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BRIEF NOTES

Motion of inclusions in a solid

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THE PAPER concerns the motion of small inclusions in a solid. The equations of their motion is given and suitable experiments are proposed.

Introduction

THE SUBJECT of this paper is the motion of small inclusions in a crystalline solid. As regards this phenomenon we know the following (cf. [1, 6]):

A. If the temperature of a body with inclusions is sufficiently high and this body is subjected to the external field of stresses, then the inclusions move through the environment (which is called "the matrix").

B. Every inclusion is surrounded by a layer of the matrix in which condensation of crystal lattice defects of the matrix and macroscopic (plastic) slip on the boundary between an inclusion and the matrix take place.

Basic data on the microscopic ground of these phenomena are as follows:

C. Forced diffusion of vacancies and atoms takes place in the matrix.

D. If the temperature of the body is not very high and inclusions are sufficiently small, then the surface diffusion of atoms of the matrix proceeds on the boundary between an inclusion and the matrix.

We shall further consider the case when:

E. The matrix and inclusions are homogeneous and linear isotropic elastic solids;

the inclusion is spherical; $K(\boldsymbol{\xi}, R_0)$ denotes the inclusion $\boldsymbol{\xi}$ — position of its centre, R_0 is its constant radius, $S(\boldsymbol{\xi}, R_0)$ — the boundary of this inclusion;

the matrix is an infinite body.

The unbounded homogeneous body characterized by elastic constants of the matrix is referred to as "the medium". An elastic field in the medium is called "the external field".

1. The phenomenological model of the matrix-inclusion relationship

In order to describe a body together with a moving inclusion by means of the displacement field $\mathbf{u}$, we should postulate, on the basis of the physical phenomena A–D and assumptions E, the following:

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1. The displacement field $\mathbf{u}$ has a spherical discontinuity $\mathbf{U}$ on the boundary of the inclusions:

$$(1.1)\qquad [\mathbf{u}] = \mathbf{U} = -U\mathbf{n} \quad \text{on} \quad S(\boldsymbol{\xi}, R_0).$$

The main element of our description of the moving inclusion is the assumption that

2. The scalar U characterizes the structural relationships between an inclusion and the matrix in the motion. This characteristics is independent of the external fields and position of the inclusion.

On the basis of these assumptions let us propose a method of finding the scalar U in accordance with the ASHBY experiment ([2]). In this experiment the body with inclusions is uniformly compressed. It was found by Ashby that the physical phenomena B had been observed for the critical hydrostatic pressure p_{cr}, and that

$$(1.2)\qquad \begin{gathered} p_{cr} = BR_0^{-n}, \quad 0 < n < 1, \quad B > 0, \\ R_0 \geqslant R_{min} \simeq 10^{-4}\,\text{mm}. \end{gathered}$$

According to this experiment scalar U has the form (cf. [2, 3, 6]):

$$(1.3)\qquad \begin{gathered} U = \frac{1}{3} e^p R_0, \quad e^p = \frac{1}{K} A p_{cr}, \quad A = \frac{K - K_1}{K - \alpha(K - K_1)}, \\ \alpha = \frac{1}{3}\frac{1+\nu}{1-\nu}, \quad p_{cr} > 0, \quad \operatorname{sgn} U = \operatorname{sgn}(K - K_1), \end{gathered}$$

where K, K_1 — the bulk moduli: K — bulk modulus of the matrix, K_1 — bulk modulus of the inclusion and ν — Poisson's ratio of the matrix.

2. Concentrated defect approximation

This approximation is based on identifying a small moving inclusion with the material point which can move in the medium under the influence of an external field. This material point is called "the (spherical) concentrated defect". The equation of motion of the (spherical) concentrated defect was obtained by H. ZORSKI [4].

Taking into account the investigation of the asymptotics of solutions of this equation, it can be written in the form [5]

$$(2.1)\qquad M\ddot{\boldsymbol{\xi}} = \mathbf{F}(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}),$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(t) \in R^3$, $t \in R^+$ — position of the centre of the inclusion, $\dot{\boldsymbol{\xi}}, \ddot{\boldsymbol{\xi}}$ — derivatives with respect to time t, $\mathbf{F}$ — force exerted by the external field on the inclusion, $M > 0$ — the "effective mass" of the inclusion (the field mass). The force $\mathbf{F}$ has the form

$$(2.2)\qquad \begin{gathered} \mathbf{F}(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}) = \mathbf{F}_s(t, \boldsymbol{\xi}) + \mathbf{F}_d(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}), \\ \mathbf{F}_s(t, \boldsymbol{\xi}) = -\frac{1}{3} S\mu U a_0(\nu) \nabla \operatorname{div} \mathbf{u}(\boldsymbol{\xi}, t), \\ \mathbf{F}_d(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}) = \frac{1}{3} S\mu c_2^{-2} U[\mathbf{A}(\boldsymbol{\xi}, t)\dot{\boldsymbol{\xi}} + \ddot{\mathbf{u}}(\boldsymbol{\xi}, t)], \end{gathered}$$

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where

$$S = 4\pi R_0^2, \quad a_0(\nu) = \frac{2(1+\nu)}{1-2\nu} > 0, \quad 0 < \nu < \frac{1}{\sqrt{2}},$$

μ — the shear modulus of the matrix, c_2 — the velocity of equivoluminal waves in the medium

$$\mathbf{A} = \nabla\dot{\mathbf{u}} - \nabla\dot{\mathbf{u}}^T \quad (\mathbf{A} = -\mathbf{A}^T),$$

$\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ — the external displacement field. The field $\mathbf{u}$ is the solution of Navier's equation for the medium. In the theory of concentrated defects, the effective mass M is an undetermined constant because it has the form

$$M = a\Delta U^2 + m, \tag{2.3}$$

where $\Delta(\Delta > 0, [\Delta] = s^{-3})$ — a undetermined constant, m, a — the determinate constants: m — the rest mass of the inclusion and $a = a(R_0, \mu, \nu, c_2) < 0$. It is seen that the effective mass M is determinable from the proper experiment to be proposed.

3. Thermodynamic interpretation

If the external field $\mathbf{u}$ is a static field (i.e. $\dot{\mathbf{u}} = \mathbf{0}$), so we can write the force $\mathbf{F}$ in the form

$$\begin{gathered} \mathbf{F} = \mathbf{F}_s(\boldsymbol{\xi}) = -\nabla\Phi(\boldsymbol{\xi}), \\ \Phi(\boldsymbol{\xi}) = \zeta d e(\boldsymbol{\xi}), \quad e = \operatorname{div}\mathbf{u}, \\ d = \frac{1}{3} S\mu|U|a_0(\nu) > 0, \quad \zeta = \operatorname{sgn}(K - K_1). \end{gathered} \tag{3.1}$$

Let us denote by $\mathbf{I}_a$ and $\mathbf{I}_v$ the diffusion flux of atoms (a) and vacancies (v) in the matrix. Basing on thermodynamic considerations (in a similar manner as [1]) and on the assumption that $\mathbf{I}_a = -\mathbf{I}_v$ ([1]), it can be found that for small inclusions (cf. [6])

$$\begin{gathered} \mathbf{I}_a(\boldsymbol{\xi}) = \frac{3D}{fkT} \nabla\sigma(\boldsymbol{\xi}), \\ \sigma(\mathbf{x}) = \frac{1}{3} \operatorname{tr}\mathbf{T}(\mathbf{x}), \end{gathered} \tag{3.2}$$

where

- $\mathbf{T}(\mathbf{x})$ the external (static) stress field,
- D the self-diffusion coefficient of atoms,
- k the Boltzmann constant,
- T the absolute temperature,
- f the correlation coefficient, cf. [1].

Finally we get

$$\begin{gathered} \mathbf{F}_s(\boldsymbol{\xi}) = \zeta E \mathbf{I}_v(\boldsymbol{\xi}), \\ E = \frac{fkTd}{3KD} > 0, \quad \zeta = \operatorname{sgn}(K - K_1). \end{gathered} \tag{3.3}$$

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COROLLARY

$\mathbf{F}_s \uparrow\uparrow \mathbf{I}_a$ when the inclusion is harder than the matrix $(K < K_1)$,
$\mathbf{F}_s \uparrow\downarrow \mathbf{I}_a$ when the inclusion is softer than the matrix $(K > K_1)$.

4. Proposal of the experiment

Let us consider the static one-dimensional motion of the inclusion:

$$\boldsymbol{\xi} = (\xi, 0, 0), \quad \dot{\boldsymbol{\xi}} = (v, 0, 0),$$
$$\mathbf{u}(\boldsymbol{\xi}) = (u(\xi), 0, 0), \quad \mathbf{F}_s(\boldsymbol{\xi}) = \big(F_s(\xi), 0, 0\big), \tag{4.1}$$
$$F_s(\xi) = -\frac{d\varphi}{d\xi}, \quad \varphi = \zeta d e(\xi), \quad e(\xi) = \frac{du}{d\xi}$$

and let us introduce undimensionless variables r and τ:

$$r = \frac{\xi}{l}, \quad \tau = \frac{t}{t_s} \quad \left(t_s = \frac{l}{v_s}, \quad v_s^2 = \frac{d}{M}\right), \tag{4.2}$$

where l — the characteristic linear parameter, e.g. the mean equilibrium distance between the inclusions. If the initial conditions have the form

$$r_0 = l^{-1}\xi(t_0), \quad v_0 = v(t_0) \neq 0, \tag{4.3}$$

then the velocity $v = v(t)$ of the inclusion has the following representation:

$$v(t) = \hat{v}\left(r\left(\frac{t}{t_s}\right)\right), \quad 0 < |t-t_0| < \varepsilon,$$
$$\hat{v}(r) = \operatorname{sgn} v_0 \sqrt{v_0^2 - 2v_s^2 \zeta \varkappa [\hat{e}(r) - \hat{e}(r_0)]}, \tag{4.4}$$
$$\hat{e}(r) = e(lr), \quad \varkappa = \operatorname{sgn}(r - r_0), \quad \zeta = \operatorname{sgn}(K - K_1).$$

If we consider the linearization of the velocity field $\hat{v}(r)$ for the small change δr of the position r_0 of the inclusion

$$\delta r = |r - r_0| = \frac{R_0}{l} \ll 1, \tag{4.5}$$

then we obtain the formula

$$\eta(r)\mathscr{R}(r) = \frac{R_0}{Mc_1} = C_0 = \text{const} \quad \text{for every} \quad r = \xi l^{-1}, \tag{4.6}$$

where c_1 is the velocity of irrotational waves in the matrix and

$$\eta(r) = \left|\frac{\hat{v}(r + l^{-1}R_0) - \hat{v}(r)}{c_1}\right|,$$
$$\mathscr{R}(r) = \left|\frac{\hat{v}(r)}{\hat{F}_s(r)}\right|, \quad \hat{v}(r) \neq 0, \quad \hat{F}_s(r) = F_s(lr). \tag{4.7}$$

The field velocity $\hat{v}(r)$ can be determined from the static one-dimensional experiment. The force $\hat{F}_s(r)$ can be determined using the formulae (3.3) or (4.1). The quantity $\mathscr{R}(r)$

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coincides with the so-called "mobility" of the inclusion which is the determinable quantity (cf. [7]). The quantity $\eta(r)$ is the measure of change of the mobility

$$\eta(r) = \text{const iff } \mathscr{R}(r) = \text{const}. \tag{4.8}$$

Experiments suggest that (cf. [7])$\mathscr{R}(r) \simeq \mathscr{R}_0 = \text{const}$ along the trajectory of the inclusion. Then $\eta(r) \simeq \eta_0 = \text{const}$ along the trajectory of the inclusion and the effective mass M can be estimated by

$$M = \frac{R_0}{C_0 c_1}, \quad C_0 \simeq \mathscr{R}_0 \eta_0. \tag{4.9}$$

The necessary condition (yet insufficient) of the correctness of our model is $M < m$(cf. (2.3)).

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