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A velocity potential panel method for the prediction of unsteady airloads on oscillating wings and bodies(*)

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A METHOD is presented to calculate unsteady airloads on oscillating three-dimensional wings and bodies in subsonic flow. This method is based on the velocity potential using distributions of harmonically pulsating doublets in the case of wings, and harmonically pulsating sources and sinks in the case of bodies. On account of a panel type method, the oscillating surfaces are divided into small surface elements, -panels-, each with a constant yet unknown singularity distribution. The unknown singularity strengths are calculated using the solution of a large system of linear equations. The method is applied to a variety of geometrical configurations and flow conditions. The results are compared with other methods as well as with experimental results.

Przedstawiono metodę obliczania nieustalonych sił parcia powietrza działających na drgające trójwymiarowe płaty i kadłuby przy prędkościach poddźwiękowych. Metoda opiera się na potencjale prędkości przy wykorzystaniu harmonicznie pulsujących dipoli w przypadku płatów oraz harmonicznie pulsujących źródeł i upustów w przypadku kadłubów. W przyjętej koncepcji drgające powierzchnie podzielone są na małe elementy powierzchniowe, tzw. panele, z których każdy scharakteryzowany jest przez pewien nieznany rozkład osobliwości; ich intensywności obliczone są z rozwiązania dużego układu równań liniowych. Opisana metoda zastosowana jest do szeregu konfiguracji geometrycznych oraz warunków przepływu. Wyniki porównane są z rezultatami otrzymanymi za pomocą innych metod oraz z danymi doświadczalnymi.

Представлен метод расчета стационарных аэродинамических нагрузок на трехмерные, совершающие колебания крылей и фюзеляжей. Метод основан на потенциалах скорости с использованием распределения гармонических пульсирующих пар (в случае крылев) и гармонически пульсирующих источников и стоков (в случае фюзеляжей). Используется метод „панельного'' типа, осциллирующие поверхности разделяются на малые поверхностные элементы „панели'', каждая с постоянным, пока неизвестным распределением особенностей. Неизвестные амплитуды особенностей расчитываются путем решения большой системы линейных уравнений. Метод применяется для различных геометрических конфигураций и условий течения. Результаты сопоставляются с полученными с помощью других методов, а также с экспериментальными данными.

Nomenclature

- x, y, z Cartesian coordinates (Fig. 1),
- x, r, φ polar coordinates (Fig. 4),
- l_0, L reference length (Fig. 2, Fig. 4),
- $l(y)$ local lift chord,
- s wing half span (Fig. 2),
- F wing surface,
- Λ aspect ratio, $\Lambda = 4s^2/F$,
- x_A location of pitching axis,
- r distance between sending point and control point,
- t time,

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ω frequency,
ξ displacement of surface point,
U_∞ mainflow velocity,
α_s steady mean incidence (Fig. 4),
Ma_∞ Mach number, $\mathrm{Ma}_\infty = U_\infty/a_\infty$,
Φ transformed velocity potential,
$\Delta\Phi$ potential difference between wing upper and lower surface,
Δc_p unsteady pressure difference between wing lower and upper surface, $\Delta c_p = \Delta c'_p + i\Delta c''_p$,
$\bar{c}_{pl}$ unsteady pressure on body surface, $\bar{c}_{pl} = \bar{c}'_{pl} + i\bar{c}''_{pl}$,
ϱ density,
C oscillation amplitude,
c_a local unsteady lift coefficient $c_a = c'_a + ic''_a = \dfrac{dA/dy}{\frac{\varrho}{2} U_\infty^2 l(y) C}$,
c_m local unsteady pitching moment coefficient

$$c_m = c'_m + ic''_m = \frac{dM/dy}{\frac{\varrho}{2} U_\infty^2 l(y)^2 C}.$$

1. Introduction

FOR THE CALCULATION of unsteady airloads on oscillating lifting surfaces a variety of methods have been developed during recent years which are known as the kernel function methods [1, 2] or the doublet lattice methods [3]. Both concepts are based on the acceleration potential or pressure function with the obvious advantage that a wake, where no pressure jumps exist, has not to be taken into consideration.

In the following a velocity potential concept is used in connection with a panel type method which was first described by JONES and MOORE [4] and which has been consequently developed to handle more complicated three-dimensional flow problems on oscillating wings and bodies [5, 6]. The advantage in applying a velocity potential concept is that the aerodynamic influence functions are rather simple and can even be handled analytically in the case of incompressible flows. The method can be used for simple planforms as well as for arbitrary thick and cambered wings and last, but not least, also for bodies. The additional semi-infinite wake integrals can also be expressed by a sum over discrete wake strips. The solution of these integrals is not simple but can be carried out numerically in a sufficient way.

2. Governing equations, numerical solution procedure

2.1. Infinite thin wings

Using Green's theorem for the linearized subsonic unsteady potential equation and taking into account only harmonic time dependency, the potential equation can be transformed in the usual way into an integral equation for the unknown velocity potential:

$$4\pi \frac{\partial \Phi}{\partial Z} = \iint_F \Delta\Phi \frac{\partial^2}{\partial Z^2}\left(\frac{e^{-i\varkappa r}}{r}\right) dX dY + \iint_{Wa} \Delta\Phi(X_t, Y) e^{-i\nu(X-X_t)} \frac{\partial^2}{\partial Z^2}\left(\frac{e^{-i\varkappa r}}{r}\right) dX dY \tag{2.1}$$

with $\Delta\Phi$ as the complex potential jump on the thin lifting surface and r as the distance between the control point and the integration point on the wing. The first term in Eq. (2.1) expresses the influence of a doublet distribution on the wing surface F. The second term is the influence of the wake Wa where the doublet strengths of the wake are related to the doublet strength at the wings' trailing edge by applying the Kutta condition.

Assuming the oscillation of the wing to be known, the downwash on the wings' surface can be given as

$$W = -\left(\frac{\partial \Phi}{\partial Z}\right) = \frac{1}{\beta}\left(\zeta' i\omega^* + \frac{\partial \zeta'}{\partial X}\right) e^{-i(\lambda X + \omega^* T)} \tag{2.2}$$

with the displacement of an arbitrary surface point

$$\zeta = \zeta'(X, Y) e^{i\omega^* T}$$

and

$$\omega^* = \frac{\omega \cdot l_0}{U_\infty}, \quad \varkappa = \frac{\mathrm{Ma}_\infty \cdot \omega^*}{\beta^2}, \quad \lambda = \varkappa \cdot \mathrm{Ma}_\infty, \quad \text{with } \omega^*$$

as the reduced frequency.

The coordinates in Eq. (2.2) and the time t are made dimensionless in the usual way by

$$X = x/l_0; \quad Y = y \cdot \beta/l_0; \quad Z = z \cdot \beta/l_0; \quad T = t \cdot U_\infty/l_0$$

with

$$\beta = \sqrt{1 - \mathrm{Ma}_\infty^2}, \quad \nu = \omega^*/\beta^2.$$

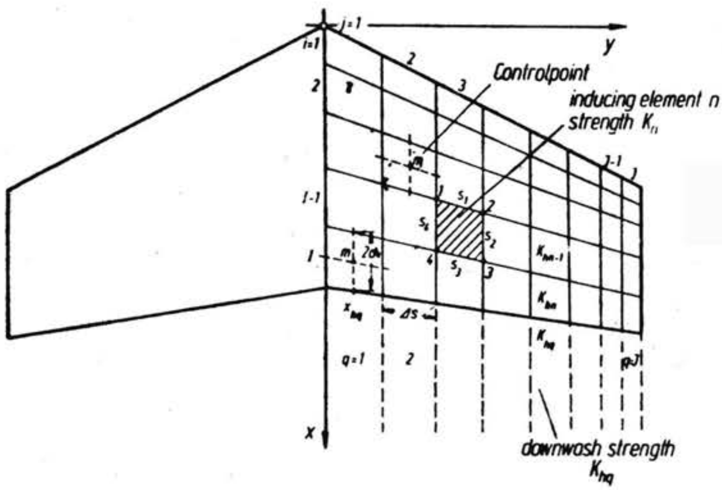

FIG. 1. Coordinate system, panel distribution for wing configuration.

The problem which has to be solved now is to find the doublet strengths of an arbitrary doublet distribution on the wing and wake surfaces, respectively, in such a way that the normal velocities induced by all doublets are equal but opposite in sign to the prescribed downwash velocity of the wing (kinematic boundary condition).

To solve this integral equation a panel type concept is used (Fig. 1) splitting up the planform of the wing into a number of small surface panels. Here, the outer edges of the wing surface as well as control surfaces etc. are taken into account as panel side edges. The wake behind the wing is represented by a corresponding number of small wake strips (Fig. 1). On each wing surface panel the potential difference $\Delta\Phi$ is assumed to be constant. The geometrical midpoint of the panel is taken as the control point where the kinematic boundary condition has to be fulfilled.

With this discretization procedure the integral equation can be rearranged in a linear system of equations:

$$(2.3)\qquad \sum_{n=1}^{I\cdot J} \Delta\Phi_n \iint\limits_{\Delta F_n} \frac{\partial^2}{\partial Z^2}\left(\frac{e^{-i\varkappa r}}{r}\right) dX\,dY + \sum_{q=1}^{J} \Delta\Phi_{tq} \iint\limits_{Wa_q} e^{-i\nu(X-X_{tq})} \frac{\partial^2}{\partial Z^2}\left(\frac{e^{-i\varkappa r}}{r}\right) dX\,dY = W$$

with I — number of panels in chordwise direction and J — number of panels and wake strips in spanwise direction. The double integrals in Eq. (2.3), the aerodynamic influence functions, must be treated in such a way that they are easy to solve by numerical means. In [5] it is shown that the surface integrals in Eq. (2.3) can be reduced to line integrals around the side edges of the panel. Evaluation of the term $e^{-i\varkappa r}$ in series leads to simple analytical expressions. It is sufficient to distinguish between a nearfield and a farfield solution of the integrals with respect to the distance of the control point from the panel under consideration. The corresponding formulae are given in [5]. Special care must be exercised in the evaluation of the wake integrals (the second term in Eq. (2.3)). The integration in spanwise direction of the wake strip can be carried out analytically. But a semi-infinite integral in streamwise direction is left. This integral is transformed into an integral with finite boundaries and then treated in a similar manner as the wing influence functions.

After the solution of the linear system of Eq. (2.3), the pressure jump on the wing surface can be calculated by applying the Bernoulli theorem:

$$(2.4)\qquad \Delta c_p = \frac{\Delta p}{\frac{\varrho}{2} U_\infty^2 C} = 2\left[i\nu\Delta\Phi + \frac{\partial\Delta\Phi}{\Delta X}\right] e^{i\lambda X} = \Delta c_p' + i\Delta c_p''.$$

The pressure coefficient is also a complex number expressed by real ($\Delta c_p'$) and imaginary ($\Delta c_p''$) parts. In Eq. (2.4) C is the oscillation amplitude.

2.2. Bodies

For the case of bodies with arbitrary thickness only the incompressible case has been treated until now. In this case a source-sink distribution is used on the real body surface. The geometric boundary condition is not linearized in the body case. A static mean angle of attack can be taken into account. On account of the nonlinearized geometric boundary condition, the solution procedure is not automatically divided into a steady and an unsteady solution as is the case for thin wings. Now the unsteady solution builds up on the steady one and therefore the latter must also be known.

On the other hand the calculation process is similar to that of the thin wing case. The body surface is split up into a number of small panels each represented by a constant yet unknown source or sink distribution of harmonically pulsating strength. The calculation of the prescribed normal velocities on the body surface representing the known right hand side of the linear system of equations is now exact.

In [6] it is shown that the pressure distributions on the body surface can be calculated as steady, first and even higher harmonic parts.

3. Results for oscillating three-dimensional wings in subsonic flow

Figure 2 shows the results for an oscillating rectangular wing of aspect ratio $\Lambda = 2$ with pitching oscillations about the midchord axis. The pressure distributions are given in a section close to the wing symmetry plane and are compared to the corresponding

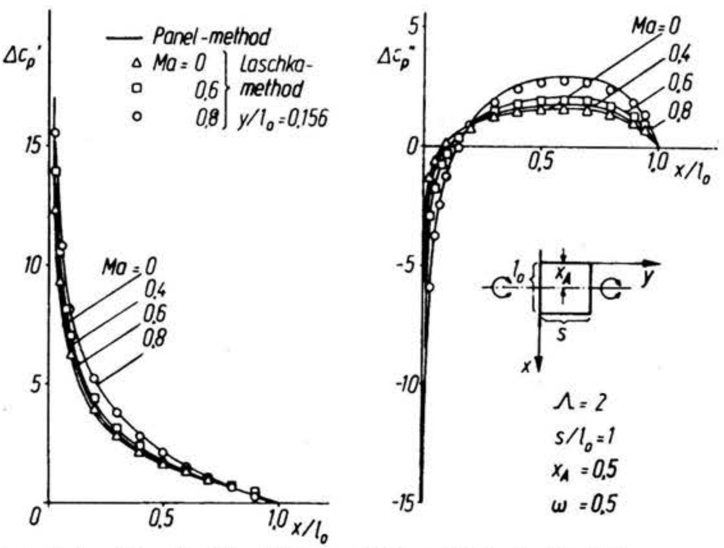

FIG. 2. Rectangular wing ($\Lambda = 2$) with pitching oscillations. Unsteady chordwise pressure distributions.

results given in [1]. For all Mach numbers the differences between the two methods are small. A similar behaviour can be observed for the spanwise pressure distributions.

Figure 3 gives local unsteady lift and pitching moment coefficients for a swept wing of aspect ratio $\Lambda = 2$ for Mach number $Ma_\infty = 0.8$. The results of the present method

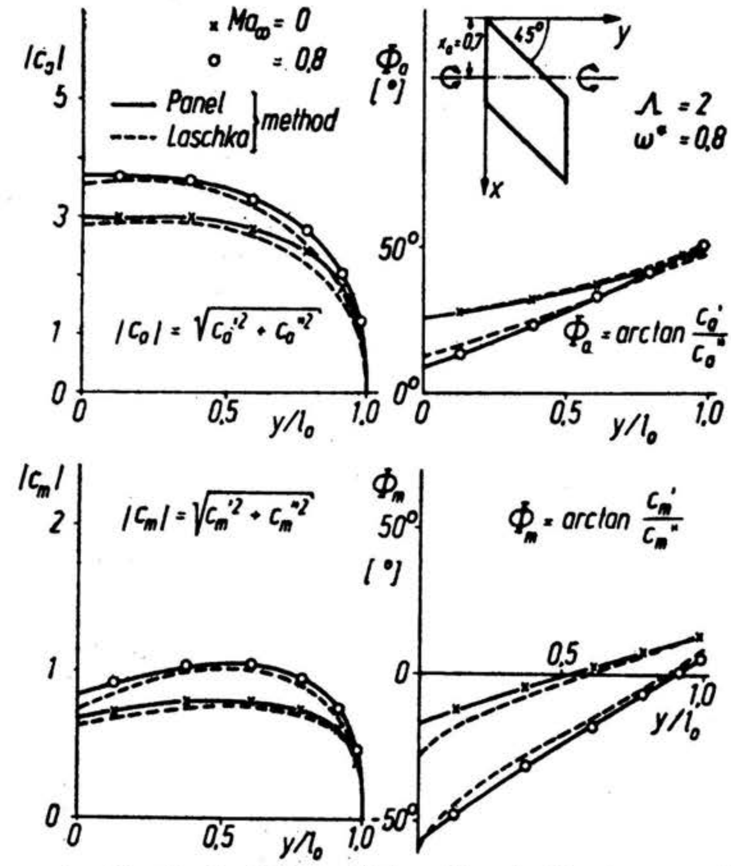

FIG. 3. Swept wing ($\Lambda = 2$) with pitching oscillations. Unsteady lift and moment distributions.

are again compared to the results of the kernel function method. The differences between the two methods which are based on quite different theoretical concepts are again sufficiently small.

4. Results for oscillating bodies including the ground effect

Figure 4 shows the coordinate system and panel arrangement. The body is allowed to make pitching or plunging oscillations. A static mean angle of attack (α_s) can be taken into account. Representing the ground, the mirror concept is used assuming a second body in double ground distance $2y_B$.

In Fig. 5 the case without ground ($y_B \to \infty$) is investigated first. The bodies' cross-section in this case is a spheroid of axis ratio 5 with a cylindrical afterbody. This more complicated cross-section can easily be represented by using a panel method. Figure 5 gives unsteady pressure distributions (real and imaginary parts) for the body undergoing plunging oscillations. The theoretical results are compared with experimental data obtained in the low speed wind tunnel of the DFVLR-AVA in Göttingen [7]. These measurements

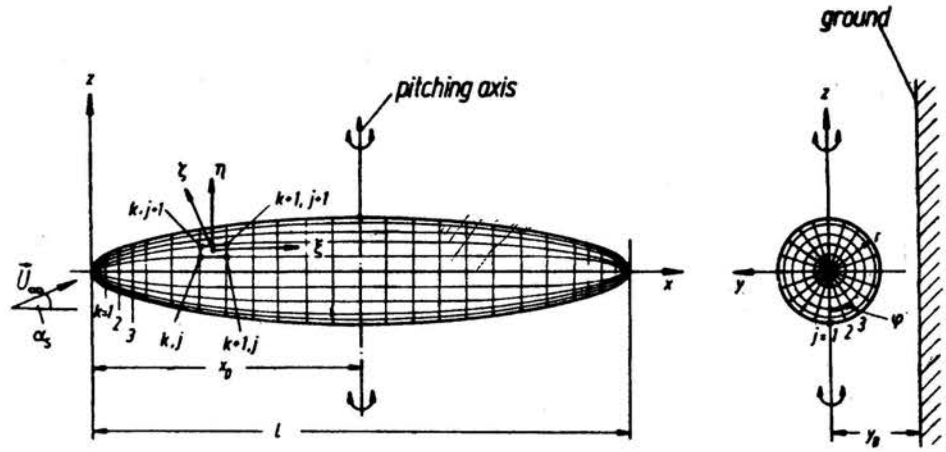

FIG. 4. Coordinate system, panel distributions for bodies.

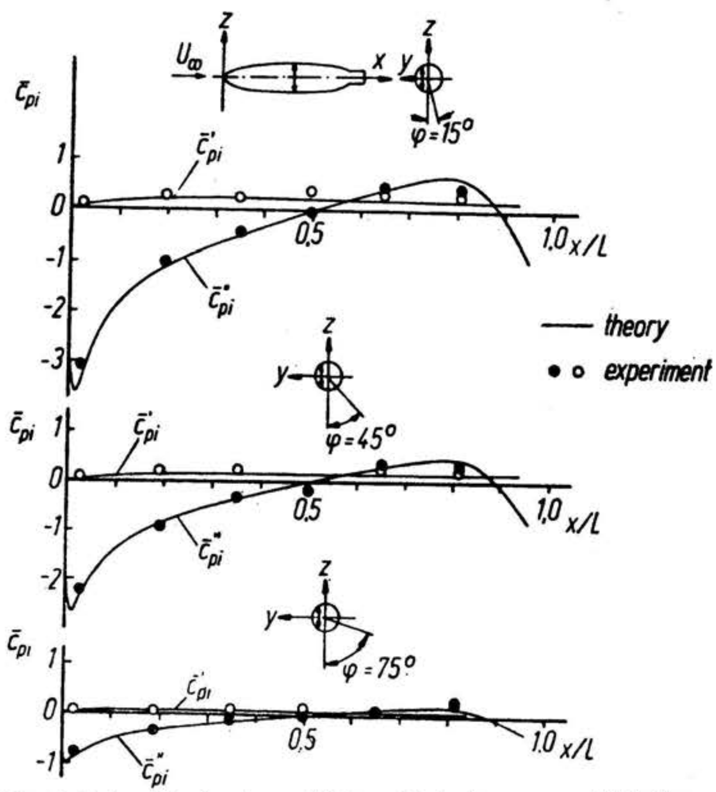

FIG. 5. Body with plunging oscillations. Unsteady pressure distributions.

were part of a wing-store investigation where some few test cases have been carried out with the store alone.

The results are given at three different circumferential positions. The differences between calculated and measured results are remarkably small.

Figure 6 gives results for a body (spheroid of axis ratio 5) with and without ground effect. The body makes pitching oscillations about the midchord axis (Fig. 6). The upper diagram of Fig. 6 gives steady results for two different circumferential positions in the ground case ($y_B = 0.15$) compared to results without ground ($y_B = \infty$). The latter results

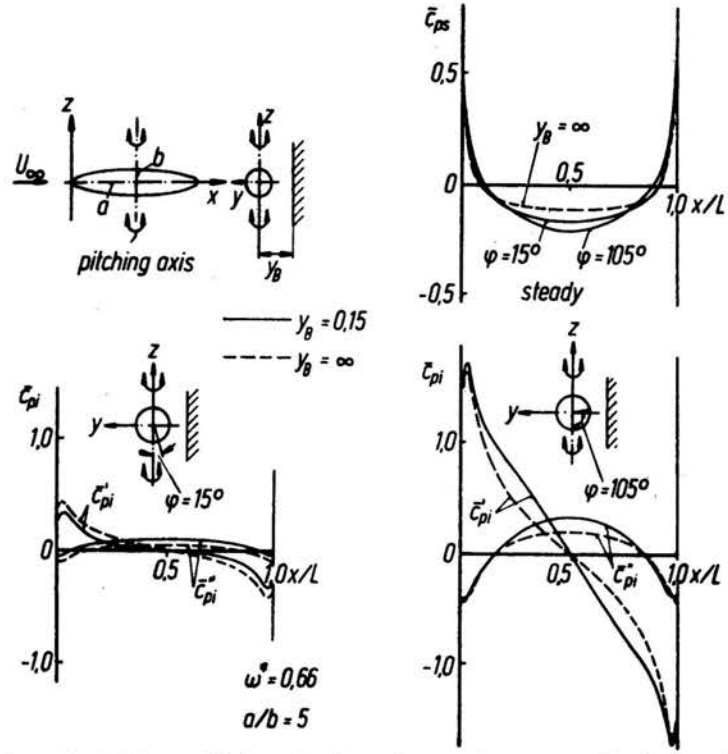

FIG. 6. Spheroid with pitching oscillations. Steady and unsteady pressure distributions including the ground effect.

are of course unchanged for different circumferential positions φ. In both φ-positions the pressure minimum is increased compared to the case without ground. The lower diagrams in Fig. 6 show unsteady pressure distributions at the same φ-positions again compared with the case without ground. There are considerable interference effects of the ground for this mode of oscillation.

In a similar manner a variety of other oscillation modes can be investigated due to the fact that these modes only influence the right hand side of the corresponding linear system of equations, whereas the aerodynamic influence functions as well as the static conditions remain unchanged in all unsteady cases.

5. Conclusion

A numerical method has been presented to calculate unsteady airloads on oscillating three-dimensional wings and bodies in subsonic flow. The velocity potential concept is used in contrast to the existing methods based on the acceleration potential or pressure function. This has the advantage of relatively simple aerodynamic influence functions and the possibility to extend the method to arbitrary body geometries. In the nonlinearized case of a finite thick body, the exact geometrical boundary condition is taken into account. The results are steady, first and even higher harmonic pressure distributions.

Some typical results for the mean surface case of three-dimensional wings are given and compared with the kernel function method for the whole subsonic flow regime.

Steady and unsteady results for oscillating bodies are given with and without ground effect. It is thought to extend the body case also to compressible subsonic flows.

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