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Nonlocal, continuum models of large engineering structures(*)

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THE AIM of the paper has been to analyse the possibilities of modelling large engineering structures by nonlocal continua. An example of rod structure is discussed in some detail. The discrete description of the structure is known and can be made use of to estimate the accuracy of the results obtained by using individual continuum models. The integral and gradient nonlocal models of the discrete structure have been constructed. The modelling maps and the associated ways of determining the forces in the rods have been discussed.

Celem pracy jest analiza możliwości modelowania dużych struktur inżynierskich przez nielokalne kontinuum. Dyskutowany jest przykład struktury kratowej. Dyskretny opis ustroju jest znany i może być użyty w celu oszacowania dokładności wyników uzyskiwanych przy stosowaniu poszczególnych modeli kontynualnych. Konstruowane są całkowy i różniczkowy nielokalny model struktury dyskretnej. Dyskutowane są odwzorowania modelowe i związane z nimi sposoby określenia sił w prętach.

Целью работы является анализ возможности моделирования больших инженерских структур через нелокальное континуум. Обсуждается пример решетчатой структуры. Дискретное описание устройства известно и может быть использовано с целью оценки точности результатов получаемых, применяя отдельные континуальные модели. Построены интегральная и дифференциальная модели дискретной структуры. Обсуждаются модельные отображения и связанные с ними способы определения сил в стержнях.

Introduction

THE PURPOSE of the paper has been to analyse the possible use of various nonlocal, continuum models for describing engineering structures. Detailed considerations have been restricted to the case of a plane rod lattice structure and to the discussion of its integral and gradient models. The gradient model of higher order is considered to be a nonlocal model, although in a weaker sense [5].

The use of nonlocal models for describing large engineering structures provides a convenient tool for analysing these structures on the one hand. On the other hand, it makes it possible to test the various aspects of constructing nonlocal models using the example of a structure about which complete information is available. The second reason has been the main purpose of undertaking the present considerations.

If, from the formal point of view, descriptions of nonlocal models of crystalline lattices [4] turn out to be analogous to those of lattice structure models, then the analysis of the latter will allow, among other things, various boundary problems to be defined, depending on the formulation of support conditions of the structure and on the basis of certain criteria for evaluating the applicability of individual models depending on the problem to be solved.

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The considerations have been presented in two parts. The present part is concerned with analysing the models of unbounded structures, whereas the second part [12] deals with the analysis of the conditions imposed at the edge of the structure.

The gradient, nonlocal theory of continuous medium has been developed by TOUPIN [10], MINDLIN and TIERSTEN [6], ROGULA [7] and KUNIN [5], whereas the integral, nonlocal theory has been put forward and extended by KRÖNER [2] and DATTA [3, 4], EDELEN [1] and ERINGEN [2]. Generally speaking, the phenomenological approach to the continuum theory has been provided by ROGULA [8], whereas some boundary problems for bounded bodies have been discussed by KUNIN [5] and RYMARZ [9].

Continuum models describing large engineering structures have been used by WOŹNIAK [11], although nonlocal models have not been considered until now.

Our considerations will be exemplified by a rod structure made up of rods articulated at their nodes (Fig. 1). It has been assumed that external load operates at the nodes only, hence the rods of the structure transmit axial forces only.

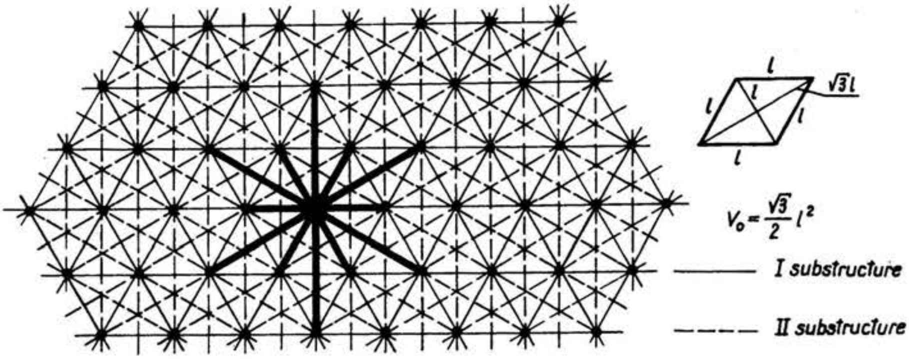

FIG. 1.

It can be easily seen that the structure can be split into two sub-structures (Fig. 1), of which the first (I) implements short-range interactions between the nodes of the structure, whereas the second (II) implements interactions of longer range. The example of the structure has been chosen so that a structure with nonlocal features may be obtained when the possible simplest problem is available.

Interactions of longer range than those implemented by substructure II can of course be introduced, but it is of no purpose at this stage of our study. However, it is essential to note that both substructures are geometrically invariant, which prevents additional complications in our work.

A homogeneous geometry of the structure and, additionally, homogeneous properties of materials have been assumed.

Nonlocal models describing unbounded structure without boundary conditions will be discussed in the present paper as follows.

Chapter 1 will describe a discrete structural model whose solution can be considered as rigorous. This solution also makes it possible to evaluate the accuracy of continuous models.

Chapter 2 will present a local model of the classical continuum providing an intermediate (less accurate) step in constructing a nonlocal integral model.

The nonlocal integral model will be described in Chapter 3 and the nonlocal gradient model in Chapter 4.

Chapter 5 will give a summary of the considerations concerning the application of the various nonlocal models.

In constructing individual structural models, the main problem reduces to obtaining an adequate correspondence between the model and the object being described. On the other hand, the problem within the model being constructed involves determining the relationship for the internal energy of the system. Given its form, it is easy to obtain structure equilibrium equations by applying the principle of minimum potential energy of the system. For determinate boundary value problems it is then possible (what will be discussed later [12]) to determine the displacement field of the structure and other quantities important from the engineering point of view.

1. Discrete structural model

Let r_i and r'_i describe the radius vectors determining the nodal points of the rod lattice structure. Assuming that the rods are made of the Hookean material, the force in the rod connecting the nodes, r_i and r'_i, of a deformed system can be related to the displacements of the nodes, r_i and r'_i, by the following relationships:

$$P_j(\mathbf{r},\mathbf{r}') = \Phi_{ji}^{\mathbf{rr}'}\big(u_i(\mathbf{r}')-u_i(\mathbf{r})\big), \tag{1.1}$$

where

$$\Phi_{ji}^{\mathbf{rr}'} = \begin{cases} \dfrac{E^{rr'}A^{rr'}}{(l^{rr'})^3}(r'_j-r_j)(r'_i-r_i) & \ldots \text{ for } \mathbf{r} \text{ and } \mathbf{r}' \text{ connected by a rod,} \\ 0\ , & \ldots \text{ for the remaining pairs of nodes,} \end{cases} \tag{1.2}$$

$E^{rr'}$, $A^{rr'}$ and $l^{rr'}$ being the Young modulus, the cross-sectional area and the length of the rod connecting the nodes $\mathbf{r}$ and $\mathbf{r}'$.

The internal elastic energy of the structure can be expressed by the equation

$$U = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} P_j(\mathbf{r},\mathbf{r}')\big(u_j(\mathbf{r}')-u_j(\mathbf{r})\big) = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} \Phi_{ji}^{\mathbf{rr}'}\big(u_i(\mathbf{r}')-u_i(\mathbf{r})\big)\big(u_j(\mathbf{r}')-u_j(\mathbf{r})\big). \tag{1.3}$$

It can be easily confirmed that for any deformed (geometrically invariant) structure the value of the energy U is positive (which means the stability of the system); since for all the rods of the system the following inequality is valid $\dfrac{EA}{l^3} > 0$.

Given the form of internal energy (1.3), the support condition of the structure and the external load, it is possible, by making use of the principle of minimum potential energy, to determine the actual state of displacements of the nodes.

This solution is considered to be exact, but rather inconvenient in the case of large structures. The purpose of further considerations will be to construct continuum models

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describing the present system not exactly, but with suitable accuracy, and more convenient to use for various reasons than the discrete model.

It can be seen that the form of internal energy (1.3) is identical with that of the energy of crystalline lattice with central interactions [4], where the condition for centrality of interactions corresponds to the assumption of the articulated nature of the nodes of the lattice system, not transmitting moment interactions. On the other hand, the distribution of mutual interactions in crystalline lattice corresponds to that of rigidity properties in the individual rods of the structure.

By including in the discussion a more general case of rod structures with rigid nodes where axial forces are accompanied by bending moments in rods, a description analogous to that of crystalline lattices with non-central two-point interactions is obtained.

An engineering system corresponding to crystalline lattice with multi-point interactions can be exemplified by a structure obtained by dividing a surface girder into finite elements. And thus for example, the introduction of finite triangular elements relates the state of stress in the element to that of displacements of its three corners.

Further considerations will be restricted to the case reported at the outset of two-point central interactions in lattice structure. Separate papers will deal with the analysis of more complex structures.

2. Local structural continuum model

The intermediate stage in constructing a nonlocal structural continuum model involves defining a (less accurate) local continuum model. Given the matrix of interactions (1.2) of discrete structure, it is possible to select the stress tensor of an equivalent classical elastic continuum. Here we take into account the criterion which claims that the local internal energy within each structural unit cell should, for the same homogeneous deformations, be identical for both the discrete and the continuum descriptions, respectively. Hence, search is instituted for the form of the stress tensor C_{ijkl} of elastic continuum such that the energy calculated for the region V_0 corresponding to a structural unit cell and related to homogeneous deformation β_{ik}:

$$U_{V_0} = \frac{1}{2} \int_{V_0} C_{lkjl} \beta_{lk} \beta_{jl} dV \tag{2.1}$$

should be equal to that of the discrete system calculated over the region V_0.

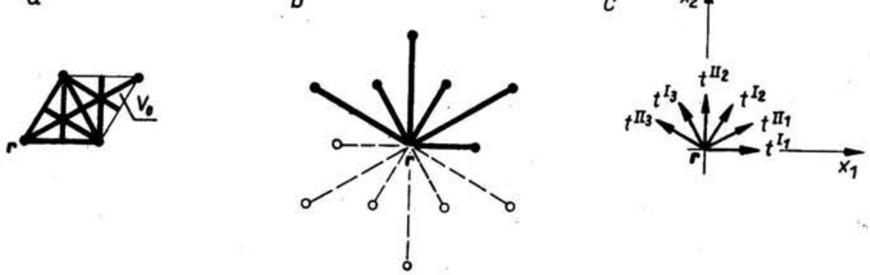

FIG. 2.

It follows from the homogeneity of the mechanical features of the structure that the tensor C_{ikjl} is constant over the entire region.

The structural unit cell — as shown in Fig. 2a, has been assumed. It can be noted that the internal energy of the unit cell is equal for homogeneous deformation to half the internal energy associated with the connection point for the node **r** (Fig. 2b).

Hence we have (cf. Eq. (1.3)):

$$U_{V_0} = \frac{1}{4} \sum_{\mathbf{r}'} \Phi_{ji}^{\mathbf{r}\mathbf{r}'} \big(u_i(\mathbf{r}') - u_i(\mathbf{r})\big)\big(u_j(\mathbf{r}') - u_j(\mathbf{r})\big). \tag{2.2}$$

Expressing the energy (2.2) with homogeneous deformation components β_{ik}, we obtain

$$U_{V_0} = \frac{1}{2}\Big[\sum_{\mathbf{r}'} \Phi_{ij}^{\mathbf{r}\mathbf{r}'}(r'_k - r_k)(r'_l - r_l)\Big]\beta_{ik}\beta_{jl}. \tag{2.3}$$

Comparing two representations (2.1) and (2.3) of the same value U_{V_0}, it is possible to determine the stress tensor of the required equivalent continuum:

$$C_{ijkl} = \frac{2}{\sqrt{3}}\Big[a^{\mathrm{I}} \sum_{\xi=1,2,3} t_i^{\mathrm{I}\xi} t_j^{\mathrm{I}\xi} t_k^{\mathrm{I}\xi} t_l^{\mathrm{I}\xi} + 3a^{\mathrm{II}} \sum_{\xi=1,2,3} t_i^{\mathrm{II}\xi} t_j^{\mathrm{II}\xi} t_k^{\mathrm{II}\xi} t_l^{\mathrm{II}\xi}\Big], \tag{2.4}$$

where the parameters a^{I} and a^{II} describe homogeneous rigidity characteristics in the substructures:

$$a^{\mathrm{I}} = \frac{E^{\mathbf{r}\mathbf{r}'}A^{\mathbf{r}\mathbf{r}'}}{l^{\mathbf{r}\mathbf{r}'}} \quad \text{for all the rods of substructure I,} \tag{2.5}$$

$$a^{\mathrm{II}} = \frac{E^{\mathbf{r}\mathbf{r}'}A^{\mathbf{r}\mathbf{r}'}}{l^{\mathbf{r}\mathbf{r}'}} \quad \text{for all the rods of substructure II,}$$

whereas the vectors $t_i^{\mathrm{I}\xi}$ and $t_i^{\mathrm{II}\xi}$ describe directional unit vectors of the rods of the first and second substructures (Fig. 2c).

The tensor C_{ijkl} can be resolved in a natural manner into two components

$$C_{ijkl} = C_{ijkl}^{\mathrm{I}} + C_{ijkl}^{\mathrm{II}} \tag{2.6}$$

describing the properties of substructures I and II, respectively:

$$\begin{aligned} C_{ijkl}^{\mathrm{I}} &= \frac{2}{\sqrt{3}}\, a^{\mathrm{I}} \sum_{\xi=1,2,3} t_i^{\mathrm{I}\xi} t_j^{\mathrm{I}\xi} t_k^{\mathrm{I}\xi} t_l^{\mathrm{I}\xi}, \\ C_{ijkl}^{\mathrm{II}} &= \frac{6}{\sqrt{3}}\, a^{\mathrm{II}} \sum_{\xi=1,2,3} t_i^{\mathrm{II}} t_j^{\mathrm{II}\xi} t_k^{\mathrm{II}\xi} t_l^{\mathrm{II}\xi}. \end{aligned} \tag{2.7}$$

Each of the stress tensors of the substructures has an axis of symmetry of the sixth order which, for a plane medium, is equivalent to its isotropy. Using the isotropy of the tensors C_{ijkl}^{I} and C_{ijkl}^{II} and the fact that they satisfy the conditions of the Cauchy symmetry relative to the permutations of all the four indices, they can be written down as follows:

$$\begin{aligned} C_{ijkl}^{\mathrm{I}} &= \lambda^{\mathrm{I}}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \\ C_{ijkl}^{\mathrm{II}} &= \lambda^{\mathrm{II}}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \end{aligned} \tag{2.8}$$

forgetting the internal structure that provides the starting point for constructing an equivalent continuous model.

Comparing Eqs. (2.7) and (2.8) and assuming (Fig. 2b),

$$
\begin{aligned}
& t^{I1} = [1, 0], && t^{II1} = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right],\\
(2.9)\qquad & t^{I2} = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right], && t^{II2} = [0, 1],\\
& t^{I3} = \left[-\frac{1}{2}, \frac{\sqrt{3}}{2}\right], && t^{II3} = \left[-\frac{\sqrt{3}}{2}, \frac{1}{2}\right],
\end{aligned}
$$

we obtain the Lamé constants $\lambda^I = \mu^I$ and $\lambda^{II} = \mu^{II}$ for classical continuum:

$$
\begin{aligned}
& \lambda^I = \frac{3}{4\sqrt{3}} a^I,\\
(2.10)\qquad & \lambda^{II} = \frac{3\sqrt{3}}{4} a^{II}.
\end{aligned}
$$

The material constants (2.10) or more generally Eq. (2.4) of equivalent elastic continuum are uniquely determined by the geometry and rigidity features of the starting discrete structure. However, many various rod structures with identical features of equivalent continuum medium can be selected, which results from reducing the number of parameters determining the system under consideration to two λ^I and λ^{II}.

The forms (2.7) of the tensors of material features of the equivalent elastic continuum are in agreement with those derived (in another manner) for structures with a homogeneous range of interactions by WOŹNIAK [11].

3. Nonlocal integral structural model

Let us now construct a nonlocal continuum model of the structure under consideration. It should make stresses at the point of the medium dependent upon deformations within the range of interactions. Such a model will give results closer to those of a strict solution, as compared with those obtained by means of the local model discussed above. The greater the differences, the more heterogeneous will the state of strain discussed be. In particular, differences in the displacement fields determined by the use of these two models will appear in boundary regions.

Let us distribute the values of the above-determined tensor components $\mathbf{C}^I$ and $\mathbf{C}^{II}$ at point $\mathbf{r}$ uniformly over the regions of interactions of the connection points of sublattices I and II:

$$
\begin{aligned}
& C^I_{ijkl}(\mathbf{r}, \mathbf{r}') = \frac{C^I_{ijkl}(\mathbf{r}) h^I(\mathbf{r}, \mathbf{r}')}{\Omega^I},\\
(3.1)\qquad & C^{II}_{ijkl}(\mathbf{r}, \mathbf{r}') = \frac{C^{II}_{ijkl}(\mathbf{r}) h^{II}(\mathbf{r}, \mathbf{r}')}{\Omega^{II}},\\
& C_{ijkl}(\mathbf{r}, \mathbf{r}') = C^I_{ijkl}(\mathbf{r}, \mathbf{r}') + C^{II}_{ijkl}(\mathbf{r}, \mathbf{r}').
\end{aligned}
$$

where

$\Omega^{I} = \pi(l^{I})^2 = \pi l^2$ } the areas of interaction regions of substructures I and II
$\Omega^{II} = \pi(l^{II})^2 = 3\pi l^2$ } (Fig. 3),

$h^{I}(\mathbf{r}, \mathbf{r}')$ } the characteristic functions of the regions of interactions in the neighbour-
$h^{II}(\mathbf{r}, \mathbf{r}')$ } hood of point $\mathbf{r}$ of substructures I and II.

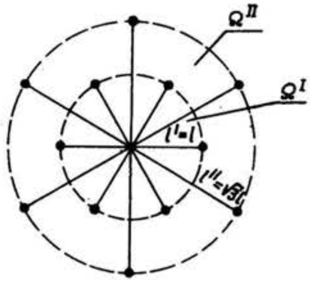

FIG. 3.

The total internal energy of the structure will be expressed by the equation

$$U = \iint_V C_{ijkl}(\mathbf{r}, \mathbf{r}')\varepsilon_{kl}(\mathbf{r}')\varepsilon_{ij}(\mathbf{r})\,d\mathbf{r}\,d\mathbf{r}' \tag{3.2}$$

describing the integral model of continuous medium with nonlocal interactions (cf. [4]).

4. Gradient structural model

Another continuum model capable of being used for describing approximately discrete rod structures of long-range interactions is the gradient model.

It is assumed that the relative displacement of two arbitrary points of structure is represented by the sum of two consecutive terms of the expansion into the exponential series, i.e. linear and quadratic

$$u_i(\mathbf{r}') - u_i(\mathbf{r}) = \beta^{r}_{ij}(r'_j - r_j) + \frac{1}{2}\gamma^{r}_{ijl}(r'_j - r_j)(r'_l - r_l), \tag{4.1}$$

where the coefficients of expansion are gradients of the first and second orders, respectively, of the displacements field $\mathbf{u}$, made continuous:

$$\begin{aligned} \beta^{r}_{ij} &= u_{(i,j)}(\mathbf{r}), \\ \gamma^{r}_{ijk} &= \beta_{i(j,k)}(\mathbf{r}). \end{aligned} \tag{4.2}$$

Elastic energy of the structure (1.3) can be expressed by the states of strain of the first and second orders β and γ

$$U = \frac{1}{2}\sum_{r,r'} \Phi^{rr'}_{ij}\left[\beta^{r}_{ik}(r'_k - r_k) + \frac{1}{2}\gamma^{r}_{ikl}(r'_k - r_k)(r'_l - r_l)\right] \times \left[\beta^{r}_{jm}(r'_m - r_m) + \frac{1}{2}\gamma^{r}_{jmn}(r'_m - r_m)(r'_n - r_n)\right]. \tag{4.3}$$

It can be noted that in view of the principle of reciprocity $\Phi_{ij}^{rr'} = \Phi_{ij}^{r'r}$, the mixed terms $\beta_{ik}\gamma_{jmn}$ cannot appear in the equation for energy and that Eq. (4.3) can be written in the form

$$(4.4)\qquad U = \frac{1}{2}\sum_{r,r'} \Phi_{ij}^{rr'}\Big[(r'_k - r_k)(r'_l - r_l)\beta_{ik}^r\beta_{jl}^r + \frac{1}{4}(r'_k - r_k)(r'_l - r_l)(r'_m - r_m)(r'_n - r_n)\gamma_{ikl}^r\gamma_{jmn}^r.$$

Using the definition (1.2), the energy U for the homogeneous structure (Fig. 1) takes the form

$$(4.5)\qquad U = \frac{1}{2}\sum_{r}\Big\{\Big[b^{\rm I}\sum_{\xi=1,2,3} t_i^{\rm I\xi}t_j^{\rm I\xi}t_k^{\rm I\xi}t_l^{\rm I\xi} + b^{\rm II}\sum_{\xi=1,2,3} t_i^{\rm II\xi}t_j^{\rm II\xi}t_k^{\rm II\xi}t_l^{\rm II\xi}\Big]\beta_{ij}^r\beta_{kl}^r + \Big[c^{\rm I}\sum_{\xi=1,2,3} t_i^{\rm I\xi}t_j^{\rm I\xi}t_k^{\rm I\xi}t_l^{\rm I\xi}t_m^{\rm I\xi}t_n^{\rm I\xi} + c^{\rm II}\sum_{\xi=1,2,3} t_i^{\rm II\xi}t_j^{\rm II\xi}t_k^{\rm II\xi}t_l^{\rm II\xi}t_m^{\rm II\xi}t_n^{\rm II\xi}\Big]\gamma_{ijk}^r\gamma_{lmn}^r\Big\},$$

where

$$b^{\rm I} = E^{\rm I}A^{\rm I}l^{\rm I}, \qquad c^{\rm I} = E^{\rm I}A^{\rm I}(l^{\rm I})^3,$$
$$b^{\rm II} = E^{\rm II}A^{\rm II}l^{\rm II}, \qquad c^{\rm II} = E^{\rm II}A^{\rm II}(l^{\rm II})^3.$$

On passing to continuum description, we obtain (just as in Chapters 2 and 3):

$$(4.6)\qquad U = \frac{1}{2}\int_V (C_{ijkl}\beta_{ij}\beta_{kl} + C_{ijklmn}\gamma_{ijk}\gamma_{lmn})\,dV,$$
$$C_{ijkl} = C_{ijkl}^{\rm I} + C_{ijkl}^{\rm II}, \qquad C_{ijklmn} = C_{ijklmn}^{\rm I} + C_{ijklmn}^{\rm II},$$

where

$$C_{ijkl}^{\rm I} = \frac{b^{\rm I}}{V_0}\sum_{\xi=1,2,3} t_i^{\rm I\xi}t_j^{\rm I\xi}t_k^{\rm I\xi}t_l^{\rm I\xi},$$

$$C_{ijkl}^{\rm II} = \frac{b^{\rm II}}{V_0}\sum_{\xi=1,2,3} t_i^{\rm II\xi}t_j^{\rm II\xi}t_k^{\rm II\xi}t_l^{\rm II\xi},$$

$$C_{ijklmn}^{\rm I} = \frac{C^{\rm I}}{V_0}\sum_{\xi=1,2,3} t_i^{\rm I\xi}t_j^{\rm I\xi}t_k^{\rm I\xi}t_l^{\rm I\xi}t_m^{\rm I\xi}t_n^{\rm I\xi},$$

$$C_{ijklmn}^{\rm II} = \frac{C^{\rm II}}{V_0}\sum_{\xi=1,2,3} t_i^{\rm II\xi}t_j^{\rm II\xi}t_k^{\rm II\xi}t_l^{\rm II\xi}t_m^{\rm II\xi}t_n^{\rm II\xi}.$$

$V_0 = \dfrac{\sqrt{3}}{2}\, l^2$ being the unit cell area.

Attention is focussed on the fact that in the case when substructure II represents interactions of a range much longer than does substructure I, ($C^{\rm II} \gg C^{\rm I}$), it is justified to neglect the term $C_{ijklmn}^{\rm I}$ as that exerting negligible influence on the accuracy of solution. This is associated with the fact that the use of the first term of the expansion (4.1) only for short-range interactions can give the accuracy of the description of displacement field similar to that obtained by using the two first terms of the development (4.1) in relation to a substructure with long-range interactions.

5. Modelling pragmatics

Two nonlocal continuum models describing an unbounded discrete structure have been described above. The determined forms of internal energy being positive for geometrically invariant structures allow the deformation area of a modelling medium to be determined by making use of the principle of minimum potential energy of structure. In addition to the knowledge of the expressions (3.2) and (4.6), it is necessary for this purpose to determine also the model representations of the external load system $\mathbf{f}$ and boundary geometric constraints $\hat{\mathbf{u}}$. In effect, it is necessary to determine their approximations $\tilde{\mathbf{f}}$ and $\tilde{\hat{\mathbf{u}}}$ made continuous:

$$\mathbf{f}, \hat{\mathbf{u}} \Rightarrow \tilde{\mathbf{f}}, \tilde{\hat{\mathbf{u}}}. \tag{5.1}$$

Here, we have adopted the principle that double arrows describe transitions between the continuous and the discrete model, whereas single arrows correspond to transitions inside one of the models.

Given the model quantities $\tilde{\mathbf{f}}$ and $\tilde{\hat{\mathbf{u}}}$, we obtain the displacement field determined in one of the continuum descriptions:

$$\tilde{\mathbf{f}}, \tilde{\hat{\mathbf{u}}} \rightarrow \tilde{\mathbf{u}}. \tag{5.2}$$

The problem of returning the solution thus obtained from the continuous model to discrete structure may involve, depending on the problem under consideration, the necessity of determining various quantities whose accuracy of determination reflects that of the model in relation to the accurate solution (in agreement with a certain standard).

In engineering problems, the displacements of the structural nodes $\mathbf{u}$ and, particularly, forces in the rods $\mathbf{P}$ are interesting as solutions. Consequently, the accuracy of determining internal forces should provide a measure of correctness (from the engineering point of view) of the model adopted.

A system of internal forces in the structure can be determined by two methods. The first of them involves the transition of the displacement model field $\tilde{\mathbf{u}}$ to nodal displacements of a discrete system:

$$\tilde{\mathbf{u}} \Rightarrow \mathbf{u}, \tag{5.3}$$

which presents no difficulty and, next, the determination of the forces in the rods of substructures $\mathbf{P}^{\mathrm{I}}$ and $\mathbf{P}^{\mathrm{II}}$ from the constitutive relationships of the discrete model (1.1):

$$\tilde{\mathbf{u}} \Rightarrow \mathbf{u} \begin{matrix} \nearrow \mathbf{P}^{\mathrm{I}} \\ \searrow \mathbf{P}^{\mathrm{II}} \end{matrix} \tag{5.4}$$

The second method involves the necessity of determining stress states in the continuous models of individual substructures $\tilde{\boldsymbol{\sigma}}^{\mathrm{I}}$ and $\tilde{\boldsymbol{\sigma}}^{\mathrm{II}}$ by using the relationships (3.1) and (4.6).

Next, by using the assumption of local uniformity of states σ^I, and σ^{II}, it is possible to determine forces in the individual rods P^I and P^{II} as the corresponding components of the states of stress $\tilde{\sigma}^I$ and $\tilde{\sigma}^{II}$.

$$(5.5)\qquad \tilde{u} \begin{array}{l} \nearrow \tilde{\sigma}^I \Rightarrow P^I \\ \searrow \tilde{\sigma}^{II} \Rightarrow P^{II} \end{array} .$$

A detailed construction of the states of stresses in substructures and discussion of boundary problems for the nonlocal models under discussion will be carried out in the second part of the paper [12].

On the one hand, it will provide the basis for analysing the applicability of individual nonlocal models to describe definite engineering systems and, on the other, to provide a mechanical interpretation for the analysis of boundary problems in nonlocal continuum.

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