Arch. Mech., 32, 2, pp. 273–283, Warszawa 1980

Incompressible boundary layer for longitudinal flow over a cylinder with an applied magnetic field

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THE FLOW, heat and mass transfer problem for a steady laminar incompressible boundary layer flow in an electrically conducting fluid over a longitudinal cylinder with an applied magnetic field has been studied. The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme. The results are found to be strongly dependent on the magnetic field and dissipation parameter. The effect of the mass transfer is more pronounced on the skin friction than on the heat transfer. The results have been compared with those of the series solution, the asymptotic solution, the Glauert and Lighthill's solution, local similarity, local nonsimilarity and difference-differential methods. Good agreement is found with all of them, except with the results of the local similarity and series solution methods.

Rozważono problem opływu, transportu masy i ciepła w ustalonym, laminarnym przepływie warstwy przyściennej wzdłuż walca i w polu magnetycznym. Równania różniczkowe cząstkowe, opisujące to zagadnienie, rozwiązano numerycznie posługując się metodą różnic skończonych. Okazuje się, że wyniki zależą w sposób istotny od pola magnetycznego i od parametru dysypacji, a transport masy wpływa bardziej na tarcie powierzchniowe niż na przewodnictwo ciepła. Wyniki porównano z rezultatami otrzymanymi metodą szeregów, rozwiązań asymptotycznych, metodą Glauerta i Lighthilla, metodą lokalnego podobieństwa oraz braku podobieństwa lokalnego, a także metodami różnicowo-różniczkowymi. Stwierdzono dobrą zgodność wszystkich wyników z wyjątkiem rezultatów otrzymanych metodami szeregów i lokalnego podobieństwa.

Рассматривается задача обтекания, переноса массы и тепла в установившимся, ламинарном течении пограничного слоя вдоль цилиндра и в магнитном поле. Дифференциальные уравнения в частных производных, описывающие эту задачу, решены численно, послуживаясь методом конечных разностей. Оказывается, что результаты зависят существенным образом от магнитного поля и параметра диссипации, а перенос массы влияет больше на поверхностное трение, чем на теплопроводность. Результаты сравнены с результатами полученными методом рядов, асимптотических решений, методом Глауэрта и Лайтхилла, методом локального подобия и отсутствия локального подобия, а также дифференциально-разностными методами. Констатировано хорошое совпадение всех результатов, за исключением результатов, полученных методами рядов и локального подобия.

Notations

- A mass transfer parameter (constant),
- B_0 applied magnetic field,
- Br Brinkman number,
- C_f skin friction coefficient,
- f dimensionless stream function,
- f_w mass transfer parameter,
- F(or f'), G dimensionless velocity and temperature, respectively,
- $F_\eta(\xi, 0)$ skin friction parameter,
- $G_\eta(\xi, 0)$ heat transfer parameter,

K thermal conductivity,
M magnetic parameter,
Nu Nusselt number,
q heat transfer rate,
r distance measured from axis of cylinder in radial direction,
R radius of the cylinder,
Re_x local Reynolds number,
T temperature,
u, v velocity components in axial and radial directions, respectively,
v_0 constant,
U velocity in axial direction at the edge of the boundary layer,
η, ξ transformed coordinates,
μ coefficient of viscosity,
ν kinematic viscosity,
ϱ density,
σ electrical conductivity,
τ_w shear stress on the wall,
ψ dimensional stream function.

Subscripts

r, x, η, ξ denote derivatives with respect to r, x, η and ξ, respectively,
w conditions at the wall,
∞ conditions in the freestream.

1. Introduction

THE STEADY laminar incompressible boundary layer flow over a longitudinal cylinder can be regarded as an extension of the Blasius solution for a flat plate; however the effect of the transverse curvature must be taken into account. This problem was considered by YOUNG [1], and JACOB and DOW [2] under certain restricted conditions using the momentum integral method. SEBAN and BOND [3] re-studied the above problem under more general conditions. They solved the equations using the series solution method and computed 3 terms of the series. Subsequently, KELLEY [4] introduced some corrections in the solution of SEBAN and BOND [3]. WANOUS and SPARROW [5] further refined the existing numerical informations and provided the fourth term of the series of [3]. GLAUERT and LIGHTHILL [6] used a different series to solve the problem for very large ξ. They also constructed an interpolation curve for the wall shear to bridge the gap between their solution and the series solution of [3]. JAFFE and OKAMURA [7] have obtained a solution for the same problem using the difference-differential approach which reduces the partial differential equations governing this problem to ordinary differential equations. Recently, SPARROW and co-workers [8–9] have investigated the same problem using the local nonsimilarity method which also reduces the partial differential equations to ordinary differential equations. More recently, NATH [10] has re-examined this problem using an approximate method based on a series expansion in derivatives of the stream function. It may be remarked that all these investigators considered only the hydrodynamic case and not the hydromagnetic case (which includes the effect of the magnetic field). Further-more, to the authors' knowledge, the solution of this problem using the finite-difference method has not been reported in the literature.

The aim of this paper is to study the steady laminar incompressible boundary-layer flow and heat transfer problem for an electrically conducting fluid over a longitudinal cylinder with an applied magnetic field. The effects of mass transfer and dissipation terms have been included in the analysis. The partial differential equations governing the non-similar problem have been solved numerically using an implicit finite-difference scheme. The results have been compared with those of local similarity, local nonsimilarity, the series solution, the asymptotic solution, the Glauert and Lighthill's solution, and difference-differential methods.

2. Governing equations

We consider an axisymmetric flow of a steady laminar incompressible electrically conducting fluid over a long thin non-conducting cylinder of radius R under the influence of an applied magnetic field B_0 imposed in a transverse direction to the flow. It is assumed that the magnetic Reynolds number of the flow is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. Further, the wall and freestream temperatures have been considered as uniform. In this problem, the non-similarity is due to the transverse curvature of the surface. Under the above conditions, the governing equations taking into account the effects of mass transfer, viscous dissipation and Joule's heating, can be expressed in dimensionless form as [8–9]:

$$(1+\eta\xi)F_{\eta\eta}+(f+\xi)F_\eta-(M/4)\xi^2F_\eta = \xi(FF_\xi-F_\eta f_\xi), \tag{2.1}$$

$$(1+\eta\xi)G_{\eta\eta}+(\mathrm{Pr}f+\xi)G_\eta+(M/4)\mathrm{Br}\,\xi^2F^2+(1+\eta\xi)\mathrm{Br}\,F_\eta^2 = \mathrm{Pr}\,\xi(FG_\xi-G_\eta f_\xi) \tag{2.2}$$

with the boundary conditions

$$F(\xi,0) = G(\xi,0) = 0, \quad F(\xi,\infty) = 2, \quad G(\xi,\infty) = 1, \tag{2.3}$$

where

$$\begin{gathered}
\xi = (4/R)(\nu x/U)^{1/2}, \quad \eta = (U/\nu x)^{1/2}(r^2-R^2)/4R,\\
u(x,r) = r^{-1}\psi_r, \quad v(x,r) = -r^{-1}\psi_x,\\
\psi(x,r) = R(\nu xU)^{1/2}f(\xi,\eta), \quad F = f_\eta = u/U,\\
G(\xi,\eta) = (T-T_w)/(T_\infty-T_w), \quad f = \int_0^\eta F d\eta + f_w,\\
M = \sigma B_0^2R^2/\mu, \quad \mathrm{Br} = \mu U^2/[4K(T_\infty-T_w)],\\
f_w = A.
\end{gathered} \tag{2.4}$$

Here $A = -Rv_w/(4\nu)$ when v_w is a constant and $A = -v_0/(\nu U)^{1/2}$ when $v_w = v_0x^{-1/2}$. It may be remarked that at $\xi = 0$, Eqs. (2.1) and (2.2) reduce to similarity equations (Blasius equation) for a flat plate and for $M = 0$ (i.e., in the absence of a magnetic field); they reduce to hydrodynamic equations for longitudinal flow over a cylinder studied in [1–10].

The skin friction coefficient and heat-transfer coefficient (Nusselt number) can be expressed as [8–9]:

$$C_f(\mathrm{Re}_x)^{1/2} = F_\eta(\xi, 0)/2, \quad \mathrm{Nu}(\mathrm{Re}_x)^{-1/2} = -G_\eta(\xi, 0)/2, \tag{2.5}$$

where

$$C_f = 2\tau_w/\varrho U^2, \quad \mathrm{Re}_x = Ux/\nu, \quad \mathrm{Nu} = qx/[K(T_w - T_\infty)]. \tag{2.6}$$

3. Results and discussion

Equations (2.1) and (2.2) under the conditions (2.3) have been solved numerically using an implict finite-difference scheme. Our method is the same as that described in [11–12] except that we have not converted the infinite interval $(0, \infty)$ for η to finite one $(0, 1)$ as has been done in [11–12]. Hence the description of this method is not given here. The computations have been carried out on a IBM 360/44 computer. The step size $\Delta\eta = 0.01$ and $\Delta\xi = 0.05$ have been used throughout the computation. Further reduction in them changes the results only in the 4th decimal place. In order to test the accuracy of the present method, the result of the similarity solution obtained by putting $\xi = 0$ in Eq. (2.) has been compared with that tabulated in [7] and it is found to agree up to the 4th decimal place.

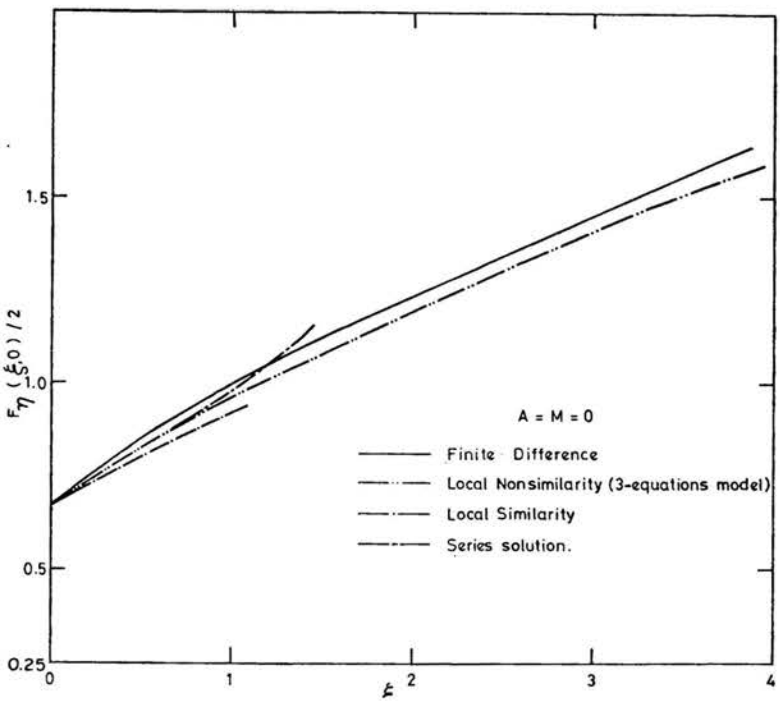

FIG. 1a. Comparison of skin friction $F_\eta(\xi, 0)/2$ with other methods.

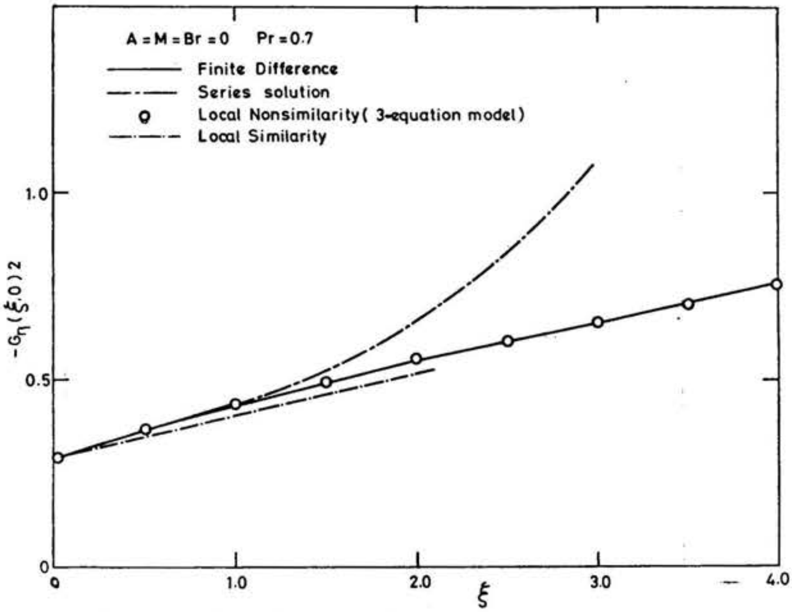

FIG. 1b. Comparison of heat transfer $-G_\eta(\xi, 0)/2$ with other methods.

Figures 1a and b show the comparison of the finite-difference results for skin friction $F_\eta(\xi, 0)/2$ and heat transfer $-G_\eta(\xi, 0)/2$ with the series solution [5], local similarity and local nonsimilarity methods [8–9] and the asymptotic method [10] for the parameters for which their results are available (i.e. for $A = M = \mathrm{Br} = 0$). Since the results obtained by the asymptotic method are nearly the same as those of the local nonsimilarity method, they are not shown in the figures. The heat transfer results (obtained by the finite-difference method) are found to be in good agreement with those of the local nonsimilarity method, however, the skin friction results differ from the corresponding results of the local nonsimilarity method maximum by about 5 per cent. The finite difference results for the skin friction and heat transfer for large ξ differ considerably from the local similarity and series solution results, indicating the inadequacy of these methods. Figure 2 gives the comparison of the finite-difference results for $F_\eta(\xi, 0)/2$ in the range $0 \leqslant \xi \leqslant 40$ with those of JAFFE and OKAMURA [7] (difference-differential results) and GLAUERT and LIGHTHILL [6] (expansion procedure results valid for $6 \leqslant \xi \leqslant 40$) and they are found to be in good agreement with both of them, except when ξ is very large ($\xi \geqslant 20$). For very large ξ, the maximum difference between the JAFFE and OKAMURA [7] results and the present results is about 7.5 per cent and between the GLAUERT and LIGHTHILL [6] results and the present ones is about 5 per cent.

The variation of the skin friction parameter $F_\eta(\xi, 0)$ with ξ for various values of the magnetic parameter M and the mass transfer parameter A when $v_w \propto x^{-1/2}$ is given in Fig. 3. Figure 4 contains the corresponding results for $F_\eta(\xi, 0)$ when v_w is a constant. $F_\eta(\xi, 0)$ increases as M or ξ increases. Similarly, the effect of suction ($A > 0$) is to increase the skin friction $F_\eta(\xi, 0)$, whereas the injection does just the reverse. This behaviour is true whether v_w is a constant or varies as $x^{-1/2}$.

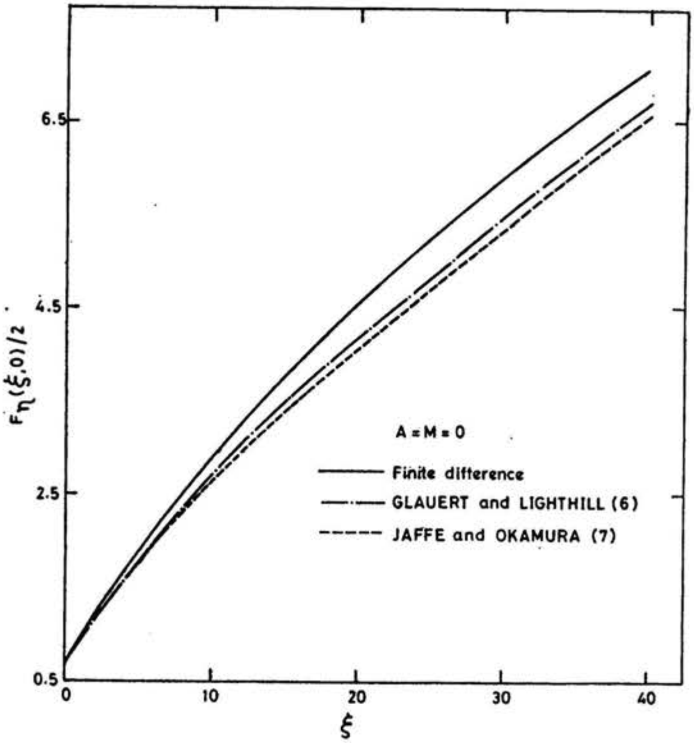

FIG. 2. Comparison of skin friction $F_\eta(\xi, 0)/2$ with other methods for large ξ.

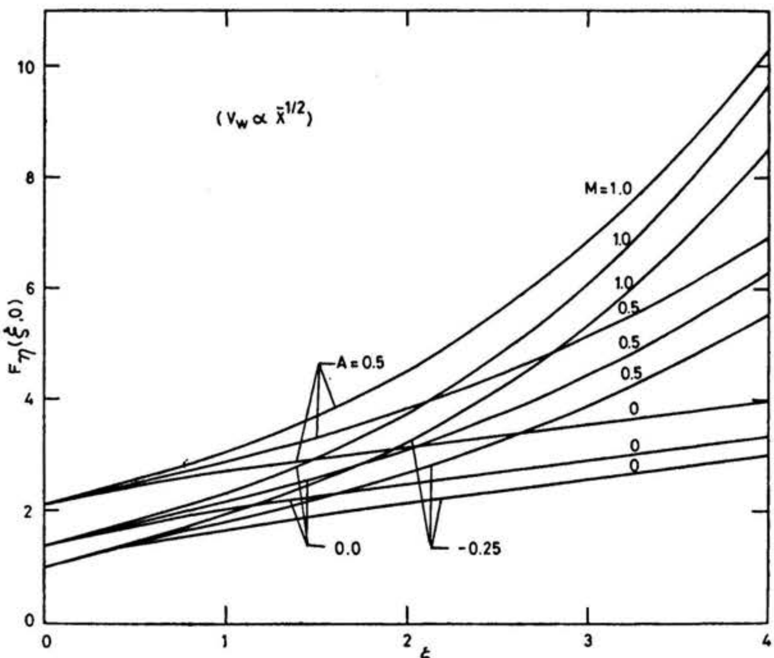

FIG. 3. Variation of $F_\eta(\xi, 0)$ with $\xi(v_w \propto x^{-1/2})$.

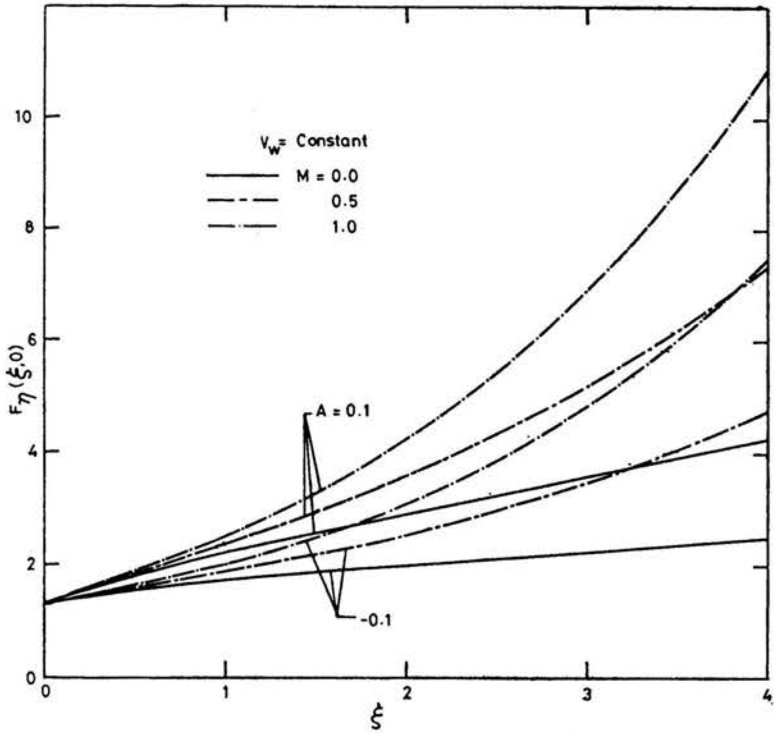

FIG. 4. Variation of $F_\eta(\xi, 0)$ with ξ (v_w = constant).

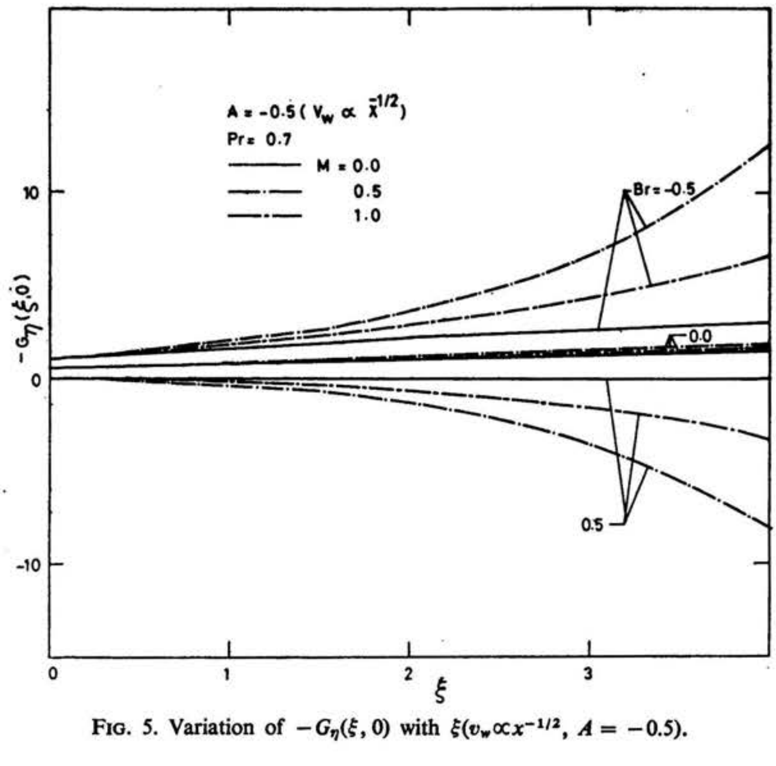

FIG. 5. Variation of $-G_\eta(\xi, 0)$ with ξ($v_w \propto x^{-1/2}$, $A = -0.5$).

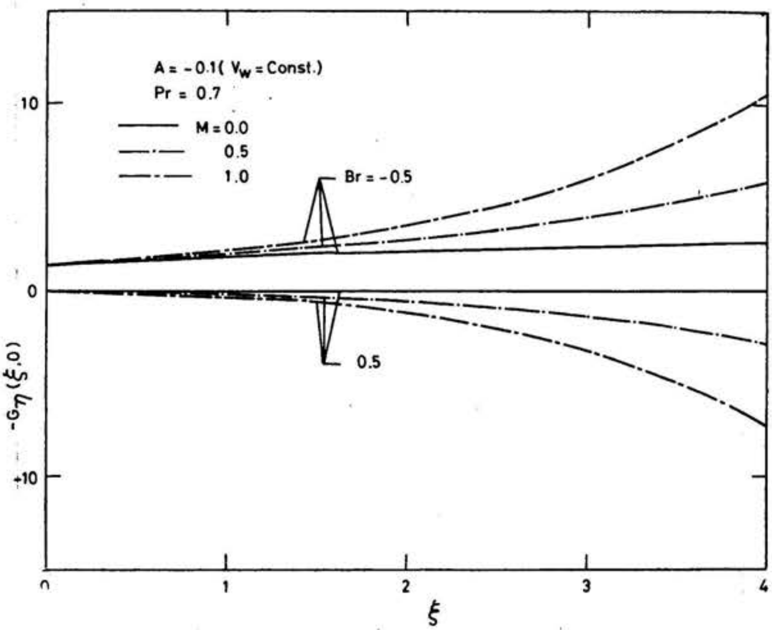

FIG. 6. Variation of $-G_\eta(\xi, 0)$ with ξ (v_w = constant, $A = -0.1$).

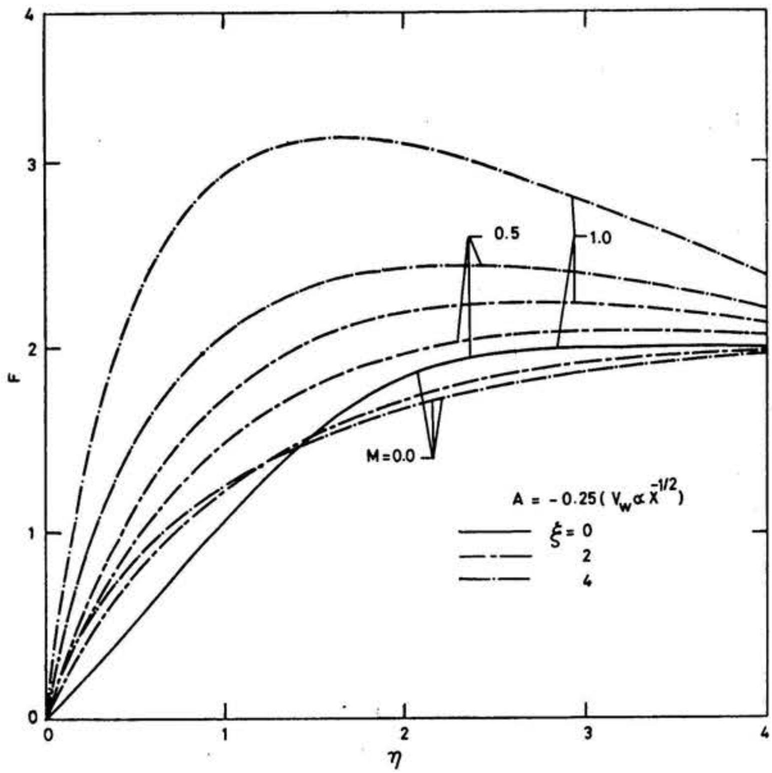

FIG. 7. Velocity profiles ($v_w \propto x^{-1/2}$, $A = -0.25$).

The distribution of the heat transfer parameter $-G_\eta(\xi, 0)$ with ξ for different values of Br, M, and A is shown in Figs. 5–6. When Br = 0, $G_\eta(\xi, 0)$ changes very little with ξ and this trend is valid whatever may be the values of M. Similarly, for $M = 0$, $-G_\eta(\xi, 0)$ changes very little with ξ irrespective of the magnitude of Br. When Br > 0 and $M > 0$, $-G_\eta(\xi, 0)$ decreases rapidly with ξ for large ξ; but for small ξ, the rate of decrease is small. On the other hand, the behaviour of $-G_\eta(\xi, 0)$ is just the opposite when Br < 0, $M > 0$, i.e. it increases with ξ (for small ξ slowly and for large ξ rapidly). For given ξ, the effect of M is to decrease $-G_\eta(\xi, 0)$ when Br > 0 but its effect is just the reverse when Br < 0. These results do not change qualitatively with mass transfer or depending on whether v_w is a constant or a variable.

The velocity profiles ($F(\xi, \eta)$) are displayed in Fig. 7. It is observed that there is a velocity overshoot when $M > 0$ and $\xi > 0$ and the velocity overshoot increases as M or ξ increases. However, there is no velocity overshoot when $\xi = 0$ whatever may be the values of M. Since there is a velocity overshoot when $M > 0$, and $\xi > 0$, there is always a point of inflexion as is evident from minimum in $F_\eta(\xi, \eta)$ (not shown in figures for the sake of brevity).

The temperature profiles ($G(\xi, \eta)$) for various values of Br are shown in Figs. 8–9. From these figures it is seen that for Br > 0, $\xi > 0$, and $M > 0$, $G(\xi, \eta)$ first increases

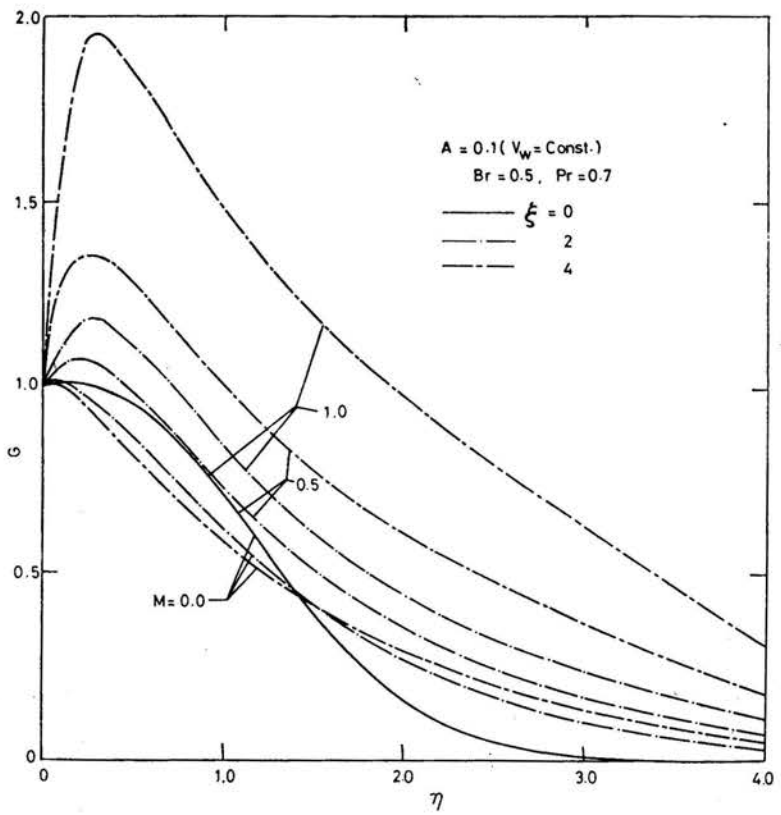

FIG. 8. Temperature profiles (v_w = constant, $A = 0.1$, Br = 0.25).

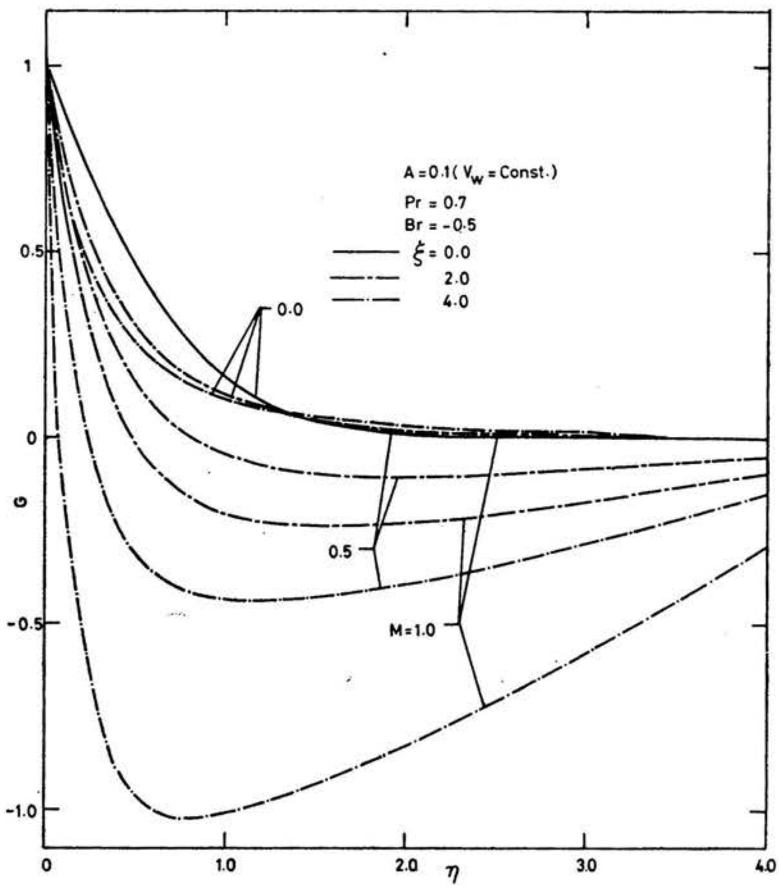

FIG. 9. Temperature profiles (v_w = constant, $A = 0.1$, $\mathrm{Br} = -0.5$).

with η, attains a maximum and then rapidly decreases to zero as η further increases. This maximum value strongly depends on the values of M, ξ and Br. It is found that the behaviour of $G(\xi, \eta)$ is the same whether v_w is a constant or a variable, hence, for the sake of brevity, $G(\xi, \eta)$ when $v_w \propto x^{-1/2}$ is not shown here. When $\mathrm{Br} < 0$, $M > 0$, and $\xi > 0$, $G(\xi, \eta)$ rapidly decreases with η, attains a minimum ($G(\xi, \eta)$ becomes negative) and then increases with η and finally tends to zero asymptotically. It can be concluded that Br exerts a strong influence on the temperature profiles.

4. Conclusions

The skin friction increases as the magnetic parameter increases, but the heat transfer decreases as the magnetic parameter increases when the Brinkman number (dissipation parameter) is positive. However, for negative values of the Brinkman number, heat transfer increases as the magnetic parameter increases. The effect of the mass transfer is more pronounced on the skin friction than on the heat transfer. The heat transfer is strongly affected by the dissipation parameter. The skin friction results differ from the corresponding results of the local nonsimilarity method by about 5 per cent, but the heat transfer results are found to be in excellent agreement with those of the local nonsimilarity method. The results differ considerably from those of the local similarity and series solution methods which imply that these methods are not suited to the present problem. For very large ξ, the results differ from difference-differential results maximum by about 7.5 per cent and from expansion procedure results due to Glauert and Lighthill by about 5 per cent.

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Received March 30, 1979.