przewodnictwo ciepła. Gdy temperatura ścianki jest wyższa od temperatury strugi swobodnej, to powyżej pewnej krytycznej wartości parametru dysypacji gorąca ścianka przestaje być chłodzona przez strugę chłodniejszego powietrza w wyniku działania „poduszki cieplnej" wytworzonej przez ciepło tarcia. Wyniki porównano z wynikami uzyskanymi za pomocą innych metod.

Рассмотрена проблема переноса тепла и массы в несжимаемых ламинарных пограничных слоях для вращающегося шара в вынужденном течении. Результаты показывают, что вихревое движение имеет большое влияние на поверхностное трение, но небольшое влияние на теплопроводность. Когда температура стенки выше чем температура свободного потока, тогда свыше некоторого критического значения параметра диссипации горячая стенка перестает охлаждаться потоком более холодного воздуха, в результате действия „тепловой подушки", образованной теплом трения. Результаты сравнены с результатами полученными при помощи других методов.

Notations

- A dimensionless constant characterizing the surface mass transfer,
- Br Brinkman number,
- c constant having a dimension $(\text{time})^{-1}$,
- $C_f, \bar{C}_f$ skin-friction coefficients in the x and y directions, respectively,
- C_p specific heat at a constant pressure,
- f dimensionless stream function,
- f_w dimensionless mass transfer parameter,
- F, s dimensionless velocity components in the x and y directions, respectively,
- F', s' shear stress functions in the x and y directions, respectively,
- g dimensionless temperature,
- g' heat-transfer function,
- L characteristic length,
- Nu Nusselt number,
- Pr Prandtl number,
- q_w heat-transfer rate at the wall,
- r distance from the axis of the body of revolution,

R radius of the body,
Re_L Reynolds number,
Re_x local Reynolds number,
T temperature,
u, v, w velocity components in the x, y and z directions, respectively,
x, y, z longitudinal, tangential and normal directions, respectively,
$\bar{x}$ dimensionless longitudinal distance,
α, λ dimensionless rotation parameters,
α_1 dimensionless parameter,
β pressure gradient parameter,
η, ξ transformed coordinates,
ν kinematic viscosity,
ϱ density,
τ_x, τ_y shear stresses along the x and y directions, respectively,
ψ dimensional stream function,
ω angular velocity of the body.

Superscript

$'$ prime denotes differentiation with respect to η.

Subscripts

e denotes conditions at the edge of the boundary layer,
w denotes conditions at the wall,
∞ denotes conditions in the free stream,
ξ denotes derivatives with respect to ξ.

1. Introduction

THE STUDY of the flow and heat transfer over a rotating body of revolution in forced flow is of considerable importance in the design of missiles, projectiles and rotodynamic machines. The flow field for the steady laminar incompressible boundary layer on a rotating sphere has been studied by HOSKIN [1] and the temperature field by SIEKMANN [2]. Both authors used four-term Blasius series to solve the governing boundary-layer equations. As pointed out by GORTLER [3], the Blasius series method does not give accurate results. Recently, CHAO and GREIF [4] re-studied the temperature field using an improved method where the velocity field is assumed to be quadratic and the temperature field is expressed as a universal function. These authors observed that for large values of the rotation parameter and for small values of the Prandtl numer, the quadratic velocity is inadequate in determining the temperature field. Subsequently, CHAO [5] improved the above method by taking more terms of the velocity profiles and applied the method for the case of a rotating disc. More recently, LEE *et. al* [6] re-studied both the flow and temperature fields using Mark's three-term series [7] as refined by CHAO and FAGHENLE [8]. Since accurate results for the skin friction and heat transfer are required, it is essential to assess the accuracy of these methods by comparing their results with those of an exact method such as a finite-difference scheme.

The purpose of the present work is twofold. First, the solution of the foregoing problem using an implicit finite-difference scheme [9–10] has been presented. Second, the effects of mass transfer and viscous dissipation which were neglected by previous investigators have been included in the analysis. The results (both for rotating and stationary bodies, but without mass transfer and viscous dissipation) have been compared with the previous theoretical results obtained by using various other methods [1–2, 4–6, 11–15]. We have also presented the skin-friction and heat-transfer results for a rotating disc which is only a special case of our results and they are also compared with the previous theoretical and experimental results [4–6, 16–19].

2. Governing equations

We consider the steady laminar dissipative constant property incompressible boundary-layer flow over a rotating porous body of revolution placed in a uniform stream with its axis of rotation parallel to the free-stream velocity. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞ and the body has a uniform temperature T_w. It has been assumed that the surface mass transfer normal to the body is uniform. Under the above conditions, the boundary-layer equations governing the flow can be expressed in dimensionless form as [6]

$$\begin{gathered} F''+fF'+\beta(\xi)(1-F^2)+\alpha(\xi)s^2 = 2\xi(FF_\xi-f_\xi F'), \\ s''+fs'-\alpha_1(\xi)Fs = 2\xi(Fs_\xi-f_\xi s'), \\ \mathrm{Pr}^{-1}g''+fg'+\mathrm{Br}(u_e/u_\infty)^2[F'^2+(r\omega/u_e)^2 s'^2] = 2\xi(Fg_\xi-f_\xi g'). \end{gathered} \tag{2.1}$$

The boundary conditions are

$$\begin{gathered} F(\xi,0)=g(\xi,0)=0, \quad s(g,0)=1, \\ F(\xi,\infty)=g(\xi,\infty)=1, \quad s(\xi,\infty)=0, \end{gathered} \tag{2.2}$$

where

$$\begin{gathered} \xi = \int\limits_0^x (u_e/u_\infty)(r/L)^2 L^{-1}dx, \\ \eta = [\mathrm{Re}_L/(2\xi)]^{1/2}(u_e/u_\infty)(r/L)(z/L), \\ \psi(x,z) = u_\infty L(2\xi/\mathrm{Re}_L)^{1/2} f(\xi,\eta); \end{gathered} \tag{2.3}$$

$$\begin{gathered} u = (L/r)(\partial\psi/\partial z), \quad v = -(L/r)(\partial\psi/\partial x), \\ w = -(r/L)[u_e/(2\xi\mathrm{Re}_L)^{1/2}][f+2\xi f_\xi+(\beta+\alpha_1/2-1)\eta F], \\ u/u_e = F = f', \quad v/r\omega = s, \quad g = (T-T_w)/(T_\infty-T_w); \end{gathered} \tag{2.3'}$$

$$\begin{gathered} \beta(\xi) = (2\xi/u_e)(du_e/d\xi), \quad \alpha(\xi) = (2\xi/r)(dr/d\xi)(r\omega/u_e)^2, \\ \alpha_1(\xi) = (4\xi/r)(dr/d\xi), \quad \mathrm{Br} = u_\infty^2/[C_p(T_\infty-T_w)]; \end{gathered} \tag{2.3''}$$

$$\begin{gathered} f = \int\limits_0^\eta F d\eta + f_w, \quad \mathrm{Re}_L = u_\infty L/\nu, \\ f_w = -(\xi)^{-1/2}(\mathrm{Re}_L/2)^{1/2}\int\limits_0^x (w_w/u_\infty)(r/L)d(x/L). \end{gathered} \tag{2.3'''}$$

The skin-friction coefficients in the x and y directions are given by

$$(2.4)\qquad \begin{aligned} C_f &= 2\tau_x/(\varrho u_\infty^2) = 2(u_e/u_\infty)^2 (r/L)(2\xi \mathrm{Re}_L)^{-1/2} F'_w, \\ \overline{C}_f &= 2\tau_y/(\varrho u_\infty^2) = 2(r/L)^2 (2\xi \mathrm{Re}_L)^{-1/2} (L\omega/u_\infty)(u_e/u_\infty) s'_w, \end{aligned}$$

where

$$(2.4')\qquad \tau_x = \mu(\partial u/\partial z)_w, \quad \tau_y = \mu(\partial v/\partial z)_w.$$

The heat-transfer coefficient in the form of the Nusselt number can be written as

$$(2.4'')\qquad \mathrm{Nu} = L(\partial T/\partial z)_w/(T_\infty - T_w) = [\mathrm{Re}_L/(2\xi)]^{1/2} (u_e/u_\infty)(r/L) g'_w.$$

It may be noted that for a stationary body $\alpha = 0$ and Eq. $(2.1)_2$ becomes inessential since s is not interesting in this case. Consequently, Eqs. $(2.1)_1$ and $(2.1)_3$ represent the classical nonsimilar flow over a stationary body of revolution which has been studied in the past.

The computations have been carried out for the case of a sphere. For a sphere the velocity at the edge of the boundary layer, curvature of the body, and surface mass transfer all being functions of $\overline{x}$ give rise to nonsimilarity. The velocity at the edge of the boundary layer and the distance from the axis of the body are given by

$$(2.5)\qquad u_e/u_\infty = (3/2)\sin\overline{x}, \quad r/R = \sin\overline{x}, \quad \overline{x} = x/R.$$

In this case we use the radius of the sphere R as a characteristic length instead of L. Using the above relations, the expressions for ξ, $\beta(\xi)$, $\alpha(\xi)$, $\alpha_1(\xi)$, C_f, $\overline{C}_f$, Nu, etc. given by Eqs. (2.3) and (2.4) can be expressed as

$$(2.6)\qquad \begin{aligned} \xi &= (1-\cos\overline{x})^2 (2+\cos\overline{x})/2, \\ \beta &= (2/3)[\cos\overline{x}(2+\cos\overline{x})/(1+\cos\overline{x})^2], \\ \alpha &= \lambda\beta, \quad \alpha_1 = 2\beta, \quad \lambda = (4/9)(R\omega/u_\infty)^2; \end{aligned}$$

$$(2.6')\qquad (r\omega/u_e)^2 = \lambda, \quad \mathrm{Re}_L = u_\infty R/\nu;$$

$$(2.6'')\qquad \begin{aligned} C_f(\mathrm{Re}_L)^{1/2} &= (9/2)\sin\overline{x}(1+\cos\overline{x})(2+\cos\overline{x})^{-1/2} F'_w, \\ \overline{C}_f(\mathrm{Re}_L)^{1/2} &= (9/2)\lambda^{1/2}\sin\overline{x}(1+\cos\overline{x})(2+\cos\overline{x})^{-1/2} s'_w, \\ \mathrm{Nu}(\mathrm{Re}_L)^{-1/2} &= (3/2)(1+\cos\overline{x})(2+\cos\overline{x})^{-1/2} g'_w; \end{aligned}$$

$$(2.6''')\qquad f_w = A[2/(2+\cos\overline{x})]^{1/2}, \quad A = -(w_w/u_\infty)(\mathrm{Re}_L)^{1/2};$$

$$(2.6'''')\qquad \xi(\partial/\partial\xi) = 3^{-1}\tan(\overline{x}/2)(2+\cos\overline{x})(1+\cos\overline{x})^{-1}(\partial/\partial\overline{x}).$$

Here we have taken the surface mass transfer (w_w/u_∞) to be constant. Hence A is a constant ($A \gtrless 0$ according to whether there is suction or injection) and the mass transfer parameter f_w will vary according to Eq. (2.6‴).

It may be noted that the flow and heat transfer characteristics of forced flow for an isothermal rotating disc can be obtained from those of a rotating sphere. For a disc [6]

$$(2.7)\qquad r = x, \quad u_e/u_\infty = 2x/\pi R, \quad \lambda = \pi R\omega/2u_\infty, \quad \beta = 0.5.$$

The local skin-friction coefficient in the radial direction is given by [16]

$$(2.8)\qquad C_f(\mathrm{Re}_x)^{1/2} = 2\{\tau_w/[\varrho(c^2+\omega^2)x^2]\}[(c^2+\omega^2)^{1/2}x^2/\nu]^{1/2} = 2^{3/2}(1+\lambda)^{-3/4}(F'_w)_{\overline{x}=0}.$$

Similarly, the heat-transfer coefficient in terms of the Nusselt number can be expressed as [17]

(2.8′) $$\mathrm{Nu} = q_w \nu^{1/2}/[K(T_w - T_\infty)(c^2+\omega^2)^{1/4}] = 2^{1/2}(1+\lambda)^{-1/4}(g_w')_{\bar{x}=0},$$

where

(2.8″) $$c = 2u_\infty/\pi R, \quad \mathrm{Re}_x = [(c^2+\omega^2)^{1/2}x^2\nu^{-1}]^{1/2}.$$

We find that Eq. (2.8′) is similar to (17) of Ref. [17] except the factor $2^{1/2}(1+$ $+\lambda)^{-1/4}$.

3. Results and discussion

We have used an implicit finite-difference scheme for the solution of the governing equations (2.1) under the boundary conditions (2.2) using the relations (2.6) or (2.7). Since the method is fully described in [9-10], its description is not repeated here. To ensure the convergence of the finite-difference scheme to the true solution, several values of the stepsize $\Delta\eta$ and $\Delta\bar{x}$ were employed. The results presented here are independent of the step size within at least three significant digits.

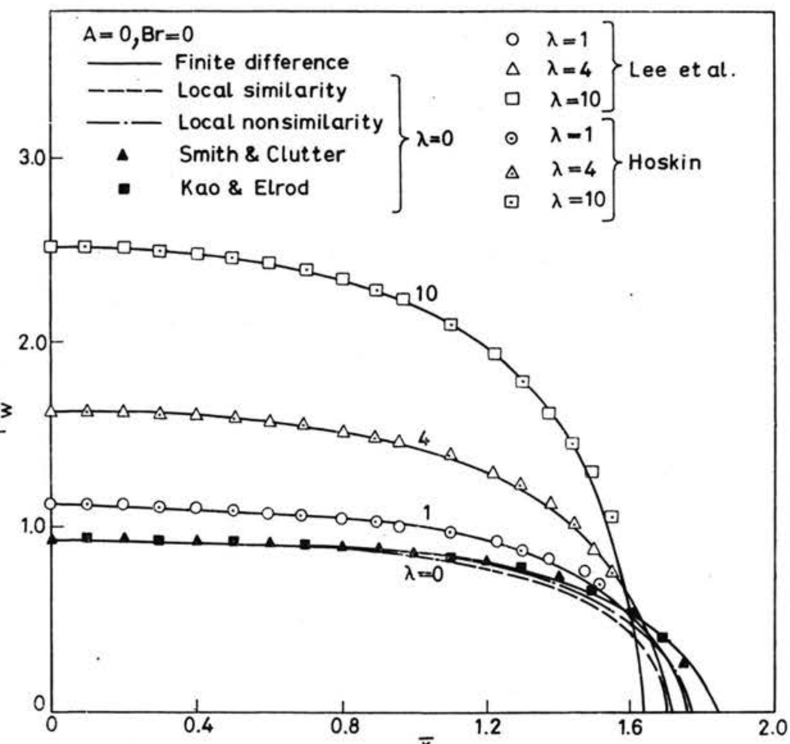

FIG. 1. Effect of rotation on the skin-friction parameter in the x direction for the sphere and comparison with that of other methods.

The skin-friction and heat-transfer results (F'_w, g'_w) for the stationary body ($\lambda = \alpha = 0$) in the absence of mass transfer ($A = 0$) and viscous dissipation ($\mathrm{Br} = 0$) have been compared with those obtained by the local similarity and local nonsimilarity methods [11, 12], asymptotic method [13, 14] and difference-differential method [15] (see Figs. 1 and 2).

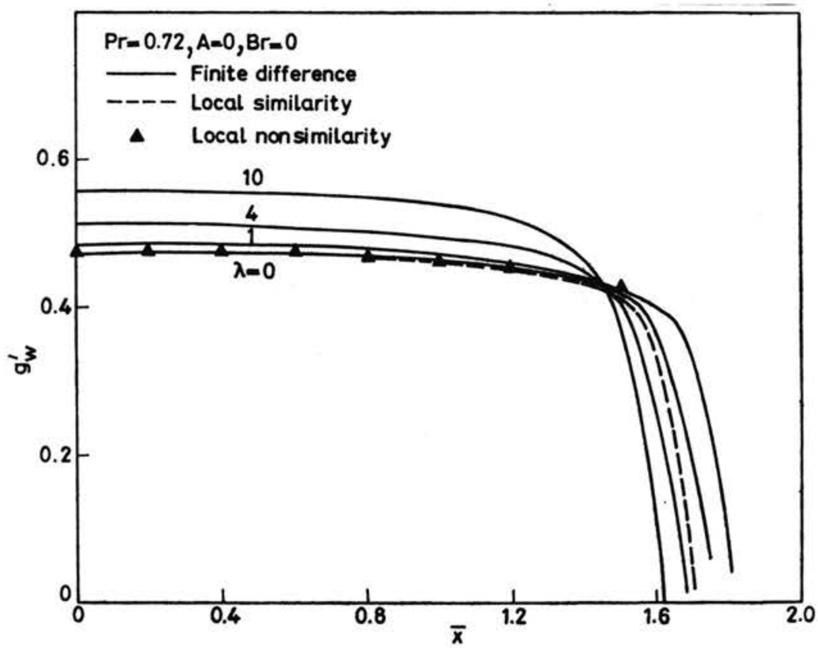

FIG. 2. Effect of rotation on the heat-transfer parameter for the sphere and comparison with that of other methods ($\mathrm{Pr} = 0.72$).

The finite-difference (present) results are almost the same as those of the difference-differential method [15]. They are also found to be in very good agreement with those of the local nonsimilarity method [11, 12] and asymptotic method [13, 14] except when $\bar{x}$ is large. However, the asymptotic method gives better results than the local nonsimilarity method for large $\bar{x}$. For small $\bar{x}$, all the methods predict nearly the same results. But the finite-difference results are found to differ significantly from the local similarity results and this difference increases as $\bar{x}$ increases.

The skin-friction and heat-transfer results (F'_w, $-s'_w$, g'_w) corresponding to rotating bodies ($\lambda > 0$) for $\mathrm{Br} = A = 0$ have also been compared with those of HOSKIN [1], SIEKMANN [2], CHAO and GREIF [4], CHAO [5], and LEE et al. [6] (see Figs. 1, 3–5]. From Figs. 1 and 3, it is clear that the skin-friction results (F'_w, $-s'_w$) are in good agreement with those of HOSKIN [1] and LEE *et al.* [6]. The heat-transfer results (g'_w) are also in good agreement with those of CHAO [5] and LEE *et al.* [6] for small values of $\bar{x}$ (Figs. 4–5). However, for large $\bar{x}$ the results differ and this difference becomes more pronounced as λ or Pr increases. It is also observed that the heat-transfer results (g'_w) obtained by CHAO and GREIF [4] differ considerably from the finite-difference (present) results even for small $\bar{x}$ and this difference increases as λ or $\bar{x}$ increases or Pr decreases. Also the results (g'_w)

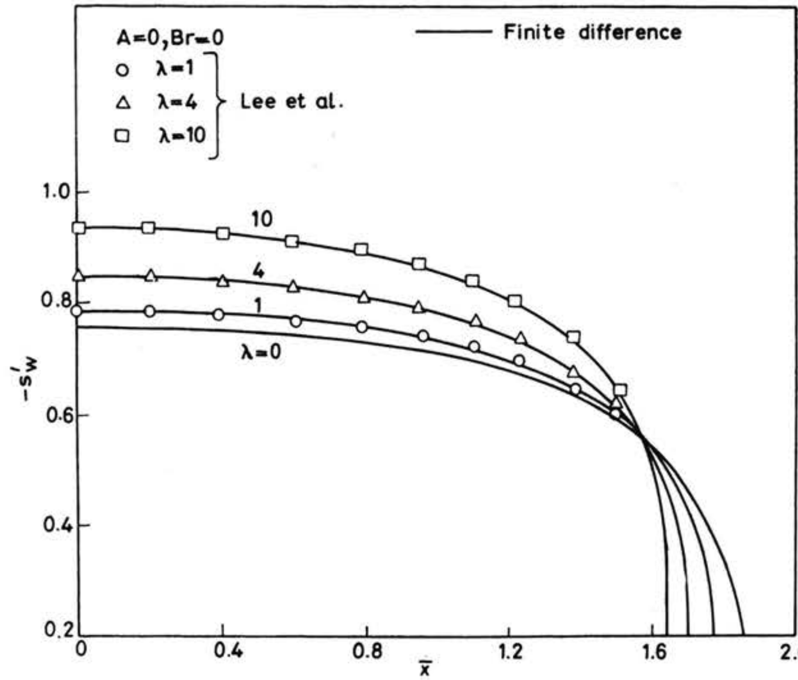

FIG. 3. Effect of rotation on the skin-friction parameter in the *y* direction for the sphere and comparison with that of other methods.

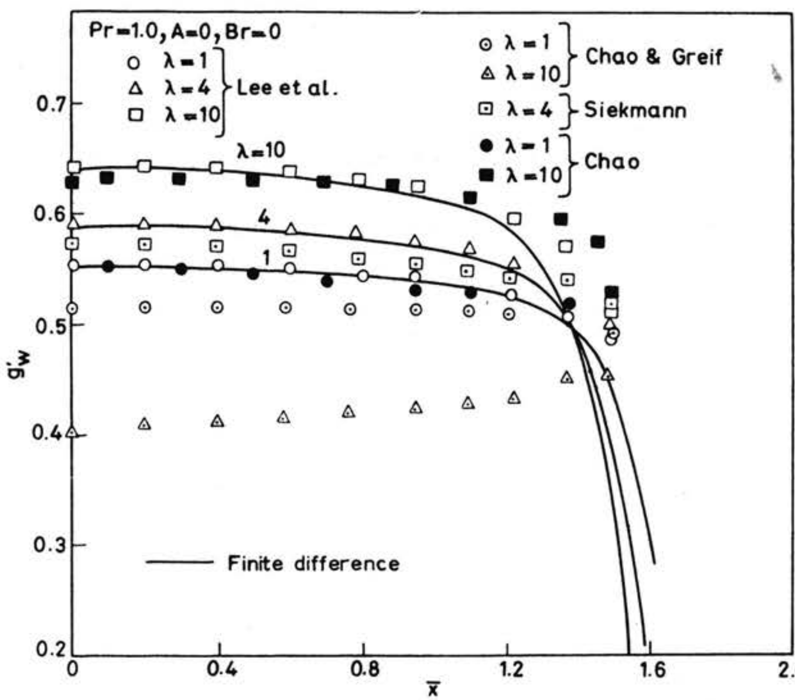

FIG. 4. Effect of rotation on the heat-transfer parameter for the sphere and comparison with that of other methods (Pr = 1.0).

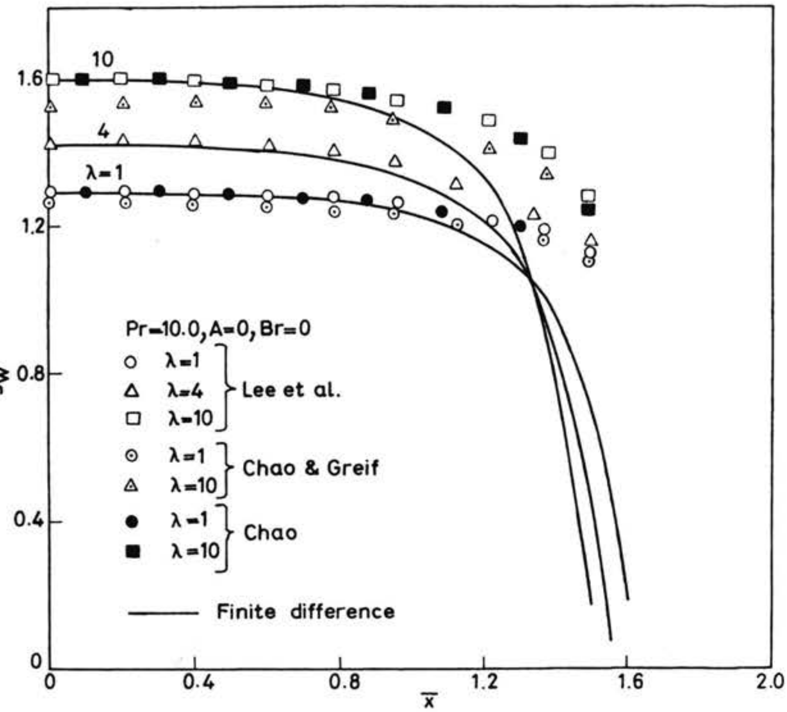

FIG. 5. Effect of rotation on the heat-transfer parameter for the sphere and comparison with that of other methods (Pr = 10.0).

obtained by SIEKMANN [2] differ from those of the finite-difference method. However, they are more accurate than those of CHAO and GREIF [4]. Furthermore the results (g'_w) obtained by LEE *et al.* [6] are found to be comparatively more accurate than those of SIEKMANN [2], CHAO and GREIF [4], and CHAO [5]. The results of CHAO [5] who used 3-term velocity profiles in the energy equation are more accurate than those of CHAO and GREIF [4] who used 2-term velocity profiles. Hence it can be concluded that for the accurate prediction of heat transfer, an exact method such as a finite-difference method has to be employed, because all approximate methods are found to give inaccurate results especially for large λ and $\bar{x}$.

As mentioned earlier, the rotating disc problem is a special case of rotating sphere

Table 1. Comparison of the skin-friction coefficient $C_f(\mathrm{Re}_x)^{1/2}$ for disc.

λ	Present analysis	LEE *et al.* [6]	TIFFORD and CHU [16]
0	2.6252	2.6239	2.61
1	1.8712	1.8717	1.83
4	1.3726	1.3934	1.38

problem. The skin-friction results $\left(C_f \mathrm{Re}^{\frac{1}{2}}\right)$ for the rotating disc without mass transfer ($A = 0$) and viscous dissipation ($\mathrm{Br} = 0$) are given in Table 1 along with the results obtained by LEE *et al.* [6] and TIFFORD and CHU [16]. Similarly, the heat-transfer results (Nu) are presented in Table 2 which also contains results obtained by CHAO and GREIF [4], CHAO [5], LEE *et al.* [6], and TIEN and TSUZI [17]. The skin-friction results are found

Table 2. Comparison of heat-transfer coefficient Nu for disc.

Pr	λ	Present analysis	LEE *et al.* [6]	TIEN and TSUJI [17]	CHAO and GREIF [4]	CHAO [5]
1	1	0.6578	0.6583	0.658	0.6113	0.659
1	4	0.5572	0.5577	0.557	0.432	0.548
10	1	1.5335	1.5354	1.535	1.518	—
10	4	1.3405	1.3410	1.340	1.297	—

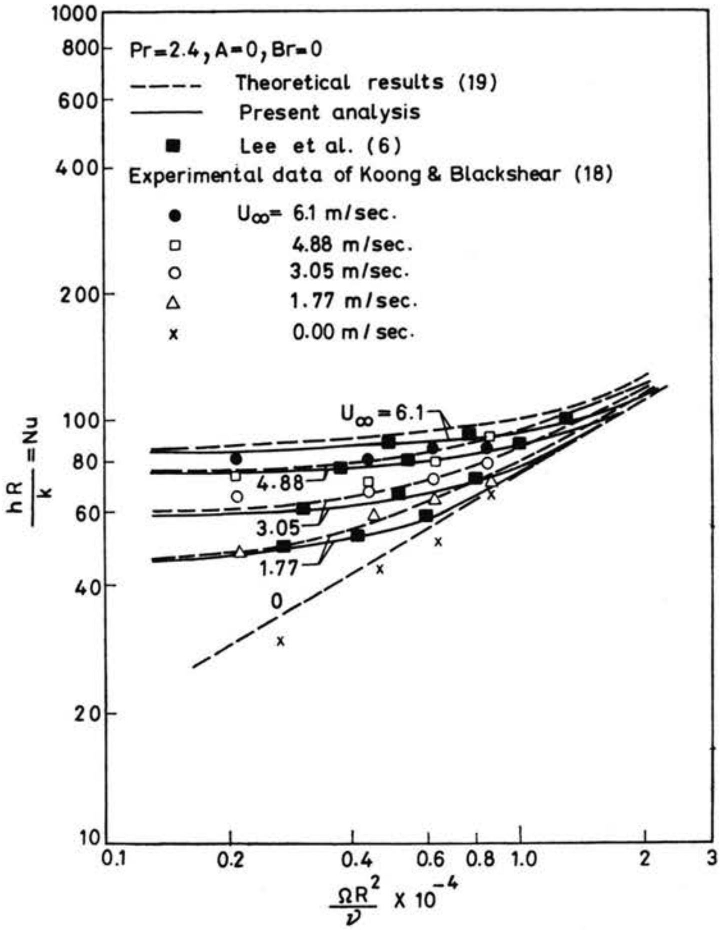

FIG. 6. Comparison of heat-transfer for the rotating disc with the experimental and theoretical results.

to be in good agreement. The heat-transfer results are also in good agreement except with those of CHAO and GREIF [4] (who used the quadratic velocity profiles in the analysis of the energy equation) for large λ and small Pr. A significant improvement was observed when the three-term velocity results were used [5]. This implies that as for the rotating sphere, the accurate prediction of heat transfer (Nu) especially for large λ and small Pr requires more accurate velocity results in the energy equation. We have also compared the heat-transfer results (Nu) with the experimental results given by KOONG and BLACKSHEAR [18]. The comparison is shown in Fig. 6 which also contains theoretical results of SCHLICHTING and TRUCKENBRODT [19] and LEE *et al.* [6]. It can be observed from the figure that the numerical results generally differ by about 10 per cent from the experimental results except at some points. The results (Nu) of LEE *et al.* [6] have been found to be in good agreement with our results. On comparing the results of SCHLICHTING and TRUCKENBRODT [19] with our results, we find that the results differ significantly especially for large values of the rotational Reynolds number.

The effect of the rotation parameter (λ) on the skin friction in the x-direction (F'_w) is shown in Fig. 1. It is evident from the results that the point of separation (i.e. the point of zero shearing stress) moves forward towards the equator ($\bar{x} = \pi/2$) due to the centrifugal acceleration on the boundary layer which tends to push fluid towards the equator.

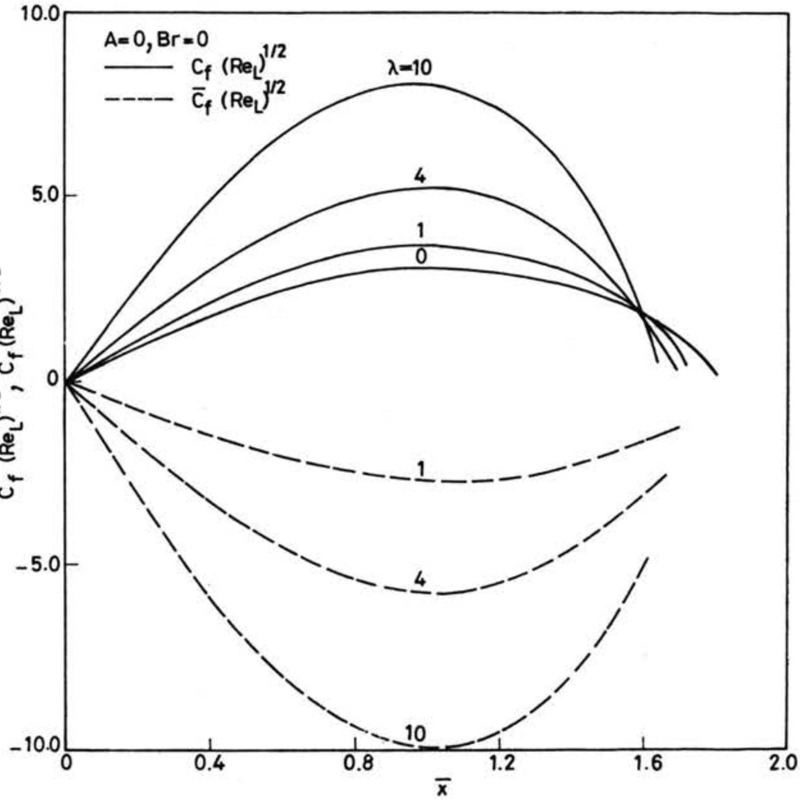

FIG. 7. Variation of the skin-friction coefficients in the x and y directions with $\bar{x}$ for the sphere.

This results in the increase of the adverse pressure gradient on the down-stream side of the equator [20]. An empirical relation to determine of the point of separation for the rotating sphere can be expressed as [20] (¹)

$$\bar{x}_{sep} = 1.853 - 0.22 \log_{10} \lambda, \quad \lambda > 1. \tag{3.1}$$

We find that the point of separation predicted by the present method is in very good agreement with that given by the relation (3.1). It is observed from Figs. 1–5 that in the range $0 \leqslant \bar{x} \leqslant \bar{x}_0$, F'_w, $-s'_w$, and g'_w increase as λ increases. In general, g'_w increases as Pr increases. It is also observed that g'_w is strongly influenced by Pr (see Figs. 2,4–5).

The skin-friction coefficients in the x and y directions ($C_f(\mathrm{Re}_L)^{1/2}$, $-\bar{C}_f(\mathrm{Re}_L)^{1/2}$) and heat transfer coefficient ($\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$) for various values of λ are given in Figs. 7–8

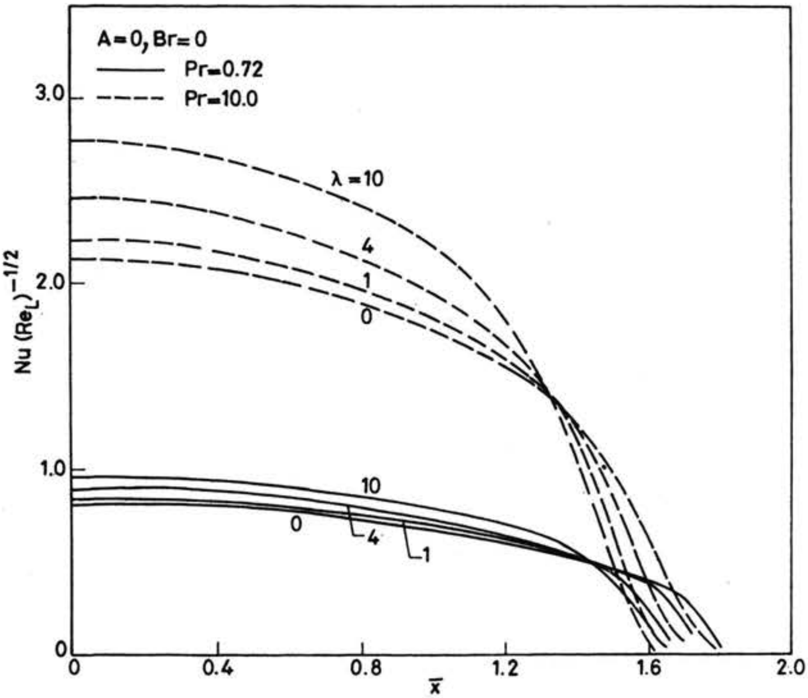

FIG. 8. Variation of the heat-transfer coefficient with x for the sphere.

and in the range $0 \leqslant \bar{x} \leqslant \bar{x}_0$ they increase as λ increases. For a given λ, $C_f(\mathrm{Re}_L)^{1/2}$ and $-\bar{C}_f(\mathrm{Re}_L)^{1/2}$ are zero at $\bar{x} = 0$ and they increase as $\bar{x}$ increases till a certain value $\bar{x}_1$ beyond which they decrease as $\bar{x}$ increases. On the other hand, $\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$ has a finite value at $\bar{x} = 0$ and it continuously decreases as $\bar{x}$ increases.

The effect of mass transfer (A) on the skin friction and heat transfer (F'_w, $-s'_w$, g'_w) is shown in Figs. 9–11. The results indicate that suction ($A > 0$) or injection ($A < 0$) exerts a strong influence on the skin friction and heat transfer whatever the values of λ may be. However, the effect is comparatively less pronounced when λ is large. Further, we find that for a given λ, F'_w, $-s'_w$, and g'_w increase as suction ($A > 0$) increases,

(¹) In [20] λ is defined as $\lambda^{1/2}$.

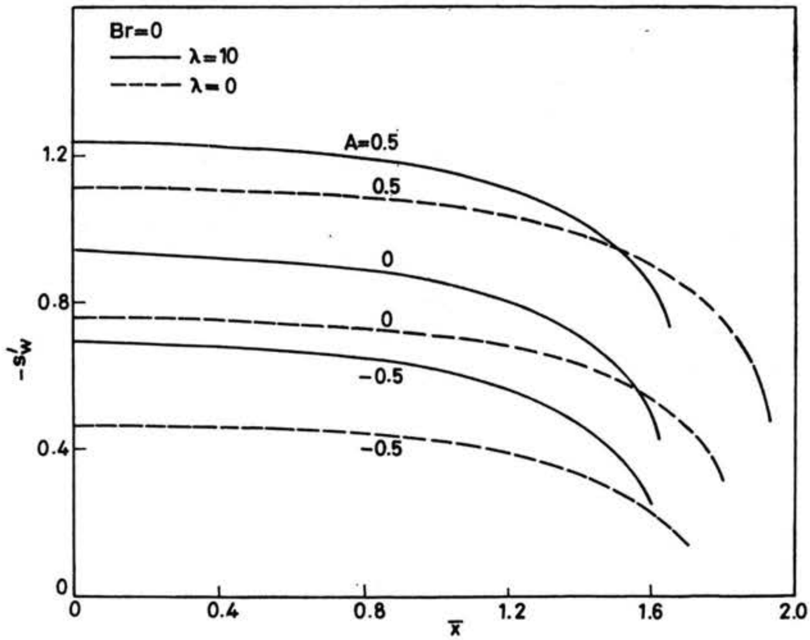

FIG. 9. Effect of mass transfer on the skin-friction parameter in the x direction for the sphere.

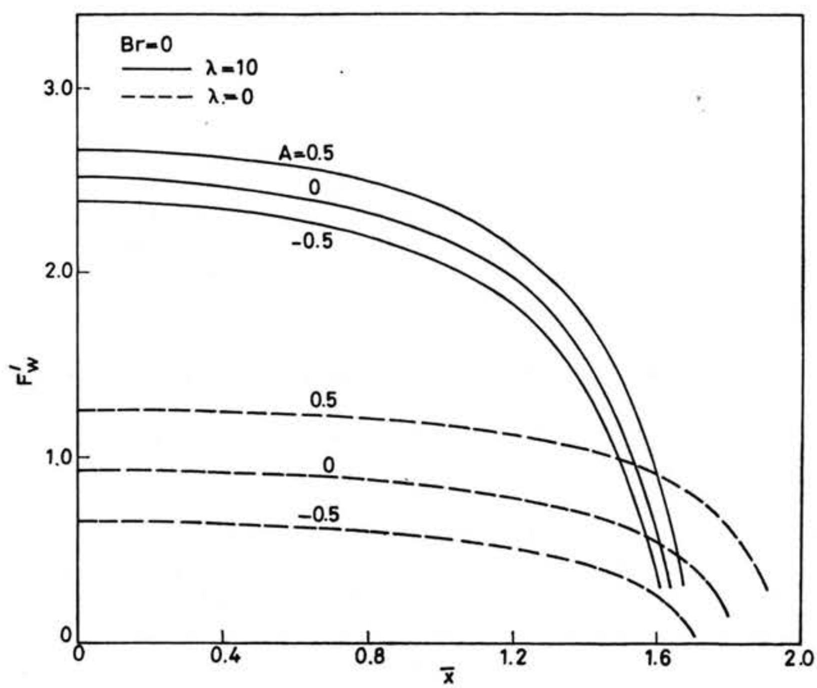

FIG. 10. Effect of mass transfer on the skin-friction parameter in the y direction for the sphere.

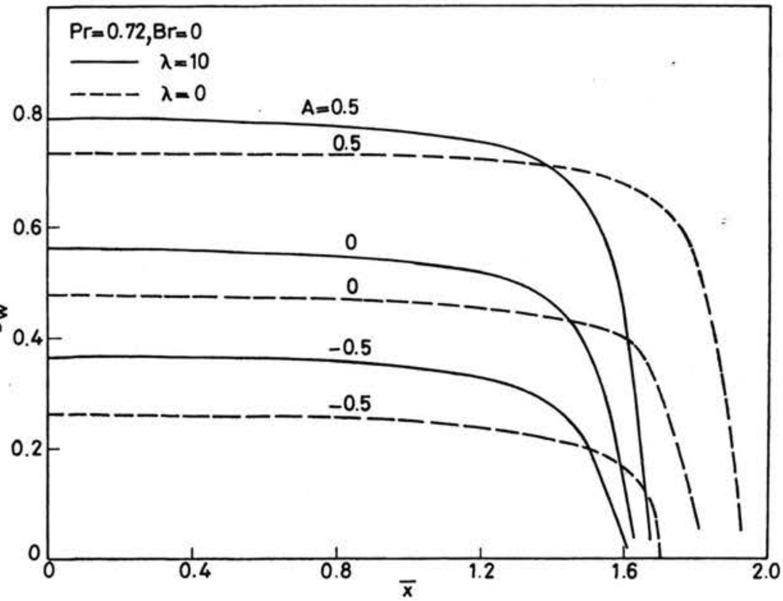

FIG. 11. Effect of mass transfer on the heat-transfer parameter for the sphere.

but the effect of injection ($A < 0$) is just the reverse. Furthermore, for a given λ, injection ($A < 0$) moves the point of separation towards the equator ($\bar{x} = \pi/2$) whereas suction ($A > 0$) does the reverse.

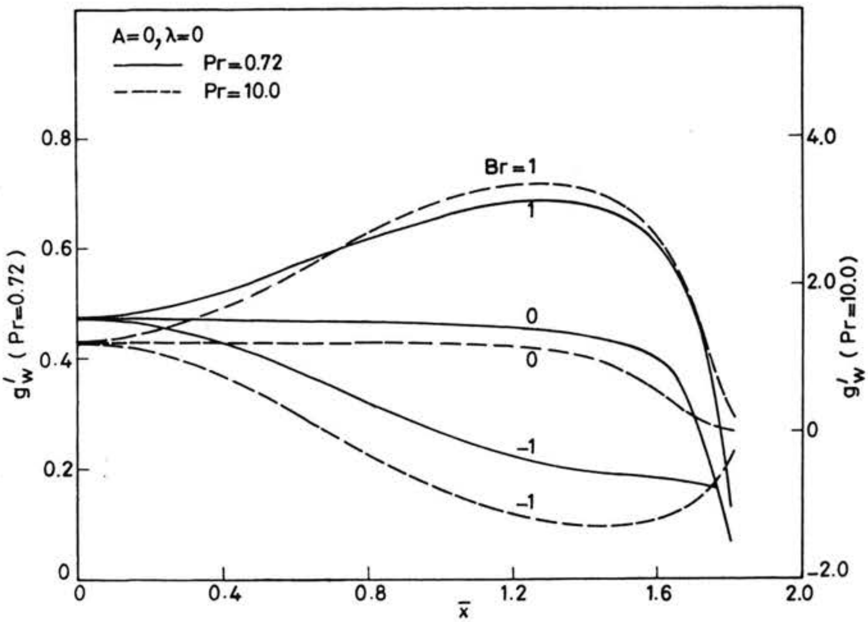

FIG. 12. Effect of viscous dissipation and Prandtl number on the heat-transfer parameter for the sphere ($\lambda = 0$).

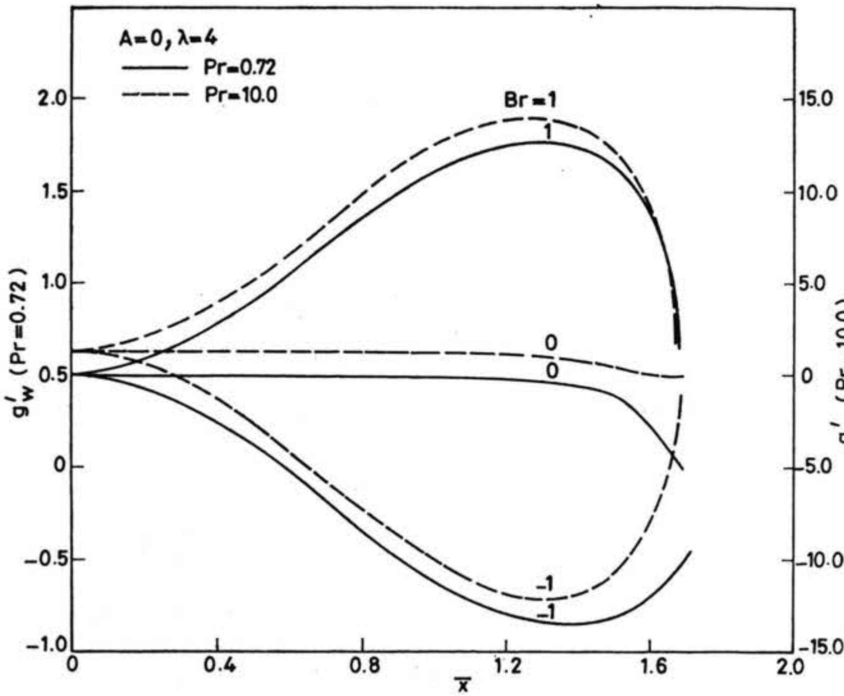

FIG. 13. Effect of viscous dissipation and Prandtl number on the heat-transfer parameter for the sphere ($\lambda = 4$).

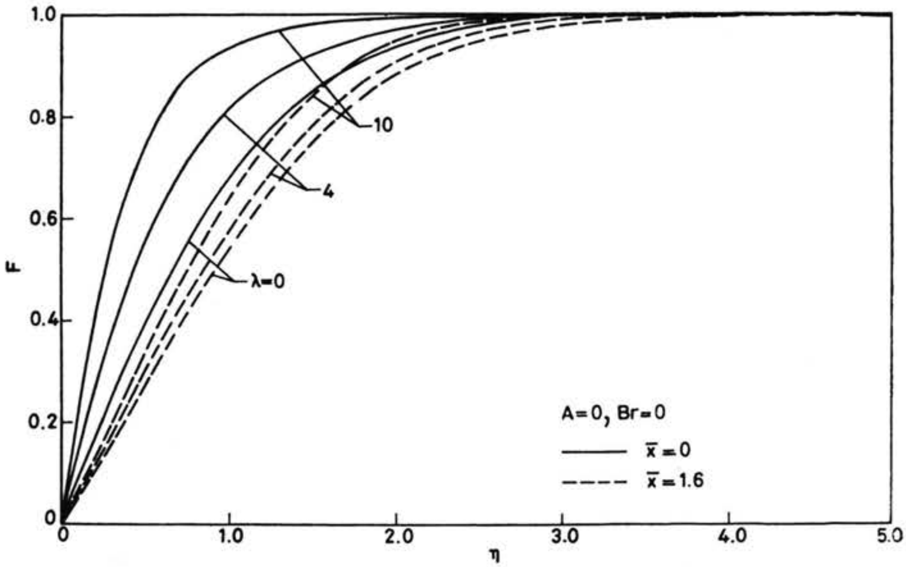

FIG. 14. x-velocity component distributions for the sphere.

The effect of the viscous dissipation ($Br \neq 0$) on the heat-transfer parameter g'_w is shown in Figs. 12–13 which also contain the effect of the Prandtl number, Pr. The results show that the parameters Br (Brinkmann number) and Pr strongly influence g'_w. When $Br \leqslant 0$ (i.e. $T_w \geqslant T_\infty$), g'_w continuously decreases as $\bar{x}$ increases, but when $Br > 0$ (i.e. $T_w < T_\infty$), g'_w first increases as x increases and then begins to decrease with x. At a given location $\bar{x}$, g'_w increases as Br increases.

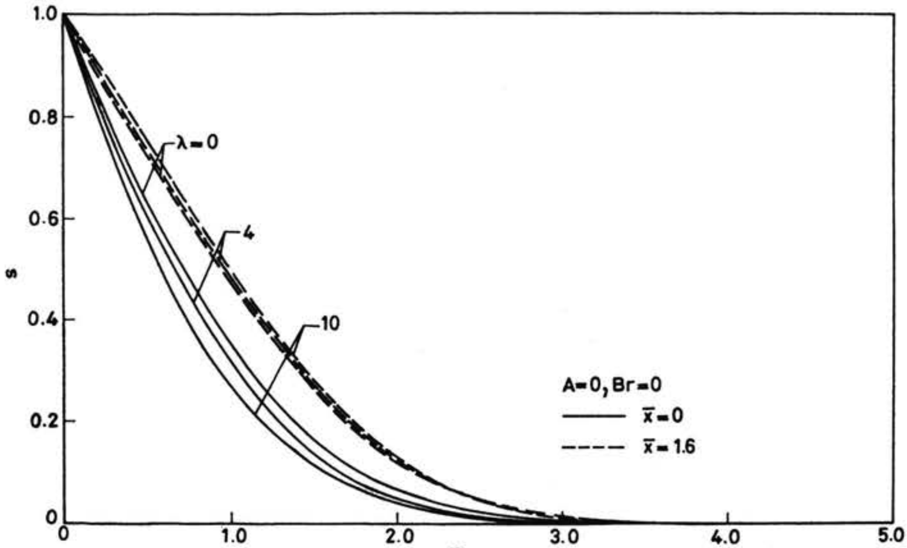

FIG. 15. y-velocity component distributions for the sphere.

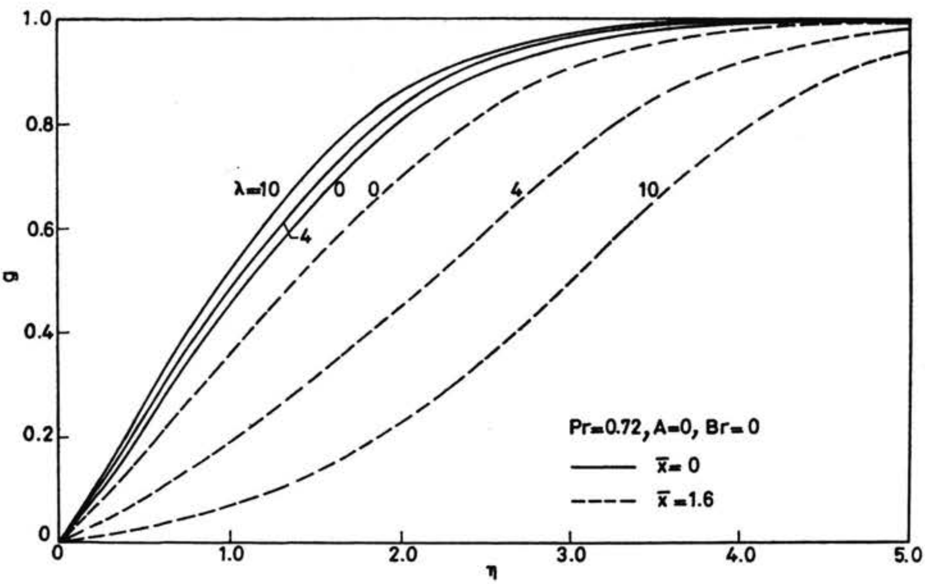

FIG. 16. Temperature distributions for the sphere ($Br = 0$; $\bar{x} = 0.\ 1.6$).

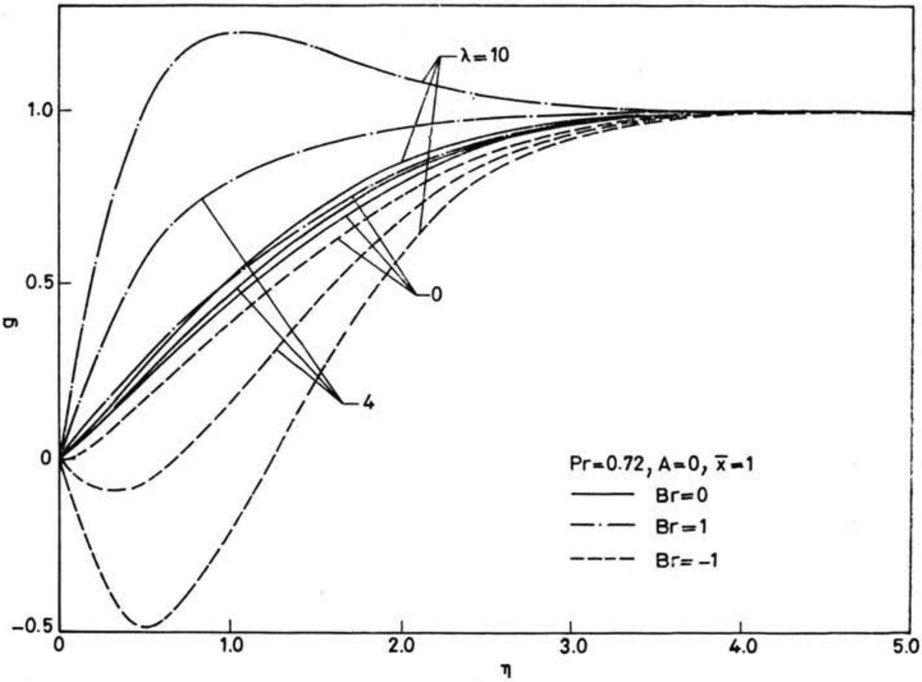

FIG. 17. Temperature distributions for the sphere (Br $= -1, 0, 1; \bar{x} = 1$).

The velocity and temperature profiles are shown in Figs. 14–17. For a given $\bar{x}$ $(0 \leqslant \leqslant \bar{x} \leqslant \bar{x}_0)$, the velocity and temperature profiles $(F, -s, g)$ become steeper when λ increases (Figs. 14–16). When $\bar{x} > \bar{x}_0$, the effect of λ on the profiles is just the opposite. This behaviour is due to the fact that F'_w or $-s'_w$ or g'_w decreases as λ increases when $\bar{x} > \bar{x}_0$. Similarly, for a given λ, the velocity and temperature profiles $(F, -s, g)$ become less steep as $\bar{x}$ increases. When Br < 0 and $\lambda > 0$, the temperature profile g first decreases to negative values and then tends to 1 asymptotically (see Fig. 17). This implies that the temperature of the fluid near the wall is greater than that at the wall. Therefore, the wall instead of being cooled will get heated. This effect becomes more pronounced as the rotation parameter λ increases. We also see from Fig. 14 that for $\lambda = 10$, $\bar{x} = 1$, $A = 0$ and Br $= 1$ the profile g does not tend to 1 monotonically, but exceeds 1 at certain η and then decreases as η increases and finally tends to 1. This implies that the combined effect of rotation and viscous dissipation tends to heat the fluid within the boundary layer to such an extent that the temperature of the fluid within the boundary layer is greater than the free-stream temperature. It may be remarked that no such effect is observed when any one of the two parameters λ or Br is equal to zero whatever the values of other parameters may be.

4. Conclusions

The results indicate that the rotation and injection tend to move the point of separation of the flow towards the equator while suction does the reverse. Further, the rotation exerts a strong influence on the skin friction, but its effect on the heat transfer is comparatively weak. On the other hand, the heat transfer is strongly dependent on the viscous dissipation parameter and also on the Prandtl number. When the temperature of the wall is greater than the temperature of the free stream, then beyond a certain critical value of the dissipation parameter, the hot wall ceases to be cooled by the stream of cooler air because the "heat cushion" provided by the friction heat prevents cooling. The skin-friction results are found to be in good agreement with the existing results, but the heat-transfer results are found to differ considerably from those of other prediction methods for large values of the longitudinal distance. This implies that for the accurate prediction of heat-transfer results, an exact method such as a finite-difference method must be used. The results for the rotating disc case are found to be a particular case of the sphere results and they are also in good agreement with the existing theoretical and experimental results except those of Chao and Greif, especially for large values of the rotation parameter.

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