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Laminar flow in an annular channel with moving permeable walls

W. KALITA and A. SZANIAWSKI (WARSZAWA)

A STEADY laminar flow of an incompressible viscous fluid in the annular channel between coaxial cylindrical permeable surfaces is investigated. The fluid of constant density and viscosity is uniformly emitted from the inner immovable cylinder and it flows to the outer moving porous wall contained between two other cylindrical surfaces. This porous wall moves axially with given velocity that may vary slowly with the channel axis. The flow analysis is based on the assumption of small variations of flow velocity components along the channel with respect to their radial variations. The flow in the channel is described through an approximated solution of the Navier-Stokes equations while the flow in the moving porous wall is obtained from the filtration theory.

W niniejszej pracy badany jest ustalony laminarny przepływ nieściśliwego lepkiego płynu w pierścieniowym kanale między współosiowymi cylindrycznymi powierzchniami przepuszczalnymi. Płyn o stałej gęstości i lepkości jednorodnie wypływa z nieruchomego wewnętrznego cylindra i przepływa do zewnętrznej ruchomej porowatej ścianki zawartej między dwoma powierzchniami cylindrycznymi. Porowata ścianka porusza się wzdłuż osi kanału z daną prędkością, która może być wolno zmienna wzdłuż kanału. Analiza przepływu opiera się na założeniu małych zmienności składowych prędkości przepływu wzdłuż kanału w stosunku do ich radialnych zmienności. Przepływ w kanale opisany jest przez przybliżone rozwiązanie równań Naviera-Stokesa, podczas gdy przepływ w ruchomej porowatej ściance otrzymano z teorii filtracji.

В настоящей работе исследуется установившееся ламинарное течение несжимаемой вязкой жидкости в кольцевом канале между соосными цилиндрическими проницаемыми поверхностями. Жидкость с постоянной плотностью и вязкостью истекает однородным образом из неподвижного внутреннего цилиндра и протекает к внешней подвижной пористой стенке, содержавшейся между двумя цилиндрическими поверхностями. Пористая стенка движется вдоль оси канала с данной скоростью, которая может медленно изменяться вдоль канала. Анализ течения опирается на предположении малого изменения составляющих скорости течения вдоль канала по отношению к их радиальным изменениям. Течение в канале описано приближенным решением уравнений Навье-Стокса, в то время как течение в подвижной пористой стенке получено из теории фильтрации.

1. Introduction

A STEADY laminar flow of an incompressible viscous fluid in the annular channel between coaxial cylindrical permeable surfaces is investigated. The fluid of constant density and viscosity is uniformly emitted in a radial direction from the inner immovable cylinder and it flows to the outer moving porous thick wall contained between two other cylindrical surfaces (Fig. 1). This porous wall moves axially with assumed velocity that, in the case of a deformable wall, may vary with the axis coordinate. The pressure outside the wall is constant.

A large number of theoretical studies has been reported on the flows through channels with porous immovable walls [1–13]. In most of these papers the distribution of the velocity component perpendicular to the wall has been assumed *a priori* at the walls. In our

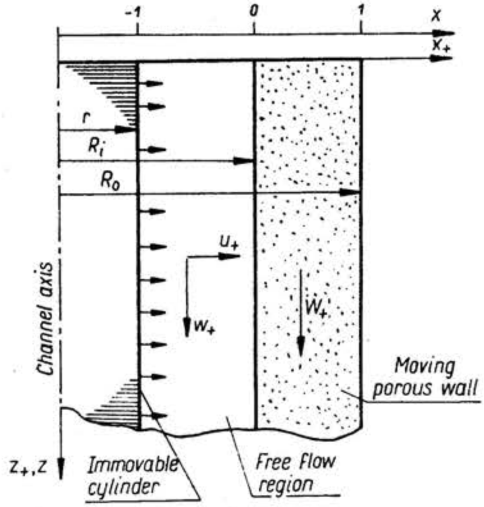

FIG. 1. Axial section of the channel.

case we determine this distribution from the filtration flow solution. The studies of the flows in porous channels are generally motivated by many different practical problems. Recently, for example, flow investigations in systems containing porous regions have been performed to evaluate hydrodynamic effects accompanied with the spinning processes of artificial fibres [14–20]. Some results of the present paper could also be applied to these problems.

The aim of this work is to determine the influence of the motion of the porous wall on the flow field in the channel described above. We assume that the fluid flux through the channel inlet is equal to zero. The channel is assumed to be long enough and we consider the case when the porous medium velocity changes slowly along the channel and, for a large distance from the channel's inlet, approaches some asymptotic value. Particular attention will be given to the plane flow between plane surfaces. This case may be considered as a limiting case of the annular flow when the radii of the cylinders tend to infinity.

Our analysis of the flow field is based on the assumption that the variations of the flow parameters along the channel are much smaller than their radial variations. Taking advantage of this assumption, we apply the method of small perturbations in searching for the flow parameters distributions in the region of a free flow (Fig. 1). The lowest order solutions are obtained (following BERMAN [6]) provided the longitudinal derivatives of the velocity components are neglected in the flow equations. The applied method allows us to determine afterwards the next approximation for the radial velocity component and to find the longitudinal pressure distribution that is undetermined in the lowest order solution. Our method is essentially based on the method introduced early by O. REYNOLDS [21] in his study of the lubrication flow in a slit.

In order to find the flow in the porous medium of the wall, we assume that the relationship between the filtration velocity (relative to the medium) and the pressure gradient is there linear according to Darcy's law. However, the pressure variability along the wall is assumed to be small in comparison with its variability across the wall. Hence the filtration velocity (with respect to the porous medium) is assumed to have only a transversal non-zero component [18]. The filtration properties of the porous medium are described by the filtration coefficient. The variations of this coefficient, in the case of a deformable medium, will be neglected here.

2. Flow equations and boundary conditions

Let r be the radius of the inner immovable cylinder and R_i and R_0 the radii of cylinders that bound the porous wall (Fig. 1). The fluid of density ϱ and viscosity μ is emitted radially from the inner cylinder with the constant flow rate Q per unit length along the channel axis. The velocity of the porous medium W_+, directed along the channel, approaches the value W_∞ at a large distance from the inlet. We assume that the pressure outside the channel is equal to p_0.

By u_+ and w_+ we denote the components of fluid velocity (in the free flow region) or the filtration velocity (in the porous region of the wall) respectively in the directions x_+ and z_+ of the cylindrical immovable coordinate system (x_+, z_+) with z_+ coincident with the channel axis. We have to mention that the filtration velocity, defined through the flow rate per the complete area of the cross section containing the porous medium, is lower than the mean flow velocity. The ratio of the filtration velocity and the mean flow velocity (the flow rate per the area of the pores) is equal to the porosity ε of the medium. On the other hand, the flow rate of the incompressible porous medium in one-dimensional motion considered remains constant in case of the deformation of the medium: $(1-\varepsilon)W_+ = \text{const}$.

We introduce the following dimensionless quantities:

$$\begin{gathered}
x = \begin{cases} (x_+ - R_i)/(R_i - r) & \text{for} \quad r \leqslant x_+ \leqslant R_i, \\ (x_+ - R_i)/(R_0 - R_i) & \text{for} \quad R_i < x_+ \leqslant R_0, \end{cases} \\
z = z_+/(R_i - r), \quad u = u_+/W_\infty, \quad w = w_+/W_\infty, \quad W = W_+/W_\infty, \\
p = (p_+ - p_0)(R_i - r)/(W_\infty \mu), \\
\alpha = (R_0 - R_i)/R_i, \quad \beta = (R_i - r)/R_i, \quad \gamma = \alpha/\beta, \\
\mathrm{Re}_w = \varrho(R_i - r) W_\infty/\mu, \quad \mathrm{Re}_u = \varrho Q(R_i - r)/(2\pi R_i \mu), \\
K = K_+/[(R_0 - R_i)(R_i - r)],
\end{gathered} \tag{2.1}$$

where p_+ is the dimensional pressure, Re_w and Re_u are the Reynolds numbers and K_+ is the dimensional filtration coefficient. K_+ depends weakly on the structure of the medium but its variation is here neglected.

The Navier-Stokes equations for the free flow region, $-1 < x < 0$, may be written as

$$\begin{gathered}
\frac{\partial[(1+\beta x)u]}{(1+\beta x)\partial x} = -\frac{\partial w}{\partial z}, \\
\frac{\partial p}{\partial x} + \mathrm{Re}_w u \frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial z}\right) + \frac{\partial^2 u}{\partial z^2} - \mathrm{Re}_w w \frac{\partial u}{\partial z}, \\
\frac{\partial p}{\partial z} + \mathrm{Re}_w u \frac{\partial w}{\partial x} - \frac{1}{1+\beta x}\frac{\partial}{\partial x}\left[(1+\beta x)\frac{\partial w}{\partial x}\right] = \frac{\partial^2 w}{\partial z^2} - \mathrm{Re}_w w \frac{\partial w}{\partial z}.
\end{gathered} \tag{2.2}$$

For the porous region, $0 < x < 1$, the system of the flow equations contains the continuity equation

$$\frac{\partial[(1+\alpha x)u]}{(1+\alpha x)\partial x} = -\gamma \frac{\partial w}{\partial z}, \tag{2.3}$$

and the equations of filtration in the radial and axial directions. Here u and w are the filtration velocity components. With the axial filtration velocity component (relative to the medium) assumed to be zero, [18], the filtration equations are

$$\begin{aligned} u &= -K\frac{\partial p}{\partial x}, \\ w &= W\varepsilon. \end{aligned} \tag{2.4}$$

The solutions, u, w, p of Eqs. (2.2)–(2.4) have to be continuous functions of the coordinates x, z and have to satisfy the following boundary and compatibility conditions:

$$\begin{aligned} x = -1: &\quad u = Q/(2\pi r W_\infty) = U, \quad w = 0; \\ x = 0: &\quad u_{\mathrm{I}} = u_{\mathrm{II}}, \quad w_{\mathrm{I}} = w_{\mathrm{II}}/\varepsilon, \quad p_{\mathrm{I}} = p_{\mathrm{II}}; \\ x = 1\ : &\quad p = 0, \end{aligned} \tag{2.5}$$

where the indexes I and II denote the free flow region and the wall region respectively.

3. Method of solution

At first let us introduce some auxiliary functions defined as follows:

$$\begin{aligned} f(x,\beta,q) &= \frac{1}{\beta}[(1+\beta x)^{q/\beta}-(1-\beta)^{q/\beta}], \\ F(x,\beta,q) &= \frac{f(x,\beta,q)}{f(0,\beta,q)} = \frac{(1+\beta x)^{q/\beta}-(1-\beta)^{q/\beta}}{1-(1-\beta)^{q/\beta}}, \\ \hat{F}(x,\beta,q) &= \frac{1}{1+\beta x}\int\limits_{-1}^{x}(1+\beta\zeta)F(\zeta,\beta,q)d\zeta \\ &= \frac{1}{f(0,\beta,q)}\left[\frac{f(x,\beta,q+2\beta)}{q+2\beta}-(1-\beta)^{q/\beta}\frac{f(x,\beta,2\beta)}{2\beta}\right]\frac{1}{1+\beta x}, \\ G(x,\beta,q) &= \frac{\hat{F}(x,\beta,2\beta)-\hat{F}(x,\beta,q)}{\hat{F}(0,\beta,2\beta)-\hat{F}(0,\beta,q)}, \\ H(x,\beta,q) &= \frac{\hat{F}(x,\beta,2\beta)\hat{F}(0,\beta,q)-\hat{F}(x,\beta,q)\hat{F}(0,\beta,2\beta)}{\hat{F}(0,\beta,2\beta)-\hat{F}(0,\beta,q)}. \end{aligned} \tag{3.1}$$

For the particular case of a plane flow ($\beta \to 0$) these functions are

$$f(x,0,q) = e^{qx}-e^{-q}, \tag{3.2}$$

$$F(x,0,q) = \frac{e^{qx}-e^{-q}}{1-e^{-q}}, \quad F(x,0,0) = x+1,$$

(3.2) [cont.]
$$\hat{F}(x,0,q) = \frac{1}{q}\frac{e^{qx}-e^{-q}}{1-e^{-q}} - \frac{x+1}{e^{q}-1}, \quad \hat{F}(x,0,0) = \frac{(x+1)^2}{2},$$

$$G(x,0,q) = \frac{\hat{F}(x,0,0)-\hat{F}(x,0,q)}{\hat{F}(0,0,0)-\hat{F}(0,0,q)} = \frac{(x+1)q[(x+1)(e^{q}-1)+2]+2[1-e^{q(x+1)}]}{q(e^{q}+1)+2(1-e^{q})},$$

$$G(x,0,0) = (x+1)^2(1-2x),$$

$$H(x,0,q) = \frac{\hat{F}(x,0,0)\hat{F}(0,0,q)-\hat{F}(x,0,q)\hat{F}(0,0,0)}{\hat{F}(0,0,0)-\hat{F}(0,0,q)}$$

$$= \frac{(x+1)[(x+1)e^{q}-x(q+1)]-x-e^{q(x+1)}}{q(e^{q}+1)+2(1-e^{q})},$$

$$H(x,0,0) = -x(x+1)^2.$$

The lowest order solutions of the system (2.2) for the free flow region are obtained provided the velocity components do not depend on z [6]. In this case all right-hand side terms in Eqs. (2.2) are equal to zero and these solutions are [6]

(3.3)
$$\bar{u} = U\frac{1-\beta}{1+\beta x},$$

(3.4)
$$w = WF(x,\beta,\mathrm{Re}_u)+P'\frac{2-\beta}{2(\mathrm{Re}_u-2\beta)}[F(x,\beta,\mathrm{Re}_u)-F(x,\beta,2\beta)],$$

$$p = -U\frac{\mathrm{Re}_u}{2}\frac{1-\beta}{(1+\beta x)^2}+P(z).$$

(By prime we denote the derivatives with respect to z).

It can be stated that the solutions (3.3) and (3.4) would satisfy Eqs. (2.2) exactly, provided the quantities U, W and P' are constant.

In the applied method we admit now a slight dependence of W and P' on z. Moreover, we take into account a small perturbation of the radial component of velocity in the free flow region, i.e. we put $u = \bar{u}+\tilde{u}$, where the basic solution $\bar{u}$ is determined by Eq. (3.3) and $\tilde{u}$ is a perturbation term.

From the continuity equation in Eq. (2.2), our approximation for the u component that satisfies the condition $u = U$ for $x = -1$ can be found as

(3.5)
$$u = U\frac{1-\beta}{1+\beta x} - W'\hat{F}(x,\beta,\mathrm{Re}_u)+P''\frac{2-\beta}{2(\mathrm{Re}_u-2\beta)}[\hat{F}(x,\beta,2\beta)-\hat{F}(x,\beta,\mathrm{Re}_u)]$$

for $-1 < x < 0$.

The solutions of Eqs. (2.3) and (2.4) for the porous region, $0 < x < 1$, satisfying the following boundary conditions in the relations (2.5): $x = 0$: $w_{\mathrm{I}} = w_{\mathrm{II}}/\varepsilon$, $p_{\mathrm{I}} = p_{\mathrm{II}}$; $x = 1$: $p = 0$, can be written in the following form:

(3.6)
$$u = \left(P-U\frac{(1-\beta)\mathrm{Re}_u}{2}\right)\frac{K\alpha}{(1+\alpha x)\ln(1+\alpha)}$$

$$+(\varepsilon W)'\frac{\alpha}{2\beta}\frac{1}{1+\alpha x}\left[\frac{2+\alpha}{2\ln(1+\alpha)}-\frac{(1+\alpha x)^2}{\alpha}\right],$$

(3.6) [cont.]

$$w = W\varepsilon,$$

$$p = \left(P - U\frac{(1-\beta)\mathrm{Re}_u}{2}\right)\left[1 - \frac{\ln(1+\alpha x)}{\ln(1+\alpha)}\right] + (\varepsilon W)'\frac{1}{4\beta K}\left[x(2+\alpha x) - (2+\alpha)\frac{\ln(1+\alpha x)}{\ln(1+\alpha)}\right].$$

In the solutions (3.4)–(3.6) we can distinguish the functions that do not depend on z. These functions are multiplied by the constant U or variable quantities W and P (or their derivatives) that depend only on the z coordinate. In the present investigation $W(z)$ has to be specified but $P(z)$ is the function we are searching for.

The function $P(z)$ characterizes the longitudinal pressure variation in the channel. This function or its derivatives enter also in expressions for the velocity components. In this way it describes the influence of the longitudinal pressure nonuniformity on the velocity field. The unknown function $P(z)$ can be found from the differential equation derived from the condition (in the relations (2.5)) concerning the continuity of the radial component of velocity at $x = 0$.

This condition has not been used until now. Setting $u_{\mathrm{I}} = u_{\mathrm{II}}$ for $x = 0$ and taking advantage of the mass conservation law of the porous medium to obtain $W' = (\varepsilon W)'$, we have the following equation for $P(z)$:

$$\delta^2 P'' - P = \omega W' - \theta U, \tag{3.7}$$

where

$$\delta^2 = \frac{(2-\beta)\ln(1+\alpha)}{2K\alpha(\mathrm{Re}_u - 2\beta)}[\hat{F}(0, \beta, 2\beta) - \hat{F}(0, \beta, \mathrm{Re}_u)],$$

(3.8)
$$\omega = \{2\ln(1+\alpha)\,[2\beta F(0, \beta, \mathrm{Re}_u) - 1] + \alpha(2+\alpha)\}/(4K\alpha\beta),$$

$$\theta = \frac{1-\beta}{2}\left[2\frac{\ln(1+\alpha)}{K\alpha} - \mathrm{Re}_u\right].$$

Introducing the function $\Pi(z)$ defined as

$$\Pi = P - \theta U, \tag{3.9}$$

Eq. (3.7) may be transformed into a simpler form:

$$\delta^2 \Pi'' - \Pi = \omega W'. \tag{3.10}$$

The solution $\Pi(z)$ of Eq. (3.10) may be determined provided the movement and deformation of the porous medium is known, i.e. the function $W(z)$ is given. The analysis of Eq. (3.10) for particular functions $W(z)$ and the boundary conditions will be given in Sect. 5.

4. Solution of the problem

With the help of Eq. (3.10) we may eliminate $P'' = \Pi''$ from Eq. (3.5). Also, the function Π defined through Eq. (3.9) may be introduced into the formulas (3.4) and (3.6). In this way the solutions describing the flow field in the channel both in the region of free flow, $-1 < x < 0$, and in the porous wall, $0 < x < 1$, can be presented in a uniform compact form:

$$
\begin{aligned}
u &= W'U_1 + \Pi U_2 + UU_3,\\
w &= \varepsilon W V_1 + \Pi' V_2,\\
p &= W'P_1 + \Pi P_2 + UP_3.
\end{aligned}
\tag{4.1}
$$

(For the free flow region the porosity is equal to 1).

The auxiliary functions $U_{1,2,3}$, $V_{1,2}$ and $P_{1,2,3}$ in Eqs. (4.1) do not depend on z and, for the cylindrical channel, are defined as follows (the upper expressions are for the free flow region, $-1 < x < 0$, the lower ones for the porous wall, $0 < x < 1$):

$$
U_1 = \begin{cases} \dfrac{\alpha(2+\alpha) - 2\ln(1+\alpha)}{4\beta\ln(1+\alpha)} G(x, \beta, \mathrm{Re}_u) + H(x, \beta, \mathrm{Re}_u), \\[2mm] \dfrac{\alpha(2+\alpha) - 2(1+\alpha x)^2 \ln(1+\alpha)}{4\beta\ln(1+\alpha)}, \end{cases}
$$

$$
U_2 = \begin{cases} \dfrac{K\alpha}{\ln(1+\alpha)} G(x, \beta, \mathrm{Re}_u), \\[2mm] \dfrac{K\alpha}{\ln(1+\alpha)} \dfrac{1}{1+\alpha x}, \end{cases} \qquad U_3 = \begin{cases} \dfrac{1-\beta}{1+\beta x}, \\[2mm] \dfrac{1-\beta}{1+\alpha x}, \end{cases}
$$

$$
V_1 = \begin{cases} F(x, \beta, \mathrm{Re}_u), \\ 1, \end{cases} \qquad V_2 = \begin{cases} \dfrac{2-\beta}{2(\mathrm{Re}_u - 2\beta)} [F(x, \beta, \mathrm{Re}_u) - F(x, \beta, 2\beta)], \\[2mm] 0, \end{cases}
\tag{4.2}
$$

$$
P_1 = \begin{cases} 0, \\[1mm] \dfrac{1}{4K\beta}\left[x(2+\alpha x) - (2+\alpha)\dfrac{\ln(1+\alpha x)}{\ln(1+\alpha)}\right], \end{cases} \qquad P_2 = \begin{cases} 1, \\[1mm] 1 - \dfrac{\ln(1+\alpha x)}{\ln(1+\alpha)}, \end{cases}
$$

$$
P_3 = \begin{cases} (1-\beta)\left[\dfrac{\ln(1+\alpha)}{K\alpha} + \dfrac{\mathrm{Re}_u}{2}\dfrac{(1+\beta x)^2 - 1}{(1+\beta x)^2}\right], \\[2mm] \dfrac{1-\beta}{K\alpha}[\ln(1+\alpha)/(1+\alpha x)]. \end{cases}
$$

5. Discussion

The expressions (4.1), describing the flow field in the channel, have been found in the form of sums of products. In these products one of the factors is a quantity that may depend only on z while the second factor (auxiliary function from Eqs. (4.2)) depends only on x. Such a form of expressions (4.1) gives an opportunity to analyse separately the radial and the longitudinal characteristics of fluid motion in the channel.

With the functions $W(z)$ and $\Pi(z)$ still undetermined, we first analyse the influence of different factors on the radial distributions of flow parameters. It may be considered that the functions U_3 and P_3 in Eqs. (4.2) take into account the effect of the forced flow from the inner cylinder. The direct influence of the velocity W of the porous medium on the radial distributions is characterized by the functions U_1, V_1 and P_1. In the same way the modifications of the radial distributions introduced by the functions U_2, V_2 and P_2 can be interpreted as the effect of the pressure nonuniformity along the channel. This nonuniformity may arise from the deformations of the porous medium or it may be the consequence of the conditions at the channel's inlet.

When the thickness (R_0-R_i) of the porous wall and the width (R_i-r) of the free flow region are much smaller than the radius R_i, the flow conditions in the channel are then close to the conditions of a plane flow. The asymptotic case, a plane flow, can be obtained when the parameters α, β tend to zero with γ being finite. For this case the functions (4.2) may be presented in a much simplified form:

$$U_1=\begin{cases}\frac{\gamma}{2}G(x,0,\mathrm{Re}_u)+H(x,0,\mathrm{Re}_u)\\ \frac{\gamma}{2}(1-2x)\end{cases},\quad U_2=\begin{cases}KG_1(x,0,\mathrm{Re}_u)\\ K\end{cases},\quad U_3=\begin{cases}1\\ 1\end{cases},$$

$$V_1=\begin{cases}F(x,0,\mathrm{Re}_u)\\ 1\end{cases},\quad V_2=\begin{cases}[F(x,0,\mathrm{Re}_u)-F(x,0,0)]/\mathrm{Re}_u\\ 0\end{cases},\tag{5.1}$$

$$P_1=\begin{cases}0\\ \frac{\gamma}{4K}x(x-1)\end{cases},\quad P_2=\begin{cases}1\\ 1-x\end{cases},\quad P_3=\begin{cases}\frac{1}{K}\\ \frac{1-x}{K}\end{cases}.$$

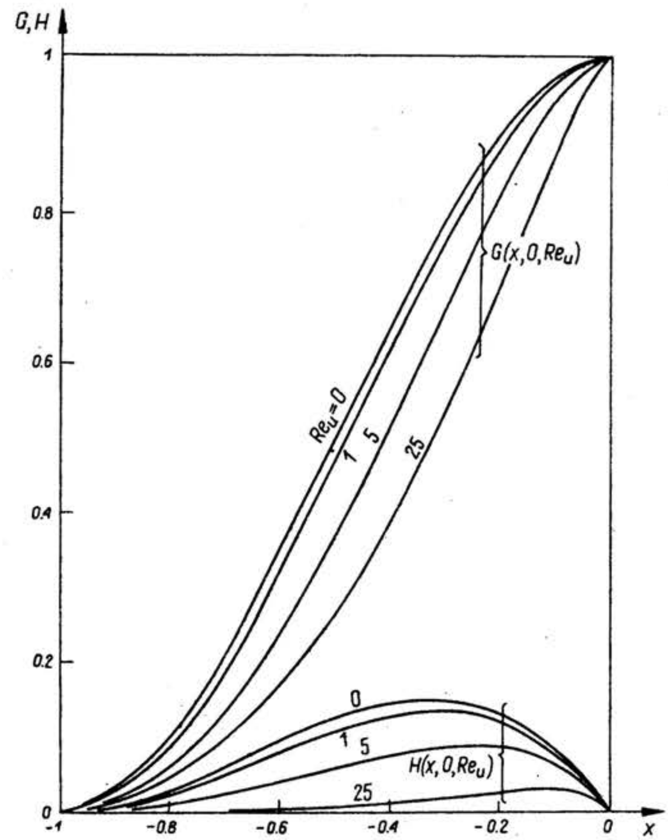

FIG. 2. Diagrams of functions $G(x,0,\mathrm{Re}_u)$ and $H(x,0,\mathrm{Re}_u)$.

The diagrams of the functions G and H, for the plane case, are presented in Fig. 2, the other auxiliary functions (5.1) are plotted against x for some values of Re_u, γ and K in Figs. 3–5.

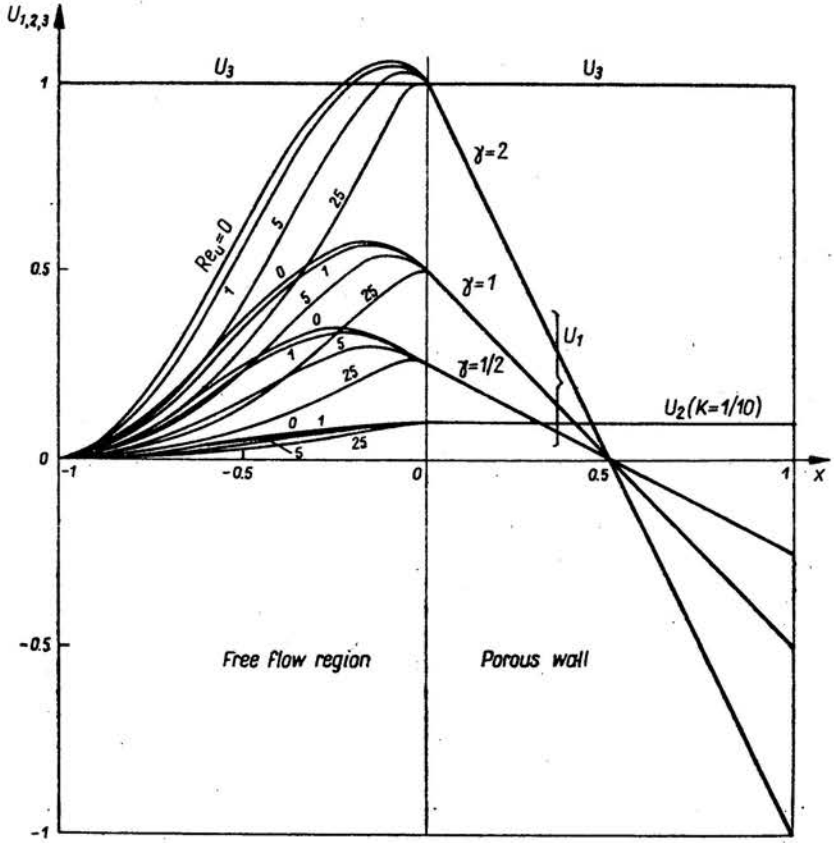

FIG. 3. Diagrams of auxiliary functions U_1, U_2, U_3 for a plane flow.

For $\mathrm{Re}_u \to 0$, in the free flow region, we obtain a superposition of the Couette flow ($V_1 = x+1$) due to the wall motion W and the Poiseuille flow $\left(V_2 = \frac{1}{4}x(x+1)\right)$ due to the existence of the pressure gradient $P' = \Pi'$ (Fig. 4). The outer blowing ($\mathrm{Re}_u > 0$) gives the velocity distribution described by the solution obtained partly in [6]. It approaches, for higher Re_u, almost uniform flow ($V_1 \approx 0$) with a thin boundary layer close to the porous moving wall. An increasing Re_u number gives also the reduction of the influence of the pressure gradient Π' (values of V_2 decrease) and leads to a strong deformation of the Poiseuille type flow.

The pressure does not change (in a plane case) across the free flow region decreasing linearly in the porous region due to the outer blowing ($U > 0$) and the pressure difference Π (Fig. 5). The deformation of the porous wall ($W' \neq 0$) may additionally provoke inside the wall suction with a parabolic pressure distribution. This suction is the cause of the linear distribution of the transversal velocity U_1 in the porous wall (Fig. 3).

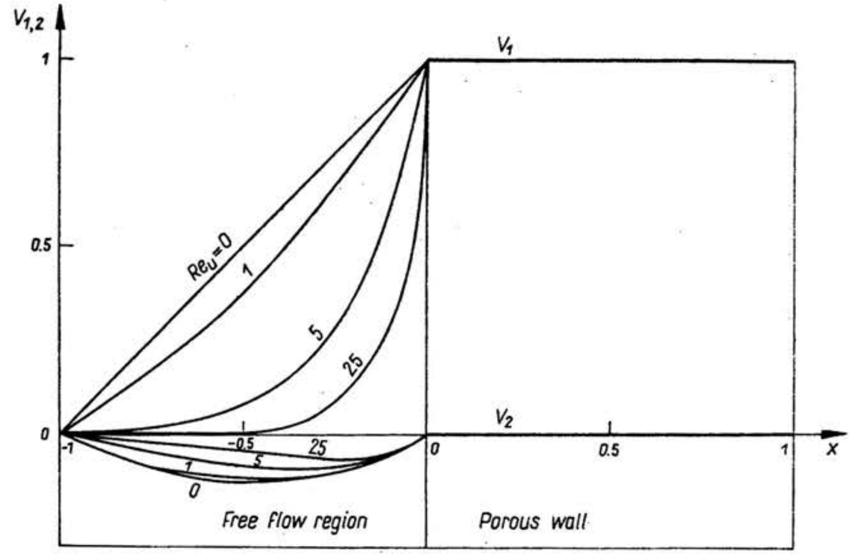

FIG. 4. Diagrams of auxiliary functions V_1 and V_2 for a plane flow.

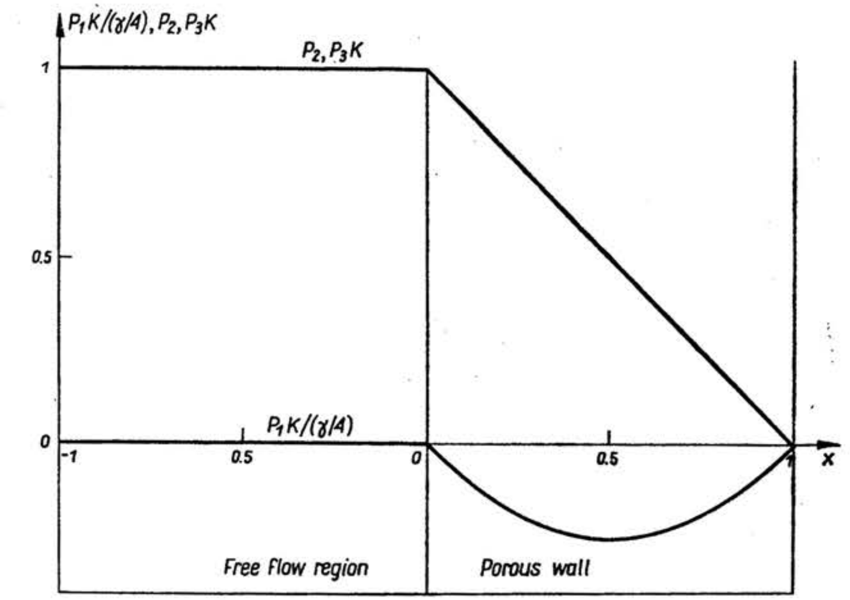

FIG. 5. Diagrams of auxiliary functions P_1, P_2 and P_3 for a plane flow.

From the diagrams of the functions U_2 and V_2 we may notice that for the free flow region the influence of the pressure nonuniformity along the channel on the radial distributions of flow parameters becomes smaller with the growing value of the Re_u number.

To complete the analysis of the flow field we have to specify the porous medium motion and to determine, from Eq. (3.10) and appropriate boundary conditions, the pressure distribution along the channel. One of the boundary conditions for Eq. (3.10) follows from the assumption that the fluid flux through the channel inlet, $-1 \leqslant x \leqslant 0, z = 0$, is equal to zero. Approximately, it is equivalent to the condition that no fluid comes through the inlet. Setting also that for $z \to \infty$ the pressure should be finite, $p < \infty$, we have

(5.2)
$$z = 0: \quad \int_{-1}^{0} w dx + \beta \int_{-1}^{0} wx dx = 0,$$
$$z \to \infty: \quad \Pi < \infty.$$

Two particular examples of the function $W(z)$ are considered, namely,

(5.3)
$$W_a = \text{const} = 1,$$
$$W_b = 1 - e^{-z/\lambda},$$

where for the second case λ may be treated as a dimensionless parameter characterizing the deformation length of the porous wall. For these particular functions (5.3), Eq. (3.10) with the conditions yields (5.2) the following distributions of the pressure, respectively:

(5.4)
$$\Pi_a = -\delta e^{-z/\delta},$$
$$\Pi_b = -\omega \frac{\delta^2}{\delta^2 - \lambda^2} (\delta e^{-z/\delta} - \lambda e^{-z/\lambda}).$$

It may be noticed that the quantity δ, in the same way as λ, can be treated as the characteristic dimensionless length of the distance of a region where the flow is strongly affected by the channel inlet conditions. It can also be noted that the ratio between δ and λ determines the length of the region where the pressure longitudinal nonuniformities are essential. With the value of δ decreasing with respect to λ, the effect of λ on the pressure distribution along the channel becomes dominant. Hence, for $\lambda \gg \delta$, the pressure distribution depends mainly on the characteristics of the deformation of the porous medium.

The value of the quantity δ decreases with the growing value of the filtration coefficient K and with the increasing value of Re_u. This means that both the growth of the porous medium permeability and the intensification of the forced flow from the inner cylinder decrease the effect of the channel inlet conditions.

In the asymptotic case, when $\text{Re}_u \gg 1$, the expression (5.4) for $\Pi_b(z)$ in a plane case may be simplified to the following:

(5.5)
$$\Pi = -\frac{\gamma}{2K} \frac{e^{-z/\lambda}}{\lambda} = -\frac{\gamma}{2K} W'.$$

In this case the nonuniformities of the pressure profile along the channel are related only with the deformation of the porous medium.

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