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Steady plane flow in a region between a porous wall and a system of moving rods

W. KALITA and J. LUBOŃSKI (WARSZAWA)

IN THIS PAPER we consider steady laminar plane flows of an incompressible viscous heat-conducting fluid and associated heat transfer in a semi-infinite channel between two parallel half-planes which is bounded by a plane perpendicular to those half-planes. From one of the half-planes the fluid is outflowing uniformly, with given velocity and temperature, into the channel. The second half-plane contains the regularly placed axes of straight parallel rods moving along their axes with a given velocity. The rods have a given initial temperature higher than that of the forced outflow. Our analysis of flow and heat transfer in the channel is based on the assumption of the small variability of flow parameters and temperature along the channel with respect to their transversal variability. The main results of this analysis concern the influence of the characteristics of rods motion and the forced outflow on the hydrodynamic and thermal conditions in the channel and the cooling of the rods.

W niniejszej pracy rozważane są ustalone laminarne płaskie przepływy nieściśliwego lepkiego i przewodzącego ciepło płynu oraz towarzysząca im wymiana ciepła w półnieskończonym kanale między dwoma równoległymi półpłaszczyznami, który ograniczony jest płaszczyzną prostopadłą do tych półpłaszczyzn. Z jednej z półpłaszczyzn jednorodnie, z daną prędkością i temperaturą, wpływa do kanału płyn. Druga półpłaszczyzna zawiera regularnie rozmieszczone osie prostych równoległych prętów poruszających się wzdłuż swych osi z daną prędkością. Pręty te mają daną początkową temperaturę wyższą niż temperatura płynu wypływającego z przeciwległej półpłaszczyzny. Analiza przepływu i wymiany ciepła opiera się na założeniu małej zmienności parametrów przepływowych i temperatury wzdłuż kanału w stosunku do zmienności tych wielkości w poprzek kanału. Główne wyniki analizy dotyczą wpływu charakterystyk ruchu prętów i wymuszonego wypływu na warunki hydrodynamiczne i cieplne w kanale oraz na przebieg chłodzenia prętów.

В настоящей работе рассматриваются установившиеся ламинарные плоские течения несжимаемой вязкой и теплопроводной жидкости, а также сопутствующий им теплообмен в полубесконечном канале, между двумя параллельными полуплоскостями, который ограничен плоскостью перпендикулярной к этим полуплоскостям. Из одной полуплоскости однородным образом, с данной скоростью и температурой, втекает в канал жидкость. Вторая полуплоскость содержит регулярно расположенные оси простых параллельных стержней, движущихся вдоль своих осей с данной скоростью. Эти стержни имеют данную начальную температуру более высокую чем температура жидкости вытекающей с противолежащей полуплоскости. Анализ течения и теплообмена опирается на предположению малого изменения параметров течения и температуры вдоль канала по отношению к изменениям этих величин поперек канала. Главные результаты анализа касаются влияния характеристик движения стержей и вынужденного вытекания на гидродинамические и тепловые условия в канале, а также на ход охлаждения стержней.

1. Introduction

THE OBJECT of the paper is a study of the laminar steady plane flows o fan incompressible viscous heat-conducting fluid and the thermal processes in a semi-infinite region between two parallel half-planes, (*1* and *2*), bounded from one side by the plane (*3*) perpendicular to those mentioned (see Fig. 1).

One of the parallel half-planes, (*1*), is an uniformly permeable wall and the uniform blow with given velocity and temperature penetrates from it into the region considered.

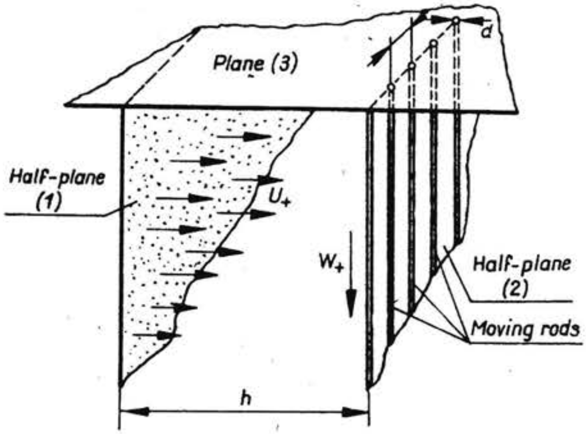


FIG. 1.

The second half-plane (*2*) contains the axes of the same thin straight rods. These axes are regularly placed and they are perpendicular to the plane (*3*).

All rods are moving along their axes outward the plane (*3*) with a given velocity which is the same for all rods in planes parallel to the plane (*3*). We consider the case when this velocity is variable along the rods (e.g. the rods may be stretched or compressed). The temperature of rods on the edge of the half-plane (*2*) is given and is higher than that of blow.

The problem stated above corresponds to a certain extent to problems concerning flow in channels with porous walls [1–15]. In the majority of these studies the porous walls of the channels were considered as immovable and non-deformable [1–12, 15]. In some of them the distribution of the velocity component transversal to the walls is a priori assumed [1–11, 15].

Studies of flows in channels having moving and deformable porous walls are particularly relevant to this paper.

In our study the half-plane (*2*) containing the axes of the rods is not to be treated as a porous wall with no slip of velocity, but as an uniformly permeable structure where the tangent component of velocity may differ from the local velocity of the rods.

The system of rods, the axes of which are placed in the half-plane (*2*), is assumed to have a given permeability for transversal flow. Despite the deformation of the rods, it is assumed that this permeability is constant.

The aim of this paper is to determine the effects the motion of the rods has on the flow field in the channel between the half-planes (*1* and *2*). We are also interested in determining the influence of the motion of rods and the resulting flow in the channel on the temperature distributions in the channel and especially along the rods themselves.

We consider the case when the velocity of the rods changes slowly along the channel and, for a large distance from the plane (*3*), approaches some asymptotic value. Also we assume that the temperature of the rods on the edge of the plane (*2*), i.e. their "inlet" temperature, is not much higher than the temperature of the blowing (on the half-plane *1*). Additionally, we assume that the variability of the flow parameters and the temperature along the channel is much weaker than their transversal variability.

The approximated method applied for the solution of the problem considered may be traced back to the early work of O. REYNOLDS [16] on lubrication flow in a narrow slit. This method is based on the asumption of small variability, both of the flow parameters and the temperature along the channel.

The conditions of flows analysed in the paper may be considered, for example, as a simplified model for so-called "melt spinning", one of the technologies exploited in the man-made fibers industry.

2. Governing equations

Following Fig. 2 we introduce the Cartesian immovable coordinate system (x_+, z_+) and denote the respective components of fluid velocity by u_+ and w_+. The fluid has

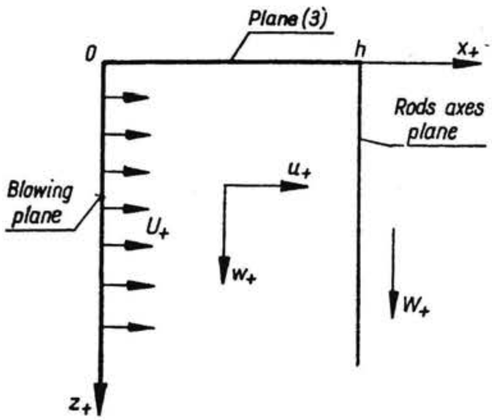


FIG. 2.

constant physical properties: density ϱ_f, viscosity μ, heat conductivity $\varkappa_f$ and capacity c_f. The fluid with the temperature T_{f0} is blowing with the velocity U_+ from the half-plane (*1*) uniformly and perpendicularly to it.

The rods have constant density ϱ_r, heat conductivity $\varkappa_r$ and capacity c_r. They are regularly placed with the distance l between their axes. The ratio of their diameter d and the distance l is assumed to be a small value. The velocity of the rods W_+ approaches the value W_∞ for $z_+ \to \infty$. From the mass conservation law for the rods, it follows that $\pi d^2 W_+/4 = Q_+$ is constant. The temperature of the rods $T_{r+}(z_+)$ is equal to T_{r0+} at the edge of the half-plane (*2*). For $x_+ > h$ (where h is the channel width) the pressure and the temperature are assumed to be constant and equal respectively to p_0 and $T_{r+}(h, z_+)$.

We introduce the following dimensionless quantities:

$$
\begin{aligned}
&x = x_+/h, \quad z = z_+/h;\\
&u = u_+/W_\infty, \quad w = w_+/W_\infty, \quad U = U_+/W_\infty, \quad W = W_+/W_\infty;\\
&p = (p_+ - p_0)h/(W_\infty \mu), \quad Q = Q_+/(W_\infty h^2);\\
&T_i = (T_{i+} - T_{f0})/T_{f0} \quad \text{where} \quad i = f\,(\text{fluid}) \quad \text{or} \quad r\,(\text{rods});\\
&\alpha = l/h, \quad \varepsilon = d/l, \quad \gamma = (c_f \varrho_f)/(c_r \varrho_r);\\
&\mathrm{Re}_w = \varrho_f W_\infty h/\mu, \quad \mathrm{Re}_u = \varrho_f U_+ h/\mu,\\
&\mathrm{Pe}_w = \mathrm{Re}_w \mathrm{Pr}, \quad \mathrm{Pe}_u = \mathrm{Re}_u \mathrm{Pr}, \quad \text{where} \quad \mathrm{Pr} = c_f \mu/\varkappa_f.
\end{aligned}
\tag{2.1}
$$

In the above definitions p_+ and T_{i+} are the dimensional pressure and temperature, Re_w and Re_u are the Reynolds numbers, Pr is the Prandtl number and Pe_w and Pe_u are the Peclet numbers.

The governing equations consist of the continuum, Navier-Stokes and energy equations. For the channel region, $0 \leqslant x \leqslant 1$, they may be written as follows:

$$
\begin{aligned}
&\frac{\partial u}{\partial x} = -\frac{\partial w}{\partial z},\\
&\frac{\partial p}{\partial x} + \mathrm{Re}_w u \frac{\partial u}{\partial x} = \Delta u - \mathrm{Re}_w w \frac{\partial u}{\partial z},\\
&\frac{\partial^2 w}{\partial x^2} - \mathrm{Re}_w u \frac{\partial w}{\partial x} - \frac{\partial p}{\partial z} = -\frac{\partial^2 w}{\partial z^2} + \mathrm{Re}_w w \frac{\partial w}{\partial z},
\end{aligned}
\tag{2.2}
$$

$$
\frac{\partial^2 T_f}{\partial x^2} - \mathrm{Pe}_w u \frac{\partial T_f}{\partial x} = \mathrm{Pe}_w w \frac{\partial T_f}{\partial z} - \frac{\partial^2 T_f}{\partial z^2} + \Phi.
\tag{2.3}
$$

The viscous terms, denoted by Φ, are to be neglected in the energy equation (2.3) as small in comparison with the others.

The solutions u, w, p and T_f of Eqs. (2.2) and (2.3) have to be the continuous functions of the coordinates (x, z) and have to satisfy the appropriate boundary conditions.

Since the fluid physical properties are assumed to be constant, the flow equations (2.2) can be solved independently of the energy equation (2.3). In the following chapters we will first determine the flow in the channel and the solutions obtained will be used next in the analysis of the thermal effects.

3. Flow determination

The boundary conditions for the flow equations (2.2) are formulated in the following way. For the half-plane (*1*), uniformly permeable for the blowing, we have:

$$
x = 0: \quad u = U, \quad w = 0.
\tag{3.1}
$$

Formulating the conditions for the half-plane (*2*), we should mention that here we shall not search for the solutions of flow equations describing the local flow in the direct vicinity of the rods. As the system of the rods has a given permeability for the transversal flow and, in general, the longitudinal component of flow velocity w on the half plane (2) differs from the rods velocity W, we postulate:

$$(3.2)\qquad x=1:\begin{cases} u = \alpha k_2 p, \\ w-W = \alpha k_3\left(\left.\dfrac{\partial w}{\partial x}\right|_{x=1+0} - \left.\dfrac{\partial w}{\partial x}\right|_{x=1-0}\right). \end{cases}$$

The condition (3.2) postulates the proportionality of u to the difference of the pressures on both sides of the half-plane (*2*). The condition (3.2) determines the slip ($w-W$) as the quantity proportional to the difference of the derivatives of the longitudinal component of flow velocity on both sides of the half-plane (*2*), the flow velocity being continuous on the half-plane (*2*).

On the basis of the consideration of the flow in the channel close to the half-plane (*2*) and for $x > 1$, we may assume additionally that the variability of the w component of velocity is much weaker outside the channel. Hence the first term in the parentheses in Eq. (3.2) will be neglected with respect to the last one. This last condition may be referred to the Beavers-Joseph slip condition [17] on the boundary of the porous medium.

The coefficients k_2 and k_3 in Eqs. (3.2) are the given [18], weakly-variable functions of ε, i.e. these coefficients depend only on the structure of a system of the rods.

The boundary condition on the plane (*3*) follows the assumption that the fluid flux through this plane into the region between the half-planes (*1* and *2*), $0 \leqslant x \leqslant 1$, is equal to zero. Approximately, it is equivalent to the condition that no fluid is coming through the boundary of the channel. As $z \to \infty$, the pressure should be finite, so the last flow boundary conditions are as follows:

$$(3.3)\qquad \begin{aligned} z = 0&: \quad \int_0^1 w\,dx = 0, \\ z \to \infty&: \quad p < \infty. \end{aligned}$$

Now let us introduce two auxiliary functions $f(x, \zeta)$ and $F(x, \zeta)$ that will be convenient in describing the solutions of the problem stated above. These functions depend on the x coordinate and the parameters ζ and αk_3 and are defined as

$$(3.4)\qquad \begin{aligned} f(x,\zeta) &= \frac{e^{\zeta x}-1}{e^{\zeta}(1+\alpha k_3\zeta)-1}, \\ F(x,\zeta) &= \int_0^x f(x,\zeta)dx = \frac{1}{\zeta}\,\frac{e^{\zeta x}-1-\zeta x}{e^{\zeta}(1+\alpha k_3\zeta)-1}. \end{aligned}$$

For the particular $\zeta \to 0$ these functions may be presented as follows:

$$(3.5)\qquad \begin{aligned} f(x,0) &= \frac{x}{1+\alpha k_3}, \\ F(x,0) &= \frac{x^2}{2(1+\alpha k_3)}. \end{aligned}$$

The method we applied for the solution of the flow problem is based on the assumption of a small variability of the flow velocity components along the z axis in comparison with their variability in the x direction.

In the first step we assume that u, w, and W do not depend on z and $\frac{\partial p}{\partial z} \equiv P'$ is constant ($P(z)$ is considered in this step as a linear function of z). Then the right hand sides of Eqs. (2.2) are equal to zero and the solutions of these equations, satisfying the conditions (3.1) and (3.2), are denoted by a bar as "basic" solutions:

$$\begin{aligned} \bar{u} &= U, \\ \bar{w} &= Wf(x, \mathrm{Re}_u) + P' \frac{1+\alpha k_3}{\mathrm{Re}_u} [f(x, \mathrm{Re}_u) - f(x, 0)], \\ \bar{p} &= P. \end{aligned} \tag{3.6}$$

In the next step we admit a slight dependence of W and P' on z and we take into account a small perturbation of the u component of velocity. Putting that $u = \bar{u} + \tilde{u}$, where $\tilde{u}$ is the perturbation, the perturbation term may be obtained from the continuity equation in the relations (2.2), with the condition $\tilde{u} = 0$ on $x = 0$, as:

$$\tilde{u} = -W'F(x, \mathrm{Re}_u) + P'' \frac{1+\alpha k_3}{\mathrm{Re}_u} [F(x, 0) - F(x, \mathrm{Re}_u)], \tag{3.7}$$

where by primes we denote the derivatives with respect to z.

In this way the solution for the transversal component of velocity is given by the formulas $(3.6)_1$ and (3.7). The solutions for the w component and the pressure are formally given by Eq. $(3.6)_2$ but with the quantities W and P' that are no longer constant and are still undetermined. The rods' velocity $W(z)$ has to be specified and the pressure function $P(z)$ should be found. This last function may be determined by the equation derived from the condition (3.2) that was not exploited until now, where $u = \bar{u} + \tilde{u}$:

$$\frac{1+\alpha k_3}{\mathrm{Re}_u} [F(1, 0) - F(1, \mathrm{Re}_u)] P'' - \alpha k_2 P = F(1, \mathrm{Re}_u) W' - U. \tag{3.8}$$

Introducing the function $\Pi(z)$ defined as

$$\Pi = P - \frac{U}{\alpha k_2}, \tag{3.9}$$

Eq. (3.8) determining the pressure distribution along the z coordinate may be transformed to:

$$\beta^2 \Pi'' - \Pi = \omega W', \tag{3.10}$$

where

$$\begin{aligned} \beta^2 &= (1+\alpha k_3)[F(1, 0) - F(1, \mathrm{Re}_u)]/(\alpha k_2 \mathrm{Re}_u), \\ \omega &= F(1, \mathrm{Re}_u)/(\alpha k_2). \end{aligned} \tag{3.11}$$

(It may be noticed that $\omega \geqslant 0$ for any $\mathrm{Re}_u \geqslant 0$).

With the help of Eq. (3.10) we can eliminate ($P'' = \Pi''$) from Eq. (3.7) and the final solutions describing the flow field in the region considered may be presented in the following form:

$$\begin{aligned} u &= U + U_1 W' + U_2 \Pi, \\ w &= \qquad V_1 W + V_2 \Pi', \\ p &= U/(\alpha k_2) + \Pi, \end{aligned} \tag{3.12}$$

where four auxiliary functions $U_{1,2}$ and $V_{1,2}$, independent of z, are defined as follows:

$$(3.13)\qquad \begin{aligned} U_1 &= \frac{F(x,0)F(1,\mathrm{Re}_u)-F(x,\mathrm{Re}_u)F(1,0)}{F(1,0)-F(1,\mathrm{Re}_u)},\\ U_2 &= \alpha k_2\frac{F(x,0)-F(x,\mathrm{Re}_u)}{F(1,0)-F(1,\mathrm{Re}_u)},\\ V_1 &= f(x,\mathrm{Re}_u),\\ V_2 &= (1+\alpha k_3)\frac{f(x,\mathrm{Re}_u)-f(x,0)}{\mathrm{Re}_u}. \end{aligned}$$

These auxiliary functions (3.13) characterize the profiles of the velocity component across the channel. In Figs. 3 and 4 they have been plotted against x for some values of Re_u and αk_3.

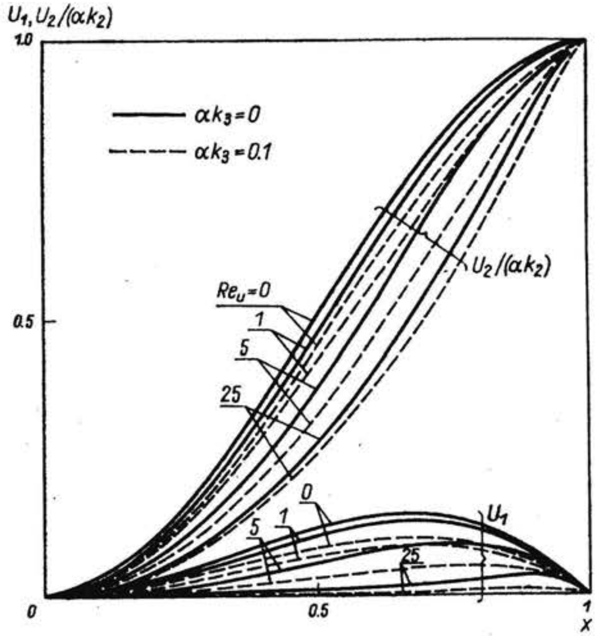


FIG. 3.

For very small values of the blowing velocity, $\mathrm{Re}_u \to 0$, and the no-slip condition, $k_3 = 0$, the distribution of the w component in the channel is given by the superposition of the Couette flow, $V_1 = x$, due to the rods' motion and the Poisseuille flow, $V_2 = \frac{1}{2}x(x-1)$, due to the existence of the pressure gradient $P' = \Pi'$.

Taking into account the slip $(w-W)$ on the rods' plane yields the deformation of this flow picture. In the case of $k_3 > 0$ and $\mathrm{Re}_u \to 0$, the w-component profile is described by the superposition of the Couette flow: $V_1 = x/(1+\alpha k_3)$, and the deformed Poisseuille flow:

$$V_2 = 1/2x\left(x-\frac{1+2\alpha k_3}{1+\alpha k_3}\right).$$

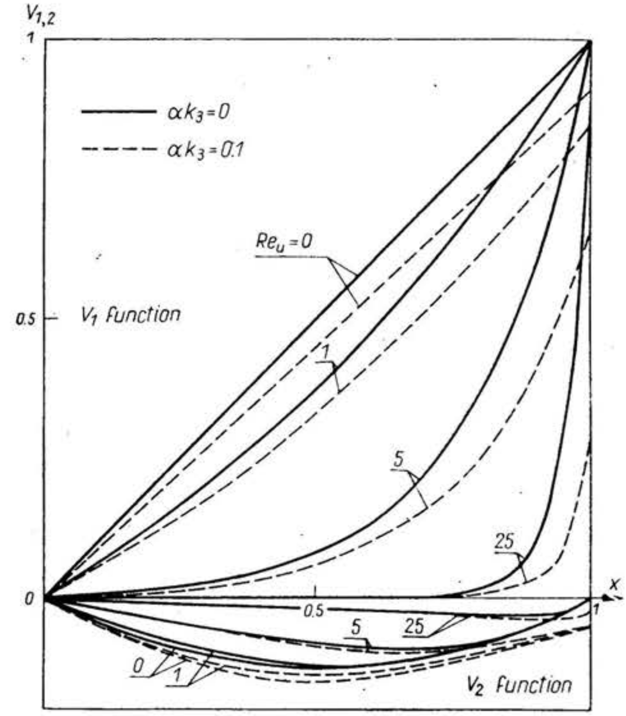


FIG. 4.

Along with the intensification of blowing (when the Re_u number increases), the formation of a flow layer near the rods may be observed. The thickness of the layer, where the intensity of flow is greatest, decreases with the growing values of the Re_u number.

Something of special interest is the influence of the slip coefficient αk_3. When this coefficient grows, the longitudinal component of velocity strongly decreases. The values of the functions U_1 and U_2 that determine the transversal component of velocity reduce in the non-zero slip case ($\alpha k_3 > 0$) only slightly. It means that if the slip of velocity on the rods' axes plane is taken into account, the streamlines may change their directions radically. Then the flow may penetrate better into the system of the rods.

In order to complete the analysis of the flow, we have to find the pressure distribution along the channel. For two particular examples of the rods' velocity function $W(z)$ chosen as

$$W_I = \mathrm{const} = 1, \tag{3.14}$$
$$W_{II} = 1 - \mathrm{e}^{-z/\lambda},$$

Eq. (3.10) together with the conditions (3.3) yields the following distributions of pressure, respectively:

$$\Pi_I = -\beta e_{\Delta}^{-z/\beta}, \tag{3.15}$$
$$\Pi_{II} = -\omega \frac{\beta^2}{\beta^2 - \lambda^2} (\beta e^{-z/\beta} - \lambda e_{\Delta}^{-z/\lambda}).$$

The quantity λ is the parameter that describes the rods' deformation. It may be noticed that the quantities β and λ can be treated as some characteristic lengths, respectively, of the distance of a region where flow is effectively influenced by the channel boundary (plane *3*) and the distance of the intensive deformation of the rods. It can also be found that when β decreases with respect to λ (in the second case of the formulae (3.14)), the non-uniformities of the pressure distribution along the channel become more affected by the rods' deformation. As β is defined through Eq. (3.11), it may be seen that its values decrease with the blowing intensification (Re_u increasing), with the growing permeability αk_2 of the rods' system and with the slip coefficient (αk_3) decreasing. All these tendencies reduce the influence of the channel boundary on the pressure distribution.

4. Thermal effects

Analysis of the thermal effects consists in determining the fluid temperature field in the channel and the temperature distribution along the rods themselves. There exists an evident coupling between the fluid temperature field and the variability of rods' temperature. Due to this coupling, it is necessary to determine simultaneously the fluid and the rods' temperature fields. The system of the equations determining these fields consists of the energy equation for the fluid (2.3) and the rods' heat balance equation. The coupling mentioned above appears in these equations and also in appropriate boundary conditions.

The rods' heat balance equation is derived on the basis of the following simplifying assumptions. First of all, we will not consider the effects due to radiation or to any physical and chemical processes that may have an influence on the heat balance of rods. Next we assume that the effects of heat conductivity along the rods are neglectable as they are small with respect to heat transfer due to the rods mass transport. However, in the cross section of each rod the temperature is assumed to be uniform and equal to the rod surface temperature. According to the above assumptions, we may write the approximate heat balance for the fixed surface element ldz_+ of the rods' axes half-plane (*2*) (Fig. 1) in the following form:

$$Q_+\varrho_r c_r dT_{r+} = -\varkappa_f \frac{\partial T_{f+}}{\partial x_+}\bigg|_{x_+=h-0}, \tag{4.1}$$

where dT_{r+} denotes the change of T_{r+} on the segment $(z_+, z_+ + dz_+)$.

The left hand side of this equality presents the difference between the heat input and output to and from the element ldz_+ due to the rods' movement. The right hand side gives the heat losses of the element ldz_+ resulting from heat exchange between the rods and the fluid. From Eq. (4.1) we have the differential equation (written below in the dimensionless form) describing the variability of temperature along the rods:

$$\frac{dT_r}{dz} = -\frac{\alpha\gamma}{Q\mathrm{Pe}_w}\frac{\partial T_f}{\partial x}\bigg|_{x=1-0}. \tag{4.2}$$

The boundary condition for this equation has the form

$$z = 0: \quad T_r = T_{r0}. \tag{4.3}$$

According to our main assumption, about the small variability of the temperature along the channel in comparison with its transversal variability, the derivatives with respect to z in Eq. (2.3) will be neglected. Then the simplified energy equation (2.3) is

$$\frac{\partial^2 T_f}{\partial x^2} - \mathrm{Pe}_w u \frac{\partial T_f}{\partial x} = 0. \tag{4.4}$$

The variable z is considered here as a parameter, and thus the sufficient boundary conditions for Eq. (4.4) have the following form

$$\begin{aligned} x = 0: &\quad T_f(0, z) = 0, \\ x = 1: &\quad T_f(1, z) = T_r(z) - \alpha k_3 \frac{\partial T_f}{\partial x}\bigg|_{x=1-0}. \end{aligned} \tag{4.5}$$

The second of these conditions is a result of the postulate [18] which claims that in the half-plane (2) there exists a difference between the mean fluid temperature and the rods temperature ("temperature jump"). The exact solution of Eq. (4.4) satisfying the condition (4.5) is

$$T_f(x, z) = T_r(z) H(x, z)\, [I_1(z) + \alpha k_3 I_2(z)]^{-1}, \tag{4.6}$$

where

$$\begin{gathered} H_1(x, z) = \int_0^x H_2(\bar{x}, z)\, d\bar{x}, \quad H_2(x, z) = \exp\left\{\mathrm{Pe}_w \int_0^x u(\bar{x}, z)\, d\bar{x}\right\}, \\ I_1(z) = H_1(1, z), \quad I_2(z) = H_2(1, z). \end{gathered} \tag{4.7}$$

It may be noticed that the expression (4.6) for T_f has until now only a formal character because it contains the unknown function $T_r(z)$.

However, this last one can be found from Eq. (4.2) with the condition (4.3), after substituting to the right hand side of Eq. (4.2) the expression of $\partial T_f / \partial x|_{x=1-0}$ obtained on the basis of Eq. (4.6). Using this expression, one obtains immediately T_r in the form

$$T_r(z) = T_{r0} \exp \int_0^z K(\bar{z})\, d\bar{z}, \tag{4.8}$$

where

$$K(z) = -\frac{\alpha\gamma}{Q\mathrm{Pe}_w} I_2(z) [I_1(z) + \alpha k_3 I_2(z)]^{-1} \leqslant 0. \tag{4.9}$$

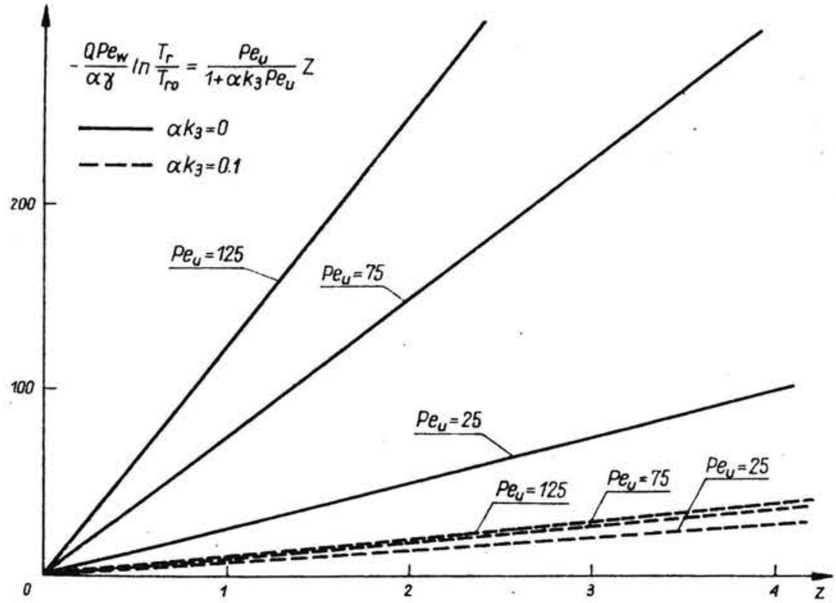
The formulae (4.6) and (4.9) taken together give the complete solution of the problem stated in frames of our simplifying assumptions.

In the following we consider the asymptotic case of a large Peclet number ($\mathrm{Pe}_u \gg 1$). It may be noticed that with growing Pe_u the degree of accuracy of heat balance (4.1) becomes higher. The asymptotic expressions for T_r and T_f have the following relatively simple forms:

$$T_r(z) = T_{r0} \exp\left\{-\frac{\alpha\gamma}{Q\mathrm{Pe}_w} \frac{\mathrm{Pe}_u}{1 + \alpha k_3 \mathrm{Pe}_u}\right\}, \tag{4.10}$$

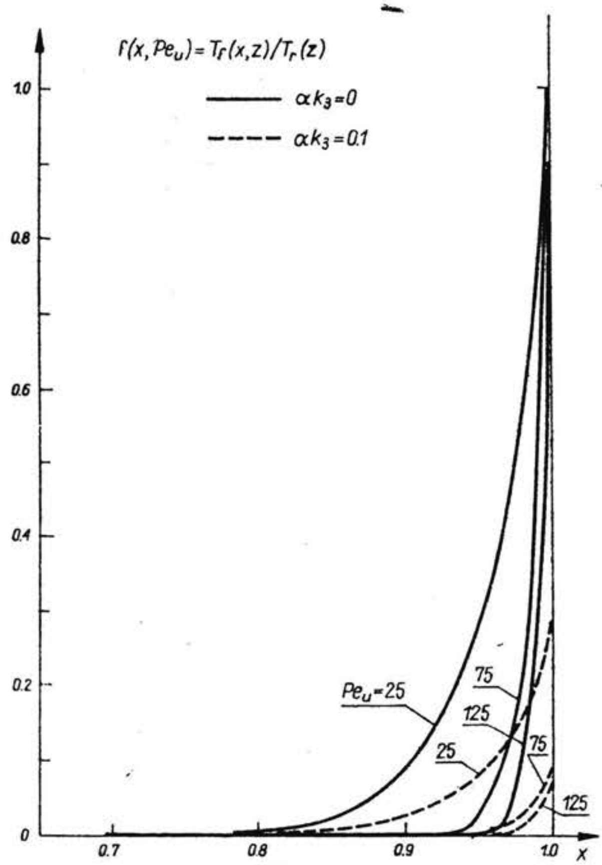
and

$$T_f(x, z) = T_r(z) \frac{e^{\mathrm{Pe}_u x} - 1}{(1 + \alpha k_3 \mathrm{Pe}_u) e^{\mathrm{Pe}_u} - 1} = T_r(z) \cdot f(x, \mathrm{Pe}_u). \tag{4.11}$$



$-\frac{QPe_w}{\alpha\gamma}\ln\frac{T_r}{T_{r0}}=\frac{Pe_u}{1+\alpha k_3 Pe_u}Z$
$\alpha k_3=0$
$\alpha k_3=0.1$
200
100
0
$Pe_u=125$
$Pe_u=75$
$Pe_u=25$
$Pe_u=125$
$Pe_u=75$
$Pe_u=25$
1
2
3
4
Z

FIG. 5.



$f(x,Pe_u)=T_f(x,z)/T_r(z)$
$\alpha k_3=0$
$\alpha k_3=0.1$
1.0
0.8
0.6
0.4
0.2
0
$Pe_u=25$
75
125
25
75
125
0.7
0.8
0.9
1.0
x

FIG. 6.

As it follows from evaluations, these expressions bear all the important features of more exact formulas for sufficiently large Pe_u. The expressions (4.10) and (4.11) are illustrated by Figs. 5 and 6. The continuous curves there correspond to the case when the half-plane (*2*) is considered as the uniformly porous wall and the "temperature jump" on it is neglected. The dotted lines present the picture in the case when the "temperature jump" does not vanish and thus the structure of the half-plane (*2*) is indirectly taken into account.

It can be concluded that although the qualitative behaviour of temperature T_r and T_f with a changing Peclet number is similar in both cases, their quantitative behaviour differs significantly.

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