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Finite-part singular integro-differential equations arising in two-dimensional aerodynamics

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A NEW METHOD is considered for the solution of the finite-part singular integro-differential equations, applied in many problems of Mathematical Physics and especially in elasticity and aerodynamic problems. This is obtained by reduction to a system of linear equations, by applying the singular integro-differential equation at properly selected collocation points. An application is given to the determination and solution of the generalized airfoil equation, which presents the pressure acting on a planar airfoil undergoing simple amplitude oscillations about the central plane of a two-dimensional ventilated wind tunnel.

Przedstawiono nową, numeryczną metodę rozwiązywania osobliwych równań różniczkowo-całkowych w zastosowaniu do wielu zagadnień fizyki matematycznej, a w szczególności do teorii sprężystości i aerodynamiki. Problem sprowadza się do układu równań liniowych spełniając równania różniczkowo-całkowe w stosownie dobranych punktach kollokacji. Podano zastosowanie metody do analizy drgań płata opływanego i podlegającego drganiom w tunelu aerodynamicznym.

Представлен новый, численный метод решения особых дифференциально-интегральных уравнений в применении к многим задачам математической физики, а в частности к теории упругости и аэродинамики. Задача сводится к системе линейных уравнений, удовлетворяя дифференциально-интегральным уравнениям в соответственно подобранных точкам коллокации. Представлено применение метода к анализу колебаний крыла, обтекаемого потоком и подлежащего колебаниям в аэродинамическом туннеле.

1. Introduction

MANY IMPORTANT problems of applied mathematics and physics can be reduced to the solution of a finite-part singular integro-differential equation.

Hence it is of interest to solve numerically these systems of singular integro-differential equations of the respective boundary value problem, instead of the problem itself.

The most effective method of solving numerically this type of singular integral equations is the direct method which consists in reducing such an equation (or system of equations) to a system of linear algebraic equations, by using an appropriate numerical integration rule on a properly selected set of collocation points.

Some studies of the generalized two-dimensional airfoil equation began in the 1930. Theories for general aerodynamic problems have been obtained by V. V. GOLUBEV [1], T. VON KÁRMÁN and J. M. BURGERS [2], H. SCHMIDT [3] and K. SCHRÖDER [4, 5].

In the 1940 the two-dimensional aerodynamic problems were advanced by the work of J. WEISSINGER [6], H. KÜSSNER and L. SCHWARZ [7], L. G. MAGNARADZE [8–10], I. N. VEKUA [11], [12] and H. SCHÖNGEN [13].

N. I. MUSKHELISHVILI [14] has given an extended study of the integro-differential equation of the aircraft wings of finite span, while V. V. IVANOV [15] has obtained some approximate methods to the numerical solution of integro-differential equations.

Over the last years some papers have been published on the application of the integro-differential equations in aerodynamics. Among them we shall mention the following authors: S. R. BLAND [16, 17], J. BLACKWELL and G. POUNDS [18], J. A. FROMME and M. A. GOLBERG [19–24], M. A. GOLBERG, M. LEA and G. MIEL [25], M. A. GOLBERG [26], W. F. MOSS [27, 28], D. J. SALMOND [29], M. H. WILLIAMS [30], E. KRAFT and C. LO [31], M. MOKRY [32] and E. NISSIN and I. LOTTATI [33].

In the present report a new technique is proposed for the numerical evaluation of the general type of the finite-part singular integro-differential equation. An application is given of the numerical solution of the airfoil equation, as a rising interest in the general problem of high subsonic and transonic aeroelasticity has made the need for improved methods of aerodynamic analysis and testing greater. Hence, the method presented in this communication is a generalization of the finite-part singular integral equations methods introduced and investigated by E. G. LADOPOULOS [34–40], and used in elasticity, plasticity and fracture mechanics problems.

In the present report the new method shall be extended to aerodynamic problems as well.

2. Finite-part singular integral equations

Let us consider the finite-part singular integral: [34, 38–40]

$$\Phi(z, \mu) = \Gamma(\mu) \int_L \frac{w(t)\varphi(t)}{(t-z)^\mu} dt, \qquad \mu = 1, 2, 3, \ldots, \tag{2.1}$$

where L denotes the interval $[a, b]$ of the real axis, $w(t)$ a given weight function defined for every $t \in [a, b]$, $\varphi(t)$ an analytic function of t in any plane domain S, containing the interval L and $\Gamma(\mu)$ the Gamma function.

Furthermore we consider the finite-part singular integral equation of the second kind with variable coefficients:

$$A(x)\varphi(x) + \Gamma(\mu) \int_L B(t) \frac{\varphi(t)}{(t-x)^\mu} dt + \int_L K(t, x)\varphi(t)dt = f(x, \mu), \qquad x \in L, \qquad \mu = 1, 2, 3, \ldots, \tag{2.2}$$

where $A(x)$, $B(t)$ and $f(x, \mu)$ are known functions, $\Gamma(\mu)$ is the Gamma function, $K(t, x)$ the Fredholm kernel, $\varphi(x)$ the unknown function and L the integration interval which may be a closed contour, a curvilinear arc, or simply a part of the real axis.

Another system of finite-part singular

Integral equations encountered in boundary value problems of two-dimensional elasticity can be written in the form

$$(2.3)\qquad f_i(x,\mu) = \sum_{j=1}^{n} C_{ij}(x)\Gamma(\mu)\int_L \frac{\varphi_j(t)}{(t-x)^{\mu}}\,dt + \sum_{j=1}^{N}\int_L K_{ij}(t,x)\varphi_j(t)dt \qquad (i = 1, 2, 3, \ldots, N)$$

in which the functions $C_{ij}(x)$, $K_{ij}(t, x)$ and $f_i(x, \mu)$ $(i, j = 1, 2, 3, \ldots, N)$ are known.

A general form of a system of finite-part singular integral equations, in which the dominant part has a generalized kernel, is as follows:

$$(2.4)\qquad A\varphi(x)+\Gamma(\mu)\int_L B\varphi(t)\,\frac{dt}{(t-x)^{\mu}}+\Gamma(\mu)\int_L \sum_{0}^{k} C_k\varphi(t)(x-a)^k\frac{d^k}{dx^k}(t-z_1)^{-\mu}dt$$
$$+\Gamma(\mu)\int_L \sum_{0}^{j} D_j\varphi(t)(b-x)^j\frac{d^j}{dx^j}(t-z_2)^{-\mu}dt+\int_L K(x,t)\varphi(t)dt = f(x,\mu), \qquad x\in L,$$

where A, B, C_k and D_j are $(N\times N)$ matrices which are generally constant, the matrix $K(x, t)$ consists of Fredholm kernels $K_{ij}(x, t)$ $(i, j = 1, \ldots, N)$ and $f = f(x, \mu)$, $(i = 1, \ldots$ $\ldots, N)$ is the input vector which satisfies a Hölder-condition in L.

Also, the variables z_1 and z_2 are given by:

$$(2.5)\qquad \begin{aligned} z_1 &= a+(x-a)e^{i\Theta_1},\\ z_2 &= a+(b-x)e^{i\Theta_2}, \end{aligned}$$

where Θ_1, Θ_2 are known constants with $0 < \Theta_1 < 2\pi$ and $-\pi < \Theta_2 < \pi$.

Moreover, let us consider the finite-part singular integral equation of the first kind:

$$(2.6)\qquad \Gamma(\mu)\int_L \frac{\varphi(t)}{(t-x)^{\mu}}\,dt+\int_L K(x,t)\varphi(t)dt = f(x,\mu), \qquad \mu = 1, 2, 3, \ldots,$$

where $\varphi(x)$ is unknown, $\Gamma(\mu)$ is the Gamma function and $f(x, \mu)$, $K(x, t)$ are known functions which are H-continuous in the closed interval L.

From Eq. (2.2) we obtain the finite-part singular integral equation of the second kind, with constant coefficients:

$$(2.7)\qquad a\varphi(x)+b\Gamma(\mu)\int_L \frac{\varphi(t)}{(t-x)^{\mu}}\int_L dt+K(x,t)\varphi(t)dt = f(x,\mu), \qquad \mu = 1, 2, 3, \ldots,$$

where the interval is again normalized to be L without any loss in generality. It will also be assumed that a, b are constants and the known functions f and K are H-continuous. The functions φ, k, f and the constants a, b may be real or complex.

Some aerodynamic problems are solved by using the equations discussed in this chapter. The same equations are used for the solution of elasticity and plasticity problems of isotropic and anisotropic solids [34–40].

3. Finite-part singular integro-differential equations

A large class of problems, in mathematical physics particularly, can be reduced to the solution of a singular integro-differential equation of the form:

$$(3.1)\qquad \sum_{j=0}^{m}\left((a_j(t)\varphi^{(j)}(t)+\Gamma(\mu)\int_L \frac{K_j(t,\tau)\varphi^{(j)}(\tau)d\tau}{(\tau-t)^{\mu}}\right)=f(t),\qquad \mu=1,2,3,\ldots,$$

$t\in L$, and a_i, $K_i, f(t)$ are given functions and $\varphi^{(j)}$ denotes the j-th derivative of φ.

Assuming that a_j, K_j and f are sufficiently differentiable and that L is a simple, closed, sufficiently smooth contour, we can reduce Eq. (3.1) to an equivalent singular or regular integral equation.

Thus, let us give a method for the reduction of Eq. (3.1) to a singular integral equation.

Let $L\equiv(a,b)$ an open smooth curve. By writing

$$(3.2)\qquad \varphi^{(m)}(t)=g(t),$$

we obtain

$$(3.3)\qquad \varphi^{(k)}(t)=\int_L \omega_{m-k-1}(t,t_1)g(t_1)dt_1+\sum_{i=0}^{m-k-1}C_{m-k-i}\frac{t^i}{i!}$$

for $k=0,1,\ldots,m-1$, where

$$(3.4)\qquad \begin{gathered}\omega_0(t,t_1)=1,\quad \text{if}\quad t_1\in(a,t),\\ \omega_0(t,t_1)=0,\quad \text{if}\quad t_1\notin(a,t),\\ \omega_{k-1}(t,t_1)=\int_L\omega_0(t,t_2)\omega_{k-2}(t_2,t_1)dt_2,\qquad k=2,3,\ldots,m\end{gathered}$$

and $C_1, C_2, \ldots, C_m$ are arbitrary constants.

Substituting into Eq. (3.1) we obtain a finite-part singular integral equation for $\mu=1$ of the form

$$(3.5)\qquad a_m(t)g(t)+\Gamma(\mu)\int_L\frac{K_m(t,\tau)g(\tau)}{\tau-t}\,dt+\int_L K(t,\tau)g(\tau)d\tau=f(t)-\sum_{k=1}^{m}C_kX_k(t),$$

where

$$(3.6)\qquad K(t,\tau)=\sum_{j=0}^{m-1}\left(a_j(t)\omega_{m-j-1}(t,\tau)+\Gamma(\mu)\int_L\frac{K_j(t,u)\omega_{m-j-1}(u,\tau)}{u-t}\,du\right)$$

and

$$(3.7)\qquad X_k(t)=\sum_{j=0}^{m-k}\left[a_jt^{m-j-k}+\Gamma(\mu)\int_L\frac{K_j(t,\tau)\tau^{m-j-k}}{\tau-t}\,d\tau\right]\Big/(m-j-k)!$$

It follows that if for any values of the constants $C_1, C_2, \ldots, C_m$, the function g is the solution of Eq. (3.5), then the function φ as given by Eq. (3.3) will be a solution of the original Eq. (3.1).

Consequently, it is obvious that if φ is the solution of Eq. (3.1), then $\varphi^{(m)}(t) = g(t)$ gives the solution of Eq. (3.5) for the specific values of $C_1, C_2, \ldots, C_m$.

Moreover, it is also possible to apply this method to the case where L is a closed smooth curve.

Thus, it is necessary to consider the function $\varphi^{(k)}$, $k = 0, 1, \ldots, m-1$ as defined by Eq. (3.3), since in general they will not be unique.

It can also be seen that for the Cauchy problem, when the values of $\varphi^{(k)}(a)$, $k = 0, 1, \ldots \ldots, m-1$ are given, we obtain

$$C_k = \sum_{j=0}^{k-1} (-1)^j \varphi^{(m-j)}(a)/j! \quad \text{for} \quad k = 1, 2, \ldots, m. \tag{3.8}$$

In the same way for the case where $\mu > 1$, we have the following finite-part singular integral equation:

$$a_m(t)g(t) + \Gamma(\mu) \int_L \frac{K_m(t, \tau)g(\tau)dt}{(\tau - t)} + \int_L K(t, \tau)g(\tau)d\tau = f(t) - \sum_{k=1}^{m} C_k X_k(t), \tag{3.9}$$

where

$$K(t, \tau) = \sum_{j=0}^{m-1} \left(a_j(t)\omega_{m-j-1}(t, \tau) + \Gamma(\mu) \int_L \frac{K_j(t, u)\omega_{m-j-1}(u, \tau)}{(u-t)^{\mu}} du \right) \tag{3.10}$$

and also

$$X_k(t) = \sum_{j=0}^{m-k} \left[a_j t^{m-j-k} + \Gamma(\mu) \int_L \frac{K_j(t, \tau)\tau^{m-j-k}}{(\tau - t)^{\mu}} d\tau \right] \Big/ (m-j-k)! \tag{3.11}$$

Equation (3.9) gives the general use of the reduction of a finite-part singular integro-differential equation to a singular integral equation.

4. Application to the determination of the two-diemesnional airfoil equation in a wind tunnel

Let us consider a planar airfoil undergoing simple amplitude oscillations about the center plane of a two-dimensional ventilated wind tunnel (Fig. 1). By removing the walls to infinity, a very important special case exists which gives free air conditions.

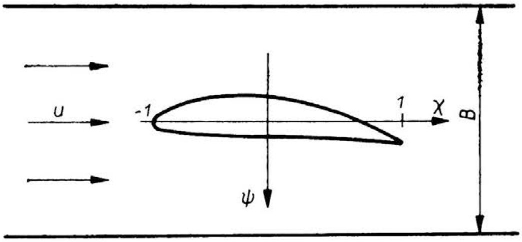

FIG. 1. A planar airfoil in a two-dimensional ventilated wind tunnel.

The flow is assumed to be inviscid and strictly subsonic, thus the following unsteady wave equation is valid: [21]

$$\nabla^2\xi - M^2\left(\frac{\vartheta}{\vartheta x}+ik\right)^2 \xi = 0, \tag{4.1}$$

where ξ denotes the perturbation velocity potential, M the freestream Mach number and k the reduced frequency:

$$k = \frac{\omega d}{u} \tag{4.2}$$

in which ω is the frequency of the simple harmonic motion of the airfoil, d its semi-chord and u the free stream velocity.

Furthermore the nondimensional pertubation pressure p is given by the following relation:

$$p = -2\left(\frac{\vartheta}{\vartheta x}+ik\right)\xi \tag{4.3}$$

with the boundary conditions:

$$p(x, 0) = \begin{cases} 0, & |x| \geqslant 1, \\ -1/2\Delta p(x), & |x| < 1, \end{cases} \tag{4.4}$$

where Δp denotes the lifting pressure jump across the airfoil. The relation between the downwash velocity w and the pressure potential ξ is valid as

$$w(x) = \left.\frac{\vartheta\xi}{\vartheta y}\right|_{y=0}, \quad |x| < 1. \tag{4.5}$$

Thus, the downwash velocity w is related to the potential ξ as follows: [16]

$$w(x, y, t) = \frac{1}{u}\int\limits_{-\infty}^{x} \xi_y\left(\mu, y, t-d\frac{x-\mu}{u}\right)d\mu \tag{4.6}$$

in which t denotes the time and μ the ventilation coefficient.

By using the Fourier transforms

$$\begin{aligned} \Xi(s, y) &= \int\limits_{-\infty}^{\infty} e^{-ixt}\xi(x,y)dx, \\ \xi(x, y) &= \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} e^{ixs}\Xi(s, y)ds, \end{aligned} \tag{4.7}$$

the pressure potential will be given by the following relation:

$$\xi(x, y) = \frac{1}{4\pi\varrho_0}\int\limits_{-\infty}^{\infty} e^{ixs}f(s)\int\limits_{-1}^{1} e^{-is\zeta}\Delta p(\zeta)d\zeta ds, \tag{4.8}$$

where

$$f(s) = \frac{\sin h\,a(B/2-y)+c\,a\cos h\,a(B/2-y)}{\sin h(aB/2)+c\,a\cos h(aB/2)} \tag{4.9}$$

in which c denotes the porosity coefficient, B the tunnel height and ϱ_0 the free stream density.

In Eq. (4.9) the parameter a is valid as

$$a(s) = (\beta^2 s^2 - 2M^2 gs - M^2 g^2)^{1/2}, \tag{4.10}$$

where g is the complex reduced frequency and $\beta = \sqrt{1-M^2}$.

By combining Eqs. (4.6) and (4.8), one has

$$\frac{w(x, y)}{u} = \frac{1}{4\pi\varrho_0 u^2} \int\limits_{-\infty}^{x} e^{-ig(x-\mu)} \frac{\vartheta}{\vartheta y} \int\limits_{-\infty}^{\infty} e^{i\mu t} f(s) \int\limits_{-1}^{1} e^{-is\zeta} \Delta p(\zeta) d\zeta ds d\mu. \tag{4.11}$$

By taking the derivative and interchanging the orders of integration, one obtains

$$w(x, y) = \frac{2}{\varrho_0 u} \int\limits_{-1}^{1} \Delta p(\zeta) K(M, g, x-\zeta, y, B, c) d\zeta, \tag{4.12}$$

where the kernel function K is given by the formula

$$K = -\frac{1}{8\pi} \int\limits_{-\infty}^{\infty} a e^{-ig(x-\zeta)} \frac{\cos h a(B/2-y) + ca \sin h a(B/2-y)}{\sin h(aB/2) + ca \cos h(aB/2)} \int\limits_{-\infty}^{x-\zeta} e^{i(s+g)\mu} d\mu ds. \tag{4.13}$$

For steady ($g = 0$), incompressible ($M = 0$) flow and in free air (no tunnel walls $B = \infty$), the kernel K takes the simple form

$$K(x) = 1/x. \tag{4.14}$$

For this case, with $y = 0$, Eq. (4.12) yields the following singular integral equation:

$$w(x) = \frac{1}{2\pi\varrho_0} \int\limits_{-1}^{1} \frac{\Delta p(\zeta)}{\zeta - x} d\zeta. \tag{4.15}$$

By using the Kutta boundary condition of a smooth flow at the airfoil trailing edge

$$\lim_{x\to 1} \frac{2\Delta p(x, t)}{\varrho_0 u^2} = 0, \tag{4.16}$$

Eq. (4.15) has the following closed form solution:

$$\Delta p(\zeta) = -\frac{2\varrho_0 u}{\pi} \left(\frac{1-\zeta}{1+\zeta}\right)^{1/2} \int\limits_{-1}^{1} \frac{w^*(x) w(x)}{x-\zeta} dx \tag{4.17}$$

with the weight function $w^*(x) = (1+x)^{1/2}(1-x)^{-1/2}$.

Thus, by putting the pressure factor

$$p(\zeta) = -\frac{4}{\pi} \int\limits_{-1}^{1} w^*(x) \frac{w(x)}{u} \frac{dx}{x-\zeta}, \tag{4.18}$$

Eq. (4.17) can be written as follows:

$$\Delta p(\zeta) = \frac{1}{2} \varrho_0 u^2 \left(\frac{1-\zeta}{1+\zeta}\right)^{1/2} p(\zeta). \tag{4.19}$$

The pressure factor $p(\zeta)$ in Eq. (4.18) is continuous on $[-1, 1]$ if $w(x)/u$ is also continuous.

5. Numerical evaluation of the airfoil equation

In order to solve numerically the airfoil equation (4.18) we shall use the Gauss–Chebyshev numerical integration rule, while solving the same problem, S. R. BLAND [16] has used the Gauss–Jacobi rule.

Let us consider the following singular integral:

$$\Phi(\zeta) = \frac{1}{2\pi i} \int_{-1}^{1} \frac{w^*(x)\varphi(x)}{x-\zeta}\, dx, \tag{5.1}$$

where $w^*(x)$ is the weight function defined in the interval $[-1, 1]$, $\varphi(x)$ is an analytic function without poles in a domain Ω containing the interval $[-1, 1]$ and $\Phi(\zeta)$ is a sectionally analytic function in the whole complex plane except $[-1, 1]$.

In order to evaluate numerically the singular integral (5.1), we consider the following contour integral on a curve C surrounding the interval $[-1, 1]$: [41, 42].

$$\Phi_0 = \frac{1}{2\pi i} \int_{C} \frac{\varphi(\zeta')}{(\zeta'-x)(\zeta'-\zeta)m_n(\zeta')}\, d\zeta', \tag{5.2}$$

where

$$m_n(\zeta) = \prod_{k=1}^{n} (\zeta - x_k) \tag{5.3}$$

in which x_k are the abscissae.

By applying the Cauchy residue theorem to the integral (5.2), one obtains

$$2\pi i\Phi(\zeta) = \int_{-1}^{1} \frac{w^*(x)\varphi(x)}{x-\zeta}\, dx = \sum_{k=1}^{n} A_k \frac{\varphi(x_k)}{x_k-\zeta} - 2\varphi(\zeta)\frac{d_n(\zeta)}{m_n(\zeta)} + E_n, \tag{5.4}$$

where the error function E_n is valid as

$$E_n = \frac{1}{\pi_i} \int_{C} \frac{\varphi(\zeta')}{\zeta'-\zeta} \frac{d_n(\zeta')}{m_n(\zeta')}\, d\zeta', \tag{5.5}$$

A_k are the weights and $d_n(\zeta)$ is given by the relation

$$d_n(\zeta) = -\frac{1}{2} \int_{-1}^{1} w^*(x) \frac{m_n(x)}{x-\zeta}\, dx. \tag{5.6}$$

By using the Gauss–Chebyshev numerical integration rule with the weight function $w^*(x) = (1+x)^{\pm 1/2}(1-x)^{\pm 1/2}$, the relation (5.4) can be written as

$$\int_{-1}^{1} \frac{w^*(x)\varphi(x)}{x-\zeta}\, dx = \sum_{k=1}^{n} A_k \frac{\varphi(x_k)}{x_k-\zeta} - 2\varphi(\zeta)R_n(\zeta) + E_n \tag{5.7}$$

for $\zeta \neq x_m$, $m = 1, 2, \ldots, n$, and

$$\int_{-1}^{1} \frac{w^*(x)\varphi(x)}{x-\zeta}\, dx = \sum_{\substack{k=1\\k\neq m}}^{n} A_k \frac{\varphi(x_k)}{x_k-\zeta} + A_m\varphi'(\zeta) - 2\varphi(\zeta)G_n(\zeta) + E_n \tag{5.8}$$

for $\zeta = x_m$, $m = 1, 2, \ldots, n$, where

$$R_n(\zeta) = -\frac{\pi U_{n-1}(\zeta)}{nT_n(\zeta)}, \quad \zeta \neq x_m, \quad m = 1, 2, \ldots, n \tag{5.9}$$

and

$$G_n(\zeta) = -\frac{\pi}{2}\frac{U_{n-2}(\zeta)}{T_{n-1}(\zeta)} + \frac{2n-1}{4}A_m\frac{\zeta}{1-\zeta^2}, \quad \zeta = x_m, \quad m = 1, 2, \ldots, n, \tag{5.10}$$

where $T_n(\zeta)$ and $U_n(\zeta)$ denote the Chebyshev polynomials of the first and the second kind and degree n, respectively, expressible in terms of trigonometric functions as follows

$$\begin{aligned} T_n(\zeta) &= \cos n\vartheta, \\ U_{n-1}(\zeta) &= \frac{\sin n\vartheta}{\sin\vartheta}, \\ \zeta &= \cos\vartheta. \end{aligned} \tag{5.11}$$

In Eqs. (5.7) and (5.8) ζ is not permitted to coincide with the endpoints -1 or 1 of the integration interval.

As an application of the airfoil equation (4.18), let us consider the case where the downwash is valid as

$$\frac{w(x)}{u} = \begin{cases} 0, & x \leqslant 0, \\ x, & x > 0. \end{cases} \tag{5.12}$$

Thus, by using the Gauss–Chebyshev numerical integration rule given by Eqs. (5.7) and (5.8), it is possible to compute the airfoil equation (4.18). The same equation was computed by S. R. BLAND [16] who used the Gauss–Jacobi rule.

Figure 2 shows the pressure distribution $p(\zeta)$ for downwash given by Eq. (5.12).

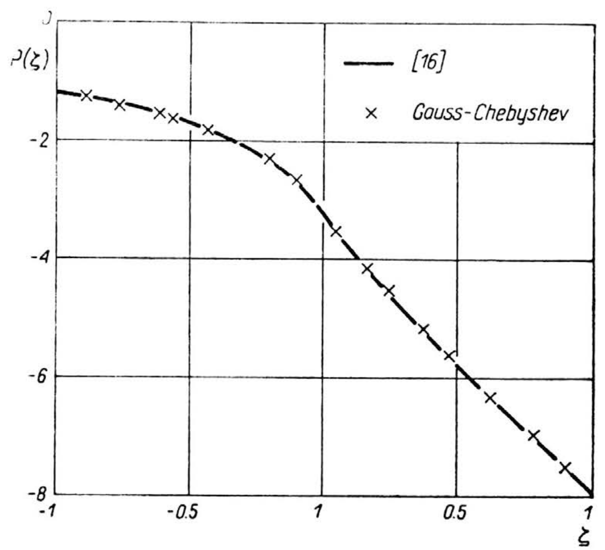

FIG. 2. Pressure distribution $p(\zeta)$ for downwash $w(x)/u = \begin{cases} 0, & x \leqslant 0 \\ x, & x > 0 \end{cases}$ for the planar airfoil of Fig. 1.

As a second application of the airfoil equation, we consider the following downwash function:

$$\frac{w(x)}{u} = \frac{1}{1+25x^2}. \tag{5.13}$$

Figure 3 shows the pressure distribution $p(\zeta)$ for downwash given by Eq. (5.13).

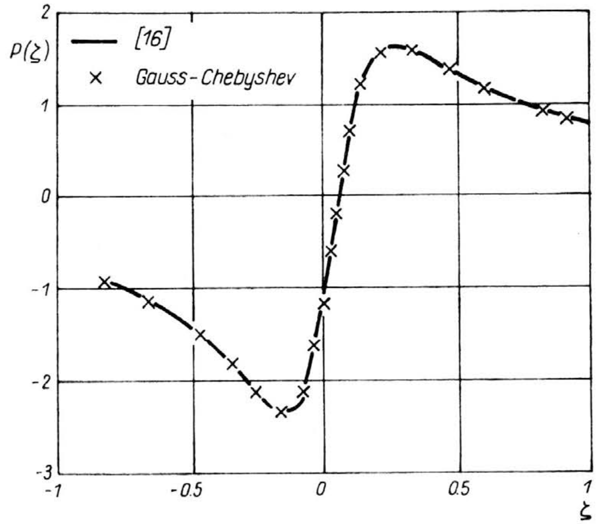

FIG. 3. Pressure distribution $p(\zeta)$ for downwash $w(x)/u = 1/(1+25\,x^2)$ for the planar airfoil of Fig. 1.

Finally, as it is easily seen from Figs. 2 and 3, the two different numerical rules, the Gauss–Chebyshev and the Gauss–Jacobi numerical integration rules coincide very well.

6. Conclusions

An effective method of numerical evaluation of the finite-part singular integro-differential equation consists in reducing such an equation to a system of linear equations after the integrals occurring in this equation are approximated by sums and the equation is applied at the abscissae used in the numerical integration rule.

In the case of finite-part singular integro-differential equations with complex singularities, it is possible, for sufficiently broad class of equations of this type, that the points of their application used lie, in general, outside the integration interval. In this case, the methods used for finite-part singular integro-differential equations with real singularities can be extended, without any modifications, to the case of finite-part singular integro-differential equations with complex singularities.

A simple form of the finite-part singular integro-differential equation has been numerically evaluated by using the Gauss–Chebyshev rule. This equation presents the pressure factor of a planar airfoil undergoing simple amplitude oscillations about the centre plane of a two-dimensional ventilated wind tunnel.

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