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BRIEF NOTES

Method of multiple scales and the problem of aerodynamically generated sound

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IN THIS note the efficiency of the method of multiple scales for a sound radiation problem is discussed. It is given by investigation the mathematical requirement of uniform validity of the solution. The latter is connected with the physical possibility of a sound radiation in the far field.

IN THE LITERATURE (see SEARS [1]), the method of matched asymptotic expansions, has been used to calculate sound radiation from low Mach-number flows. In this method, the acoustic field is found by matching a wavefield scaled on the wavelength of sound, to an incompressible flow scaled on the wavelength of sound, to an incompressible flow scaled on a typical distance associated with the source field. Another approach is to use the method of multiple scales (see NAYFEH [2]). It turns out that the efficiency of the latter becomes operative through its ability to link the mathematical requirement of uniform validity of the solution directly to the physical possibility of a sound radiation in the far field.

For linearised unsteady potential flow, in usual notations

$$(1)\qquad \frac{1}{c_\infty'^2}\frac{\partial^2\Phi'}{\partial t'^2}+\frac{2M_\infty}{c_\infty'}\frac{\partial^2\Phi'}{\partial x'\partial t'}+M_\infty^2\frac{\partial^2\Phi'}{\partial x'^2}=\nabla'^2\Phi'.$$

Put,

$$(2)\qquad \Phi'(x',y',z',t')=\Phi'(x',y',z')e^{-i\omega' t'},$$

so that Eq. (1) becomes

$$(3)\qquad \phi'_{y'y'}+\phi'_{z'z'}+\beta^2\phi'_{x'x'}+2iM_\infty^2\frac{\omega'}{c_\infty^2}\phi'_{x'}+\frac{\omega'^2}{c_\infty'^2}\phi'=0,$$

where

$$\beta^2=1-M_\infty^2.$$

Put

$$(4)\qquad \phi'(x',y',z')=\psi'(X',y',z')e^{i\frac{M_\infty x'\omega'}{c_\infty'\beta}},$$

where

$$X'=\frac{x'}{\beta},$$

so that Eq. (3) becomes

(5) $$\psi'_{x'x'}+\psi'_{y'y'}+\psi'_{z'z'}+\chi^2\psi' = 0,$$

where

$$\chi' = \frac{\omega'}{c'_\infty \beta}.$$

Put

$$x = \frac{X'}{b'}, \quad y = \frac{y'}{b'}, \quad z = \frac{z'}{b'}, \quad \psi = \frac{\psi'}{2b'^2\omega'},$$

(6) $$\varepsilon = \frac{\omega' b'}{c'_\infty \beta} \ll 1, \quad b' \Rightarrow \text{a reference length from source field}$$

and seek solutions of the form

(7) $$\psi(x, y, z, \varepsilon) = \psi_0(x, \tilde{x}, y, \tilde{y}, z, \tilde{z})+\varepsilon^2\psi_1(x, \tilde{x}, y, \tilde{y}, z, \tilde{z})+0(\varepsilon^4),$$

where

$$\tilde{x} = \varepsilon x, \quad \tilde{y} = \varepsilon y, \quad \tilde{z} = \varepsilon z,$$

so that there follow

(8) $$\psi_{0xx}+\psi_{0yy}+\psi_{0zz} = 0,$$

(9) $$\psi_{1xx}+\psi_{1yy}+\psi_{1zz} = -(\psi_{0\tilde{x}\tilde{x}}+\psi_{0\tilde{y}\tilde{y}}+\psi_{0\tilde{z}\tilde{z}}+\psi_0).$$

According to Eq. (9) ψ_1, would behave asymptotically as

(10) $$\psi_1 \sim \frac{1}{4\pi r}\tilde{\Box}^2 \int \psi_0(x, \tilde{x}, y, \tilde{y}, z, \tilde{z})dV(x, y, z),$$

where

$$r = (x^2+y^2+z^2)^{1/2}, \quad \tilde{\Box}^2 = \left(\frac{\partial^2}{\partial\tilde{x}^2}+\frac{\partial^2}{\partial\tilde{y}^2}+\frac{\partial^2}{\partial\tilde{z}^2}+1\right),$$

so that ψ_1 will at least be of the same order as ψ_0 in the far field. This means that Eq. (7) will not provide a uniformly valid approximation to the properties of the flow. In order to preclude this, one requires

(11) $$\tilde{\Box}^2\psi_0 = 0$$

or if

(12) $$\Psi_0 = \psi_0 c^{-i\tilde{t}}, \quad \tilde{t} = \omega' t'$$

Eq. (11) implies

(13) $$\left(\tilde{\nabla}^2-\frac{\partial^2}{\partial\tilde{t}^2}\right)\Psi_0 = 0,$$

where

$$\tilde{\nabla}^2 = \frac{\partial^2}{\partial\tilde{x}^2}+\frac{\partial^2}{\partial\tilde{y}^2}+\frac{\partial^2}{\partial\tilde{z}^2}.$$

Equation (13) simply implies the existence of a sound radiation in the far field.

One then has the following program: ψ_0 can be represented in terms of transformed incompressible singularities. In order to make ψ_0 uniformly valid, one replaces the latter by the corresponding acoustic singularities.

Thus for a source in motion, one obtains

$$\Phi' = \frac{1}{r'} f'\left(t' - \frac{r'}{c'_\infty \beta^2} + \frac{M_\infty x'}{c'_\infty \beta^2}\right), \tag{14}$$

where

$$r' = [(x'+U't')^2 + \beta^2(y'^2+z'^2)]^{1/2},$$

U' being the velocity of the source.

References

1. W. R. SEARS, AIAA J., 7, 577, 1969.
2. A. H. NAYFEH, *Perturbation methods*, Wiley Interscience, 1973.

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