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Material functions of creep of nonlinear viscoelastic anisotropic plastics

S. OCHELSKI (WARSZAWA)

THE PRESENT paper presents an analysis of results of wide experimental studies of creep of three plastics with anisotropy of different degree and kind, in a plane stress state and with a complex history of loading. An approximation of the relation between deformations and a functional of speed of change of stress for a viscoelastic material, by means of a polynomial representation of material functions, was proposed. A method of determining material functions was given and then, basing on experimental results, the functions were determined for the three plastics taken for testing. The influence of various parameters on the properties of the material functions were studied in detail. The material functions determined by this method display first of all the nonlinear properties of materials and allow to determine the degree of their nonlinearity.

Praca przedstawia analizę wyników z przeprowadzonych obszernych, doświadczalnych badań pełzania w płaskim stanie naprężenia i złożonej historii obciążenia trzech tworzyw o różnym stopniu i rodzaju anizotropii. Zaproponowano aproksymację prawa między deformacjami a funkcjonałem z prędkości zmiany naprężenia materiału lepkosprężystego, reprezentacją wielomianową funkcji materiałowych. Podano metodę wyznaczania funkcji materiałowych, a następnie na podstawie wyników doświadczalnych, wyznaczono je dla trzech tworzyw przyjętych do badań. Szczegółowo zbadano wpływ różnych parametrów na własności funkcji materiałowych. Funkcje materiałowe określone tą metodą charakteryzują przede wszystkim własności nieliniowe materiałów i pozwalają określić stopień ich nieliniowości.

Работа представляет анализ результатов, из проведенных обширных, экспериментальных исследований ползучести в плоском напряженном состоянии и со сложной историей нагружения, трех материалов с разной степенью и родом анизотропии. Предложена аппроксимация закона между деформациями и функционалом скорости изменения напряжения вязкоупругого материала, многочленным представлением материальных функций. Приведен метод определения материальных функций, а затем на основе экспериментальных результатов, они определены для трех материалов принятых для исследований. Подробно исследовано влияние разных параметров на свойства материальных функций. Материальные функции, определенные этим методом, характеризуют прежде всего нелинейные свойства материалов и позволяют определить степень их нелинейности.

1. Introduction

IN THE PRESENT paper a method is proposed for the determination of material functions; it is concerned with creep tests in a plane state of stress in a complex history of loading of materials, anisotropic and viscoelastic, of nonlinear characteristics. In describing the creep effect it was assumed that this method would be additive in terms of three experimentally determined functions: material functions, creep functions and a function describing anisotropic properties.

As it is well known, nearly all the real materials after a certain history of creep inducing loading behave in a different way than in the case where there is no loading history, that is, with a zero loading history. Thus materials remember the effects of former actions of loading. In connection with this fact, in order to obtain a better agreement of theoretical curves with experimental ones, it is proposed in papers [1, 2] to improve the theory by introducing reinforcement parameters.

On the other hand, in [3] it is assumed that the velocity of creep deformation at a jump change of loading depends not only on the actual value of deformation but also on the scale of the accumulated damage at the time of the change of loading. In [4] a method was proposed for dividing the material sensitive to change in the loading path into several groups by analyzing the forms of the constitutive functional.

In the case of viscoelastic materials, these problems have been considered by STAFFORD [5]; here the results from creep tests at constant loading are extrapolated to tests with a jump change of loading. In order to describe more adequately the results from tests with a jump change of loading, the mentioned author introduces into the equations various forms of combination of stress acting before and after a jump is completed.

For elastic-plastic materials, the influence of plastic pre-deformations on mechanical properties was studied in [6], in other words, the reaction of a material which underwent plastic pre-deformations was analysed. In [7] a possibility of describing the decay of memory of material is analysed by introducing into the constitutive equations a parameter slowing down the deformation history.

Although, many authors have worked on the problem of effects of the loading history of a material, the material functions have not been determined in a unique way for any real material.

In the present paper an attempt has been made to determine the material functions of viscoelastic anisotropic plastics basing on results obtained from study projects carried out for the case of a jump change of loading.

2. Program and results of experimental investigations

For experimental investigations three types of plastics were used with different anisotropy degree and kind. The plastics were obtained from polyester and epoxy resins and glass fibres in the form of continuous bands, cloth and mat. Hardened resins and glass fibres differ substantially in physical-mechanical properties.

The first plastic was composed of layers of glass cloth and of polyester resin, and the second — of glass cloth layers interchanged with layers of glass mat and epoxy resin. The directions of warp in all these plastics were the same. The third plastic was composed of continuous glass bands paralelly interchanged with epoxy resin bands.

The plastic number 1 is strongly anisotropic and the plastic number 2 — only slightly. The plastics 1 and 2 have in their structure three mutually perpendicular symmetry planes and the plastic number 3 — only one isotropic plane.

The technology of production of the plastics and samples and devices used for tests were described in papers [8] and [9].

The programs of study of creep of plastics included a jump change of loading at various values of stress and at various increments of stress jump. Changes in loading occurred at different time spans. Therefore, elaborated programs make it possible to study in a precise way different responses of the material to the given loading history; this is necessary for the determination of the material functions.

The programs of study included axial tension of samples and tension with twist. The tension was carried out for $\varphi = 0$; $\pi/12$; $\pi/6$; $\pi/4$; $\pi/3$; $5\pi/12$; and $\pi/2$, where φ denotes the angle between the main axis of anisotropy (x-axis) and the direction of normal stress. The tension with twist was carried out for the following values of the ratio of the normal to tangent stress $k = 0.25$; 0.5; 1.0; ∞ which correspond to $\varphi = 2\pi/27$; $\pi/8$; $3\pi/17$; $\pi/4$, where φ denotes the angle between the principal axis of anisotropy and the principal direction of stress. The axial tension of samples in any direction $\varphi \neq 0$ was considered to be a plane state of stress (σ_{xx}, σ_{yy}, σ_{xy}).

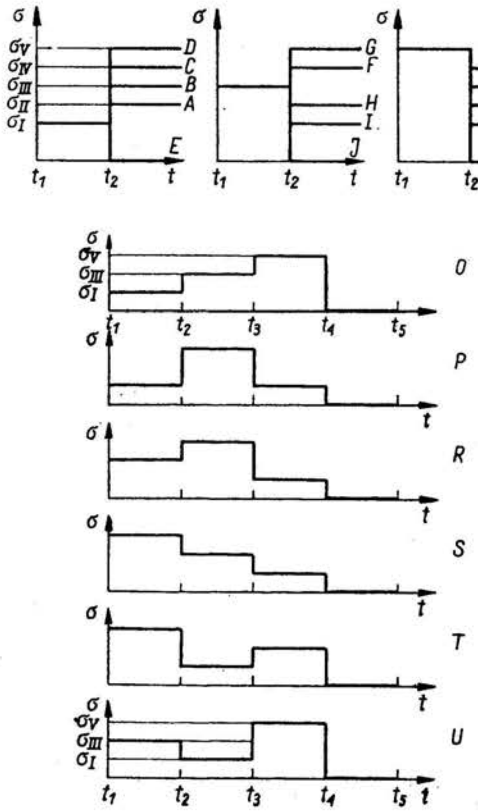

FIG. 1. Programs of loading.

Samples were loaded by various values of constant stress during 3600 kS and by a one-step and a three-step change of stress according to the programs presented in Fig. 1. A jump change of stress for a n-step test can be described by the equation

$$\sigma(t) = \sum (\sigma_j - \sigma_{j-1}) H(t-t_j), \tag{2.1}$$

where $H(t)$ — Heaviside function; $i = 1, 2, ..., n$; $\sigma_0 = 0$; $t_1 = 0$. In Fig. 1 the stress values $\sigma_{\mathrm{I}}, \sigma_{\mathrm{II}}, ..., \sigma_{\mathrm{V}}$ correspond to the following values of destructing stress: 0.2; 0.3; 0.4; 0.5; 0.6. In the programs the following values of time of stress change were provided:

$$t_2 = 9.0,\ 90.0,\ 360.0,$$
$$t_3 = 18.0, 180.0,\ 720.0,$$
$$t_4 = 27.0, 270.0, 1080.0 \text{ kS}.$$

In the studied plastics the stress velocity ($\dot{\sigma}$) influences substantially the relations between stress, deformations and time $(\varepsilon(\sigma, t))$, and thus in the tests a constant speed of pre-stressing and of stress change was assumed 0.4905 MPa/S.

All the samples to be tested according to the programs of Fig. 1 had a zero loading history. Moreover, no stress states in the samples, which could have been created during their production, were observed. Tests for checking those states were carried out by the reflection elastooptic method.

The principal mechanical indicators, for destructing stress and elastic constants, were represented in terms of the angle φ. The angle φ is formed by the principal axis of anisotropy (the warp direction in cloth or roving bands) and the principal direction of stress. Hence, by means of the angle φ the anisotropic properties of plastics in the target plane have been described.

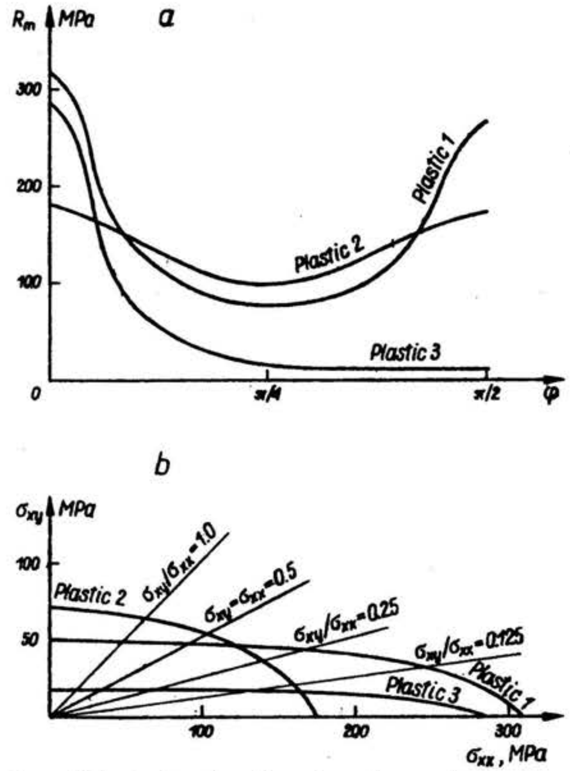

FIG. 2. Strength of plastics: a) The tension strength as depending on the direction of loading. b) Strength of plastics in a plane state of stress.

Figure 2 presents the results of strength tests of plastics, and Figs. 3, 4 and 5 the elastic constants of the plastics as dependent on the φ angle.

Tests of three plastics in a plane state of stress and with different assumed loading histories (see Fig. 1) resulted in a deformation history described by the following relations:

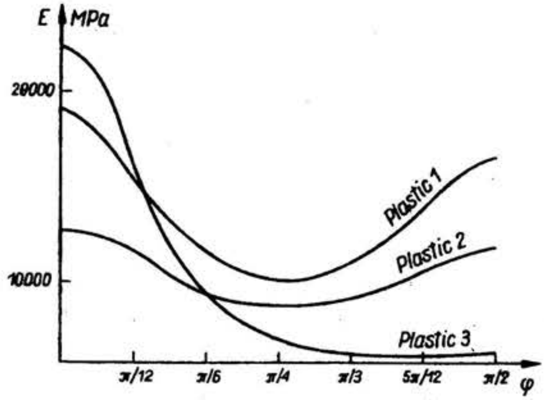

FIG. 3. Coefficients of longitudinal elasticity as depending on the direction of tension.

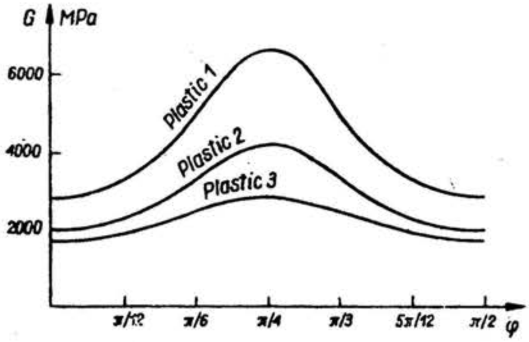

FIG. 4. Coefficients of transversal elasticity as depending on the direction of loading.

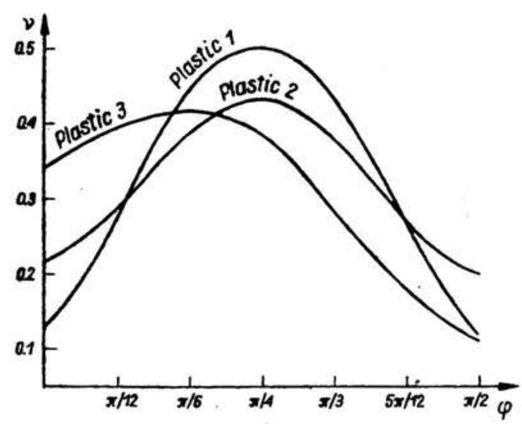

FIG. 5. Poisson's coefficients as depending on the direction of tension.

$$
(2.2)\qquad
\begin{aligned}
\varepsilon_{xx} &= \varepsilon_{xx}(\sigma_k, \varphi_k, t_k, t) \quad &\text{for}\quad \sigma_k &= \text{const}, \quad k = 1, 2, ..., 4,\\
\varepsilon_{yy} &= \varepsilon_{yy}(\sigma_k, \varphi_k, t_k, t) & \varphi_k &= \text{const}, \quad k = 1, 2, ..., 11,\\
\varepsilon_{zz} &= \varepsilon_{zz}(\sigma_k, \varphi_k, t_k, t) & t_k &= \text{const}, \quad k = 1, 2, ..., 9,\\
\varepsilon_{xy} &= \varepsilon_{xy}(\sigma_k, \varphi_k, t_k, t). & &
\end{aligned}
$$

$\varepsilon_{xx}, ..., \varepsilon_{xy}$ are components of the deformation tensor including instant deformations (ε_0) and creep deformations (ε_p), i.e.

$$
(2.3)\qquad \varepsilon_{ij} = \varepsilon_{ij0} + \varepsilon_{ijp}, \quad i, j = x, y, z.
$$

As it has already been mentioned, xyz represent the axis of a rectangular coordinate system and they coincide with the principal axis of anisotropy of the tested plastics. In a coordinate system xyz, defined in such a way, a plane state of stress $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ cannot generate any deformations ε_{yz} or ε_{xz}.

The values of deformations, as mean values of 3 to 5 tests, served for the construction of creep diagrams resulting from the realization of particular loading programs. In all,

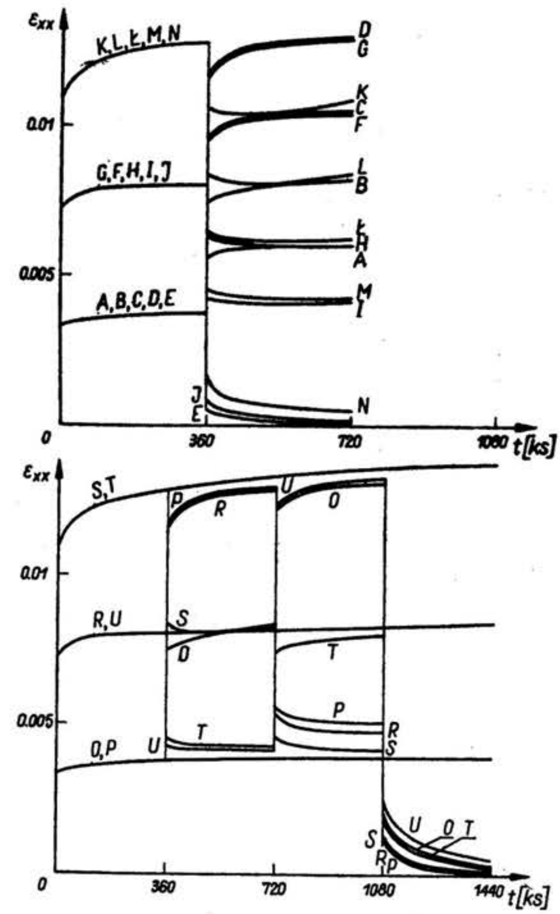

FIG. 6. Deformations along the x-axis (ε_{xx}) as functions of time, obtained at tension of Plastic 1 in direction $\varphi = 0$, at loadings as in Fig. 1.

351 sets of diagrams were obtained. In Figs. 6 and 7, as an example, two sets were presented which concern results obtained at tension in the directions $\varphi = 0$ and twist of Plastic 1 for $t_2 = 360$, $t_3 = 720$ and $t_4 = 1080$ kS. The form of the relations presented here is typical for viscoplastic bodies.

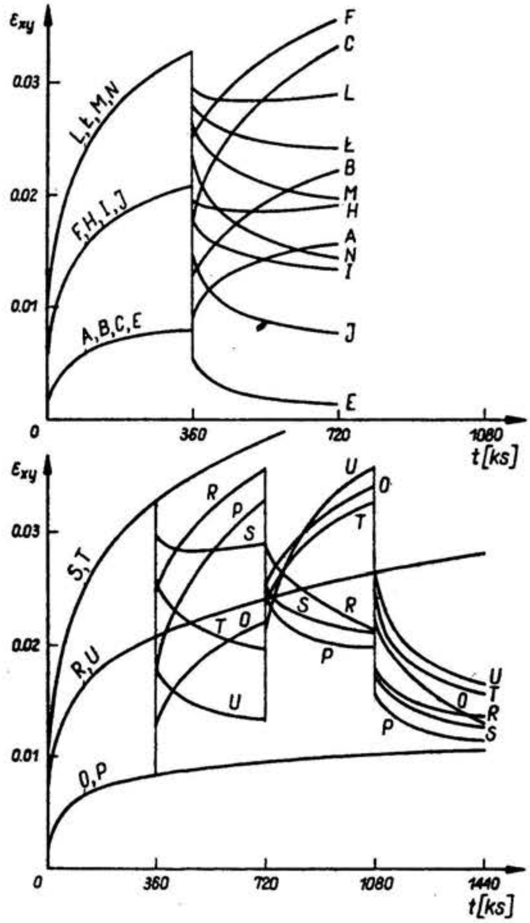

FIG. 7. Functions $\varepsilon_{xy}(t, \sigma)$ resulting from twist of Plastic 1 at loading as in Fig. 1.

Strongly anisotropic properties of Plastic 1 are described in Fig. 8. In Fig. 9 isochronic creep curves are presented and from them it follows that the relations between stress and deformation are nonlinear and the nonlinearity is stronger at larger stress and at stress acting in the directions $\varphi \neq 0$.

The anisotropic structure of the tested plastics and their viscoelastic properties make certain factors influence strongly the results of rheological tests. A precise analysis of the influence of the sample shape, of sign of normal stress, of glass concentration in a plastic and of temperature, on the rheological characteristics for a plastic of a composition similar to that of Plastic 1, were given in paper [10].

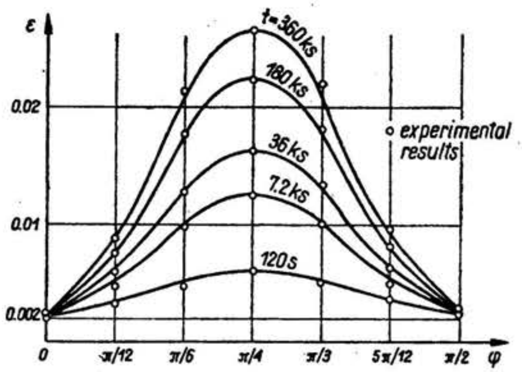

FIG. 8. Along-axis elongations as isochronic functions of the direction of tension of Plastic 1 by means of a 39.24 MPa stress.

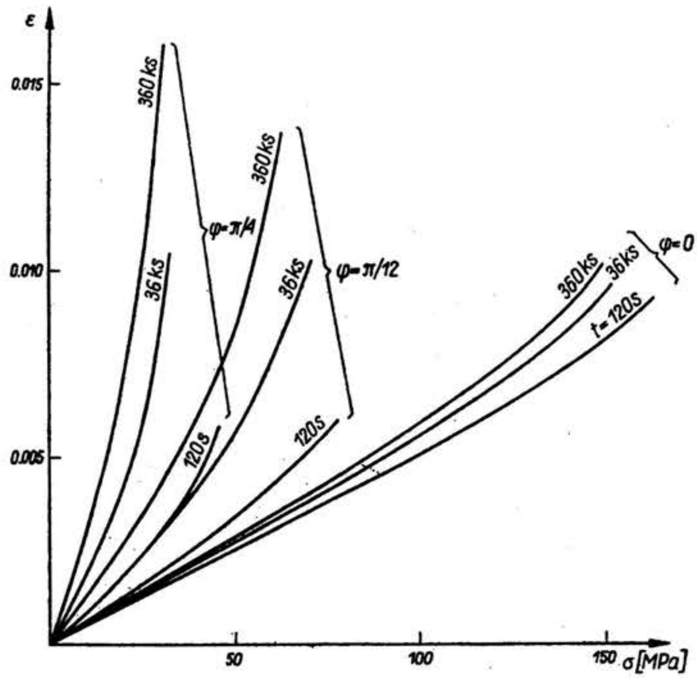

FIG. 9. Isochronic curves of creep resulting from tension of Plastic 1 in the direction $\varphi = 0, \pi/12, \pi/4$.

3. Proposed method of determining material functions

For a simple viscoelastic material with memory, the constitutive equation expressing the most general relation between deformations and the functional of the speed of stress change, for the one-dimensional case, has the form

$$\varepsilon(t) = \mathscr{F}\left(\frac{d\sigma(\tau)}{d\tau}\right)_0^t, \tag{3.1}$$

where $\mathcal{T}$ is a functional; if it is continuous and linear, then we can obtain an integral Boltzmann representation of a linear viscoelastic material.

For anisotropic viscoelastic materials of nonlinear characteristics the dependence of the intensity of deformation on the intensity of stress was assumed to be one-dimensional. When the process is isothermal and the loading history is composed of $n-1$ steps, Eq. (3.1) can be approximated by means of a polynomial representation and it can take the form

$$C_n(\sigma_1, \sigma_2, \ldots, \sigma_n; t_1, t_2, \ldots, t_n, t; \varphi) \equiv \sum_{j=1}^{n} W_j C_1(\sigma_j, t_j, t, \varphi), \tag{3.2}$$

where:

1) C_n is a constitutive functional which remembers full information on the jump history of stress and satisfies the requirements of PIPKIN and ROGERS from [11];

2) in the general case

$$W_j = W_j(\sigma_1, \sigma_2, \ldots, \sigma_j;\; t_1, t_2, \ldots, t_j; \varphi) \tag{3.3}$$

with $W_1 \equiv 1$.

The functions W_j fulfilling the condition (3.3) are material functions and they determine the sensitivity of the material to the size and time of stress jumps. Assuming a specific creep law for a material, we can obtain a corresponding form of the relation (3.2).

The function $C_1(\sigma, t_1, t, \varphi)$ represents a creep law found from tests at:

$$\sigma_k = \text{const} \quad \text{for} \quad k = 1, 2, \ldots, 4,$$
$$\varphi_k = \text{const} \quad \text{for} \quad k = 1, 2, \ldots, 11$$

and this law for the relation between the deformation intensity and the stress intensity was assumed in form of the equation

$$\varepsilon_i - \varepsilon_{i0} = A(\sigma_i, \varphi) t^{M(\sigma_i)} \tag{3.4}$$

or

$$C_1(\sigma, t_1, t, \varphi) = A(\sigma, \varphi)\,(t-t_1)^{M(\sigma)}. \tag{3.5}$$

Equation (3.2) for the creep deformation intensity takes then the form

$$\varepsilon_p(t) = \varepsilon(t) - \varepsilon_0 = \sum_{j=1}^{n} W_j A(\sigma, \varphi)_j (t-t_j)^{M(\sigma)}. \tag{3.6}$$

The relation (3.4) describes well the results of tests at stationary loadings. In the relation (3.4) the values of the functions $A(\sigma_i, \varphi)$ and $M(\sigma_i, \varphi)$ were determined by the method of least squares using the creep tests results of the three plastics taken for testing. The stress intensity was calculated from a formula which is correct for isotropic materials. The function $A(\sigma_i, \varphi)$ characterizes anisotropic properties. The method of determining creep functions and anisotropic functions was described in detail in [13].

Anisotropic properties can also be accounted for by determining the coefficients of anisotropy in the equation of stress intensity as proposed in [13].

For a creep test with a two-step loading change, assuming that $t_1 = 0$, $\varphi_k = \text{const}$, we can write Eq. (3.6) in the form

$$\varepsilon_p(t) = A(\sigma_{\text{I}})t^{M(\sigma_{\text{I}})} + W_2(\sigma_{\text{I}}, \sigma_{\text{II}}, t_2)A(\sigma_{\text{II}} - \sigma_{\text{I}})(t - t_2)^{M(\sigma_{\text{II}} - \sigma_{\text{I}})} + W_3(\sigma_{\text{I}}, \sigma_{\text{II}}, \sigma_{\text{III}}, t_2, t_3)A(\sigma_{\text{III}} - \sigma_{\text{II}})(t - t_3)^{M(\sigma_{\text{III}} - \sigma_{\text{I}})}. \tag{3.7}$$

In Fig. 10 a method is presented for the extrapolation of results of creep tests at stationary loadings onto results obtained at jump changes of loading. It follows from

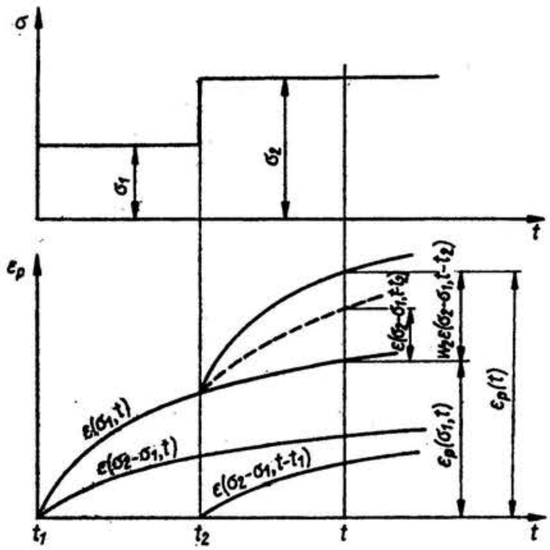

FIG. 10. A graphic interpretation of the creep description by means of material functions.

that figure that actual deformations at time t present the sum of results of stress increases from former time intervals.

The material functions described by Eq. (3.3) were determined from the condition that the sum of squares of differences between experimental deformations and those calculated from Eq. (3.6) be the smallest possible one. For the program with a one-step stress change we obtain

$$\sum_{j=1}^{n} \big(\varepsilon_p(t_j)_{\text{exp}} - \varepsilon(\sigma_{\text{I}}, t_j) - W_2\varepsilon(\sigma_{\text{II}}, t_j - t_2)\big)^2 = \text{min}. \tag{3.8}$$

The expression in Eq. (3.8) reaches its minimum value at

$$W_2 = \frac{\sum\limits_{j=1}^{n} \big(\varepsilon_p(t_j)_{\text{exp}} - A(\sigma_{\text{I}})t^{M(\sigma_{\text{I}})}\big)\big(A(\sigma_{\text{II}} - \sigma_{\text{I}})(t_j - t_2)\big)^{M(\sigma_{\text{II}} - \sigma_{\text{I}})}}{\sum\limits_{j=1}^{n} \big(A(\sigma_{\text{II}} - \sigma_{\text{I}})(t_j - t_2)^{M(\sigma_{\text{II}} - \sigma_{\text{I}})}\big)^2}. \tag{3.9}$$

Similarly, for results of the program with a two-step change of stress we determine W_3 using the values of W_2 already found, and so on.

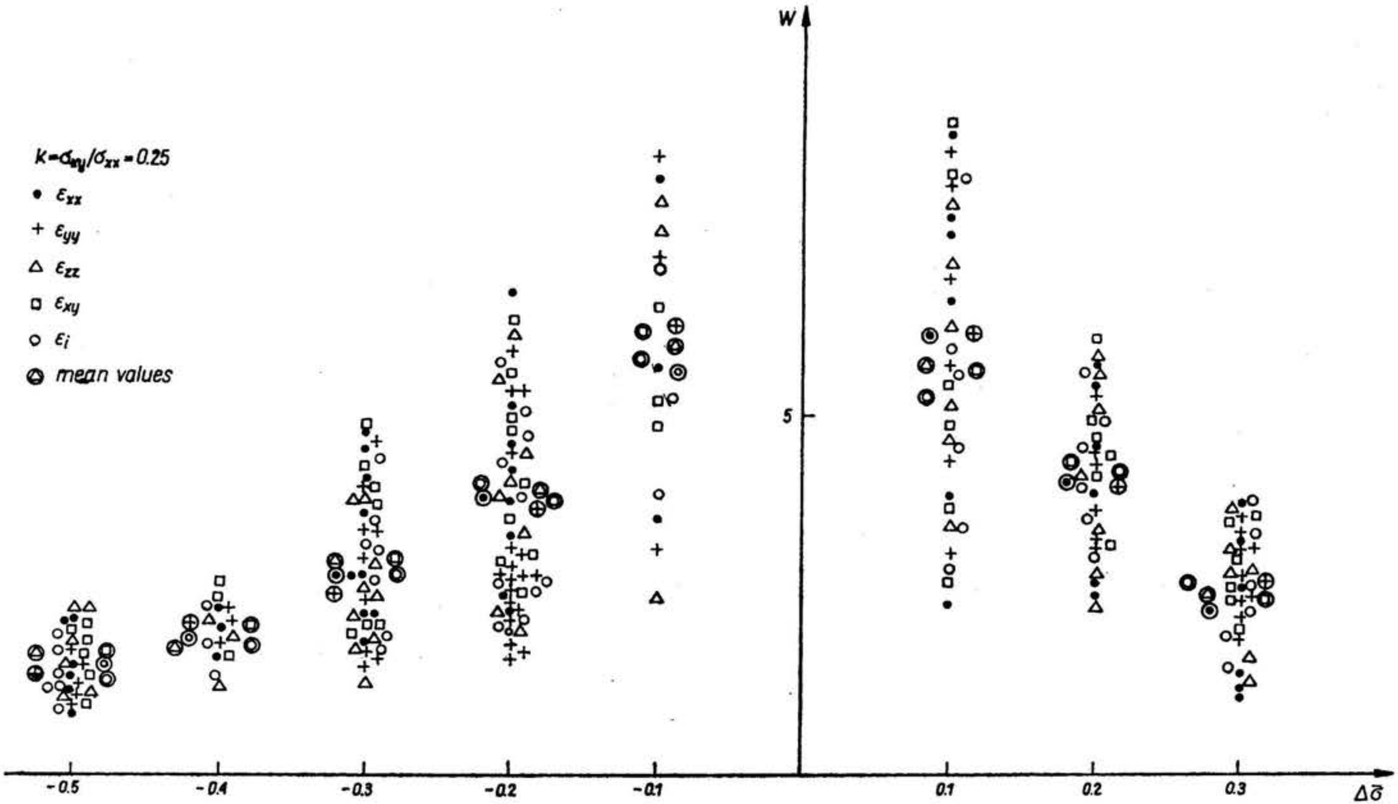

FIG. 11. Dependence of material functions on values of the stress jump for Plastic 1 under load $\sigma_{xy}/\sigma_{xx} = 0.25$, determined from elements of the deformation tensor and the intensity of deformation.

For experimentally found values of deformations (Eq. (2.2)), for times $t_1, t_2, \ldots$ $\ldots, t_j, \ldots, t_n$ values of W were to be found according to programs of loading with a three-fold change of loading time and for three kinds of plastic and that made 11 583 values found. The calculations were carried out on a computer.

The numerical analysis of the obtained values of material functions W was reduced to determining the dependence of W on the following parameters:

type of the elements of the deformation tensor and the intensity of deformation;

direction of the loading (angle φ);

value of the increment of stress jump $\Delta\bar{\sigma} = (\sigma_k - \sigma_{k-1})/\sigma_{\text{destr.}}$, where σ_k — stress values for $t_{k-1} < t < t_k$, $\sigma_{\text{destr.}}$ — destructing stress values;

stress values $\bar{\sigma} = \sigma_k/\bar{\sigma}_{\text{destr.}}$ in the interval $t_{k-1} < t < t_k$;

time of the interval $\Delta t = t_k - t_{k-1}$;

time of the stress jump t_k;

deformation increment $(\Delta\varepsilon)$ in the interval $t_{k-1} < t < t_k$;

mean values of deformation (ε_m) in the interval $t_{k-1} < t < t_k$;

work of the full history of stress $(\Sigma\sigma_j(\varepsilon_m)_j)$;

increment of the stress work $(\Delta\sigma_j(\varepsilon_m)_j)$ in the interval $t_{k-1} < t < t_k$.

In order to study the dependence of the function W on the elements of the stress tensor $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}$ and on the stress intensity, the values of the function W, determined from Eqs. (3.9) using the relations (2.2), are presented in diagrams $W(\Delta\bar{\sigma}, \varepsilon_{xx})$, $W(\Delta\bar{\sigma}, \varepsilon_{yy})$, $W(\Delta\bar{\sigma}, \varepsilon_{zz})$, $W(\Delta\bar{\sigma}, \varepsilon_{xy})$. These relations do not differ from each other substantially. Also, the functions $W(\Delta\bar{\sigma}, \varepsilon_i)$ obtained from the relations between the deformation intensity and the stress intensity had similar diagrams. These relations are presented in Fig. 11 for the case of a loading $\sigma_{xy}/\sigma_{xx} = 0.25$ for Plastic 1.

In Fig. 12 the function W was displayed as dependent on the direction of loading (φ) of the Plastic 1. Every point of the diagram corresponds to a mean value calculated

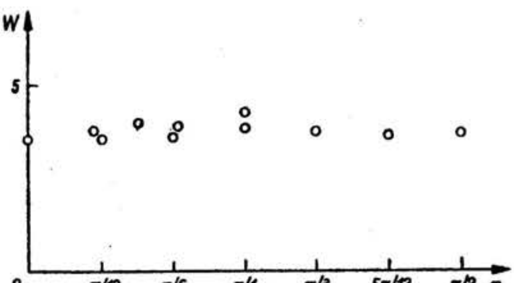

FIG. 12. Functions W as depending on direction of loading for Plastic 1.

from 216 values of W obtained for φ = constant. No clear change of W with φ can be seen; it was obtained for the studied plastics that $W(\varphi)$ = const although a small increase of W with φ growing from zero to $\pi/4$ can be observed.

Substantial changes of W with jump changes of stress ($\Delta\bar{\sigma}$), as displayed in Fig. 13, reveal the sensibility of the studied plastics to jump changes of loading. The jump changes of stress in accordance with programs of Fig. 1 in time began with larger stress at $\Delta\bar{\sigma}$

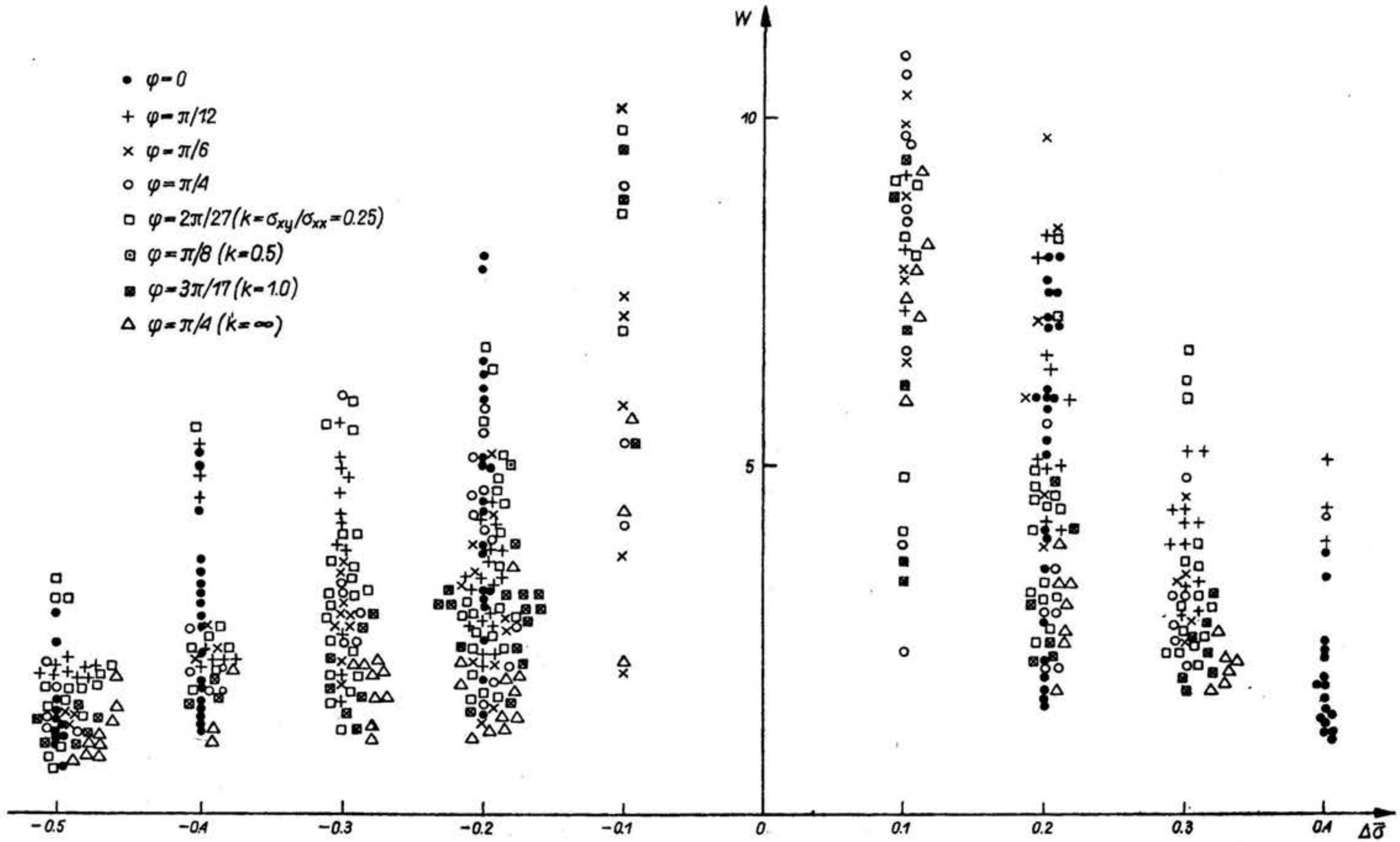

FIG. 13. Functions W as depending on values of the stress jump ($\Delta\bar{\sigma}$) for Plastic 1.

and continued to a smaller one at $-\Delta\bar{\sigma}$, what resulted in larger values of $W(\Delta\bar{\sigma})$ than $W(-\Delta\bar{\sigma})$ calculated for the same absolute values of arguments. It was observed that the relations $W(\Delta\bar{\sigma})$ and $W(-\Delta\bar{\sigma})$ follow the same pattern when the jumps of stress end at the same values of stress. Thus, in further considerations the same dependence of $W(\Delta\bar{\sigma})$ for $\Delta\bar{\sigma} > 0$ and $\Delta\bar{\sigma} < 0$ was assumed.

The function W depends strongly on the stress value within the interval $t_{k-1} < t < t_k$, what indicates strong nonlinearity of the deformation history in terms of loading. The relations $W(\bar{\sigma})$ for the studied plastics are displayed in Fig. 14.

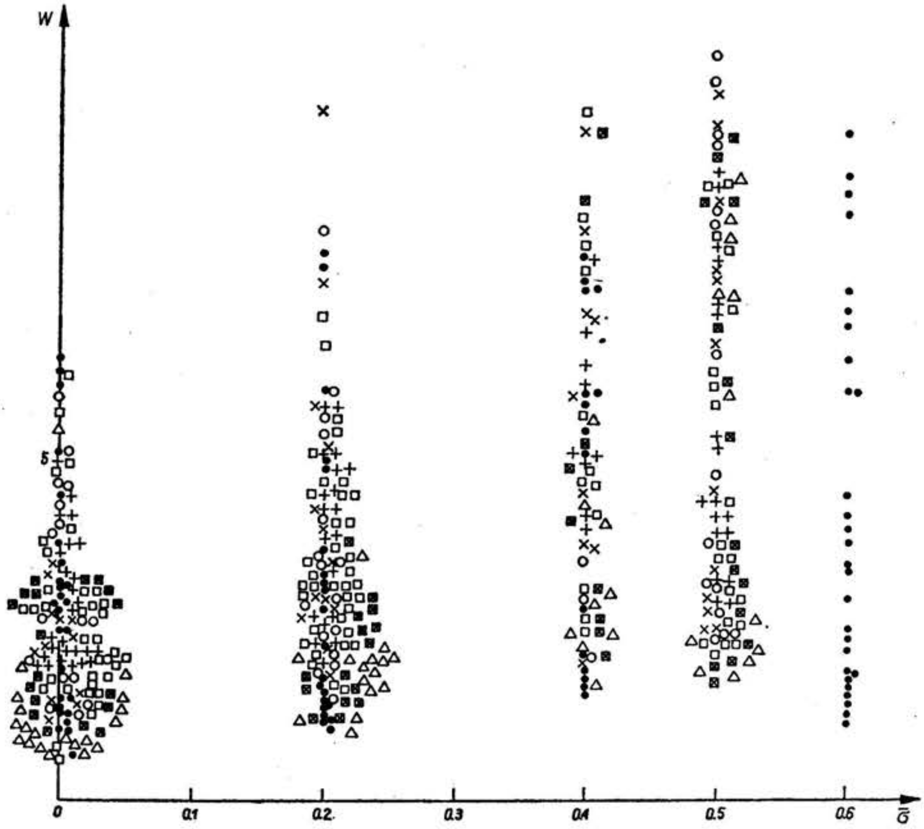

FIG. 14. Function W as depending on the actual value of stress $\bar{\sigma} = \sigma/\sigma_{destr.}$ in the interval $t_{k-1} < t < t_k$ for Plastic 1.

The decrease of values of W with growing time of (Δt) and time of the stress jump (t_k) are shown in Figs. 15 and 16.

The functions W as depending on the increment of creep deformation within the interval $t_{k-1} < t < t_k$ and on the mean values of deformation in the interval have a similar form as the functions $W(\Delta\bar{\sigma})$ and $W(\bar{\sigma})$. Qualitative differences in relations presented in Figs. 13 and 17 as well as 14 and 18 stem first of all from differences in the definition of coordinates ($\Delta\bar{\sigma}$ and $\Delta\varepsilon$ as well as $\bar{\sigma}$ and ε_m).

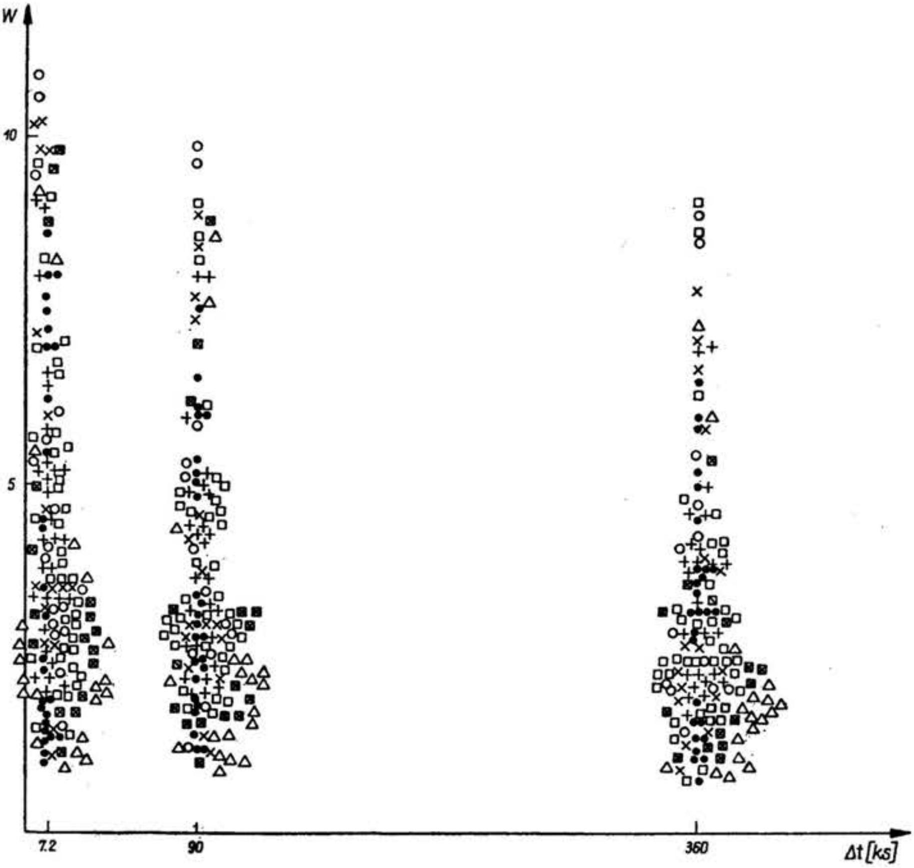

FIG. 15. Function W as depending on $t = t_k - t_{k-1}$ for Plastic 1.

It follows from the dependence of W on the increment of stress work and on the sum of works of stress from precedent time intervals that the function W does not possess the properties of a consolidation parameter. If W were only a function of the work of stress, then the points on the diagrams of Figs. 19 and 20 would be distributed more uniformly.

From comparison of the relations $W(\Delta\bar{\sigma})$, $W(\bar{\sigma})$, $W(\Delta t)$, $W(t)$ as presented in Figs. 13 through 16, obtained for the three studied plastics, it can be seen that they are qualitatively similar. However, a quantitative comparison shows that the functions W determined for Plastic 1 assume the largest values depending on the structure and the type of components of the plastic.

It is not difficult to note that the larger the values of W the stronger the nonlinearity of the material. For a linear viscoelastic Boltzmann material: $W = 1$, but it is not true that the material is nonlinear when $W = 1$, unless an additional assumption is made, namely that the relation (3.4) is linear.

In order to simplify the mathematical description of the function W it can be assumed that

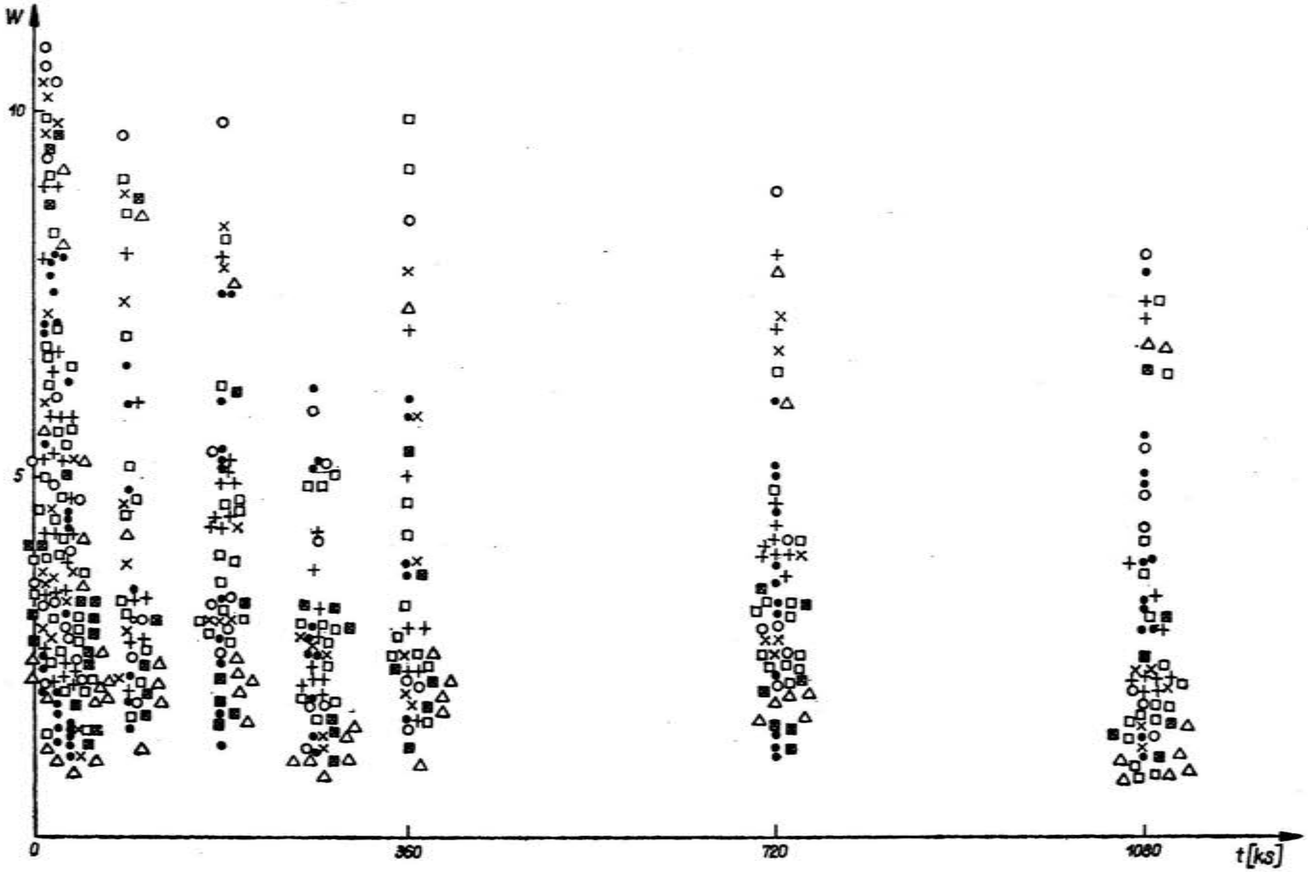

FIG. 16. Function W as depending on the time of the stress jump for Plastic 1.

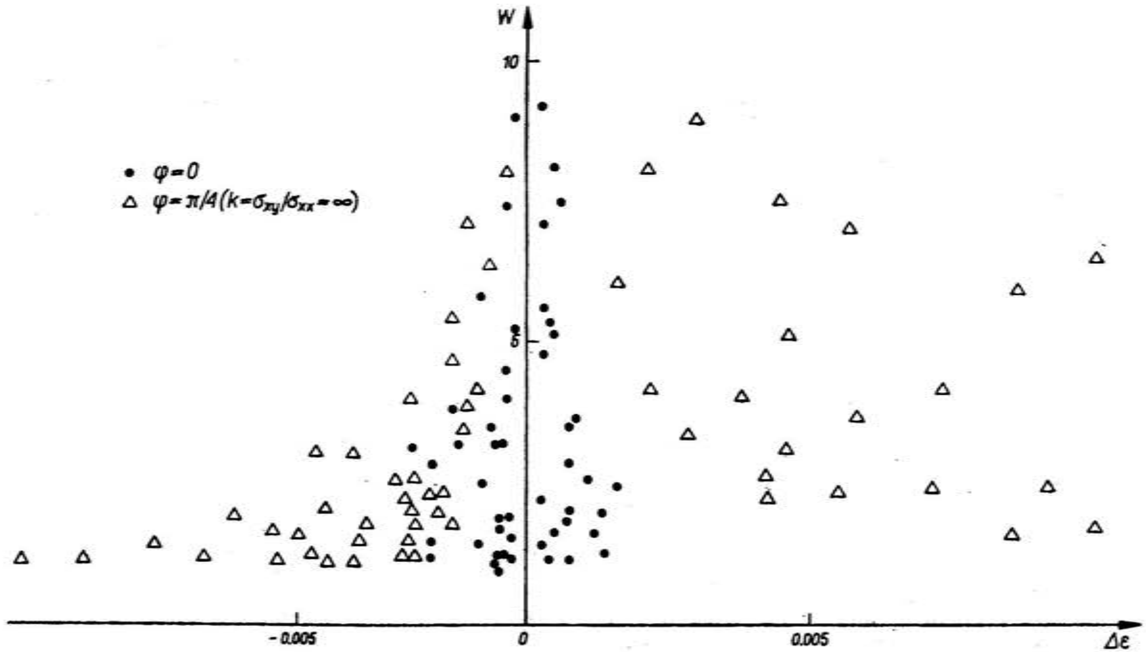

FIG. 17. Function W as depending on the deformation increment in the interval $t_{k-1} < t < t_k$ for Plastic 1.

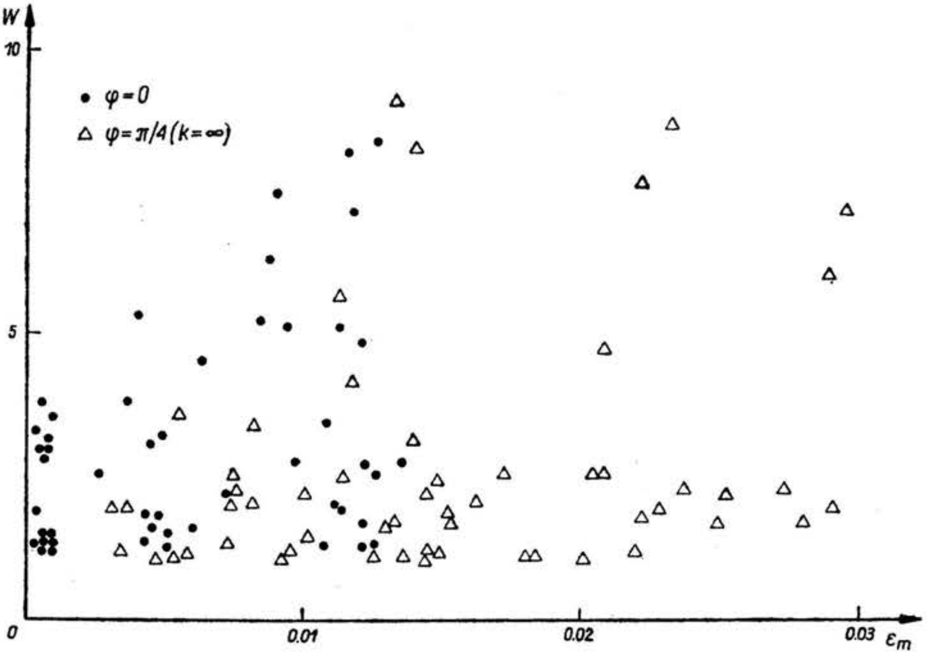

FIG. 18. Function W as depending on mean values of deformation in the intervals $t_{k-1} < t < t_k$ for Plastic 1.

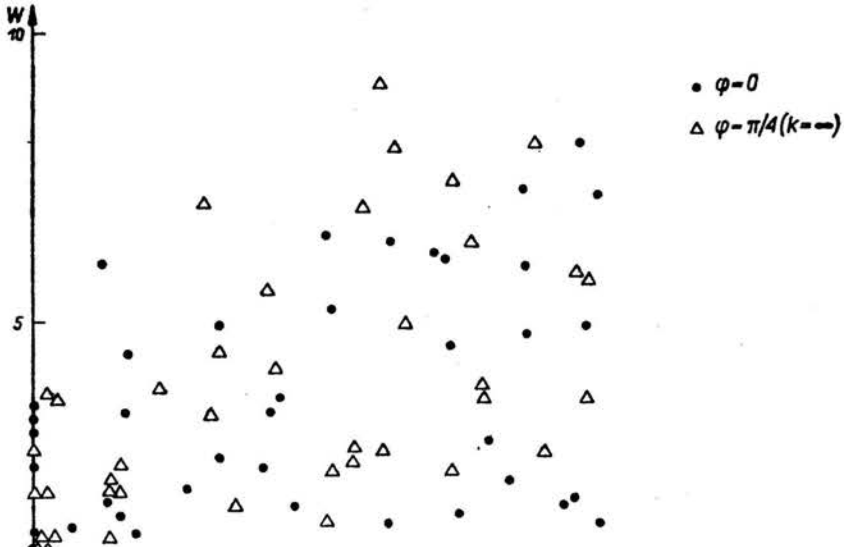

FIG. 19. Function W as depending on the increment of stress work in intervals $t_{k-1} < t < t_k$ for Plastic 1.

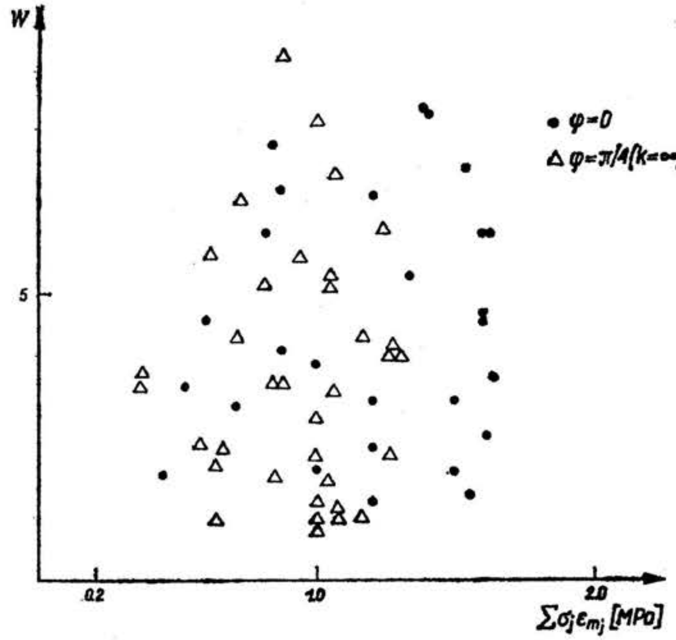

FIG. 20. Function W as depending on the sum of stress works for the intervals $t_2, t_3, \ldots, t_{k-1} < t_k$ for Plastic 1.

$$W(\varphi) = \text{const},$$
$$W(\Delta\bar{\sigma}) = W(-\Delta\bar{\sigma}), \tag{3.10}$$
$$W(t_k) = \text{const},$$

because they vary only slightly. It was assumed that the function W depends only on the jump of stress, the end value of stress after a jump and on the time of the interval of stress.

Among a large number of studied forms of the function the best description of the obtained relations $W(\Delta\bar{\sigma}, \bar{\sigma}, \Delta t)$ is given by the following equation:

$$W = F\exp(k|\Delta\bar{\sigma}|+p|\bar{\sigma}|+m\Delta t), \tag{3.11}$$

$$W = F|\Delta\bar{\sigma}|^k(\bar{\sigma})^p t^m. \tag{3.12}$$

The constants F, k, p, m in Eq. (3.11) were determined by the method of least squares and the results are given in [8].

For a mathematical description of creep effects of plastics obtained from the realization of the tests' programs (Fig. 1), the constitutive relations valid for the case of stationary loadings [14, 12, 8] can be used taking into account the relations (3.6) and (3.12).

We describe now the determination of material functions for Plastic 1 in the case when the relation (3.4) is linear.

If a division of rheological deformations into linear and nonlinear ones according to paper [15] is possible, then the linear deformations can be described by means of the Boltzmann integral representation of a linear viscoelastic material, and the nonlinear deformations by means of material functions. The material functions, as defined in the present paper, describe first of all the nonlinear properties of a material. For linear materials the functions take the value of 1 and thus such a description is physically meaningful and practically relevant.

As a boundary between the linear and nonlinear regions for the studied plastics, stress values equal to 20% of the destructing stress were assumed and this corresponds to a horizontal form of the curves (Fig. 21). Assuming an exponential form of the creep

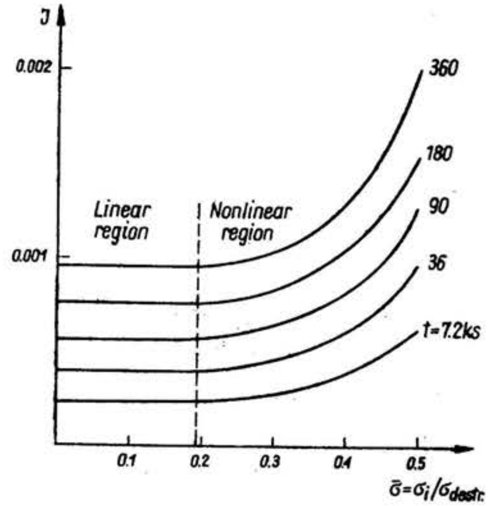

FIG. 21. Creep function as depending on stress intensity for Plastic 1 under loading $\sigma_{xy}/\sigma_{xx} = 0.25$.

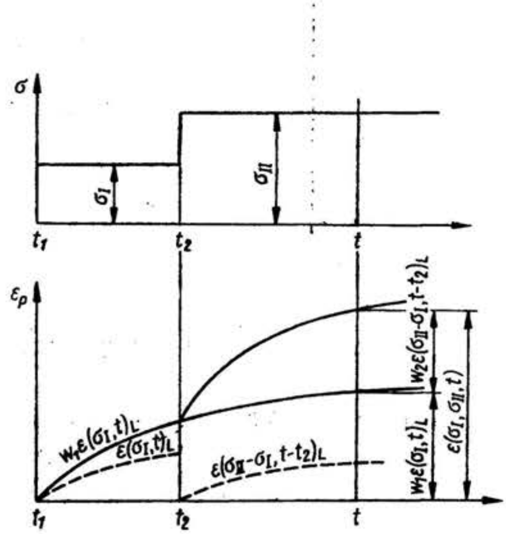

FIG. 22. Graphic interpretation of the relation (3.14).

kernel in the Boltzmann equation, we obtain a relation between the intensity of stress and the intensity of deformation in the linear form

$$\varepsilon_i - \varepsilon_{i0} = \sigma_i A(\varphi) t^M. \tag{3.13}$$

The material functions were determined in a similar manner as in the preceding case with a change that, in general, $W_1 \neq 1$. After the relation (3.13) has been taken into account we write Eq. (3.6) in the form

$$\varepsilon_i(t) - \varepsilon_i(0) = \sum_{j=1}^{l} W_j Y(\varphi)(\sigma_j - \sigma_{j-1})(t - t_j)^M. \tag{3.14}$$

Figure 22 explains in a graphical way the method consisting in summing up the deformations of particular steps of loading. Actual deformations for time t present a sum of deformations of former intervals from the linear region $(\varepsilon(\sigma_j, t)_L)$, enlarged by corresponding values of the material functions W_j.

The found relations $W(\Delta\bar{\sigma})$, $W(\bar{\sigma})$, $W(\Delta t)$, $W(t)$, which were presented in [8], are similar in diagrams as those given in Figs. 13, 14, 15 and 16, but the functions W in Eq. (3.14) assume larger values.

4. Final remarks

The approximation of the constitutive functional (3.1) by the polynomial representation (3.6) assumed in the present paper and the isolating of the anisotropic and creep material functions in these equations is in agreement with present day tendencies in describing certain classes of materials. These functions describe in a unique way the properties of the tested material and thus supply directions for comparing materials.

The material functions determined for the three plastics characterize the relevant properties of materials, and first of all, the degree of their nonlinearity. By means of the material functions an extrapolation of data from creep tests at stationary loadings onto multi-step tests was carried out.

The presented method of determining material functions can be successfully applied to determine functions related to the memory effect of other classes of materials.

Comparatively simple constitutive equations can be obtained due to the coincidence of diagrams of material functions as determined from the relation between the deformation intensity and the stress intensity as well as those resulting from the relations obtained from the elements of the stress tensor (Fig. 11).

For the purpose of determining the material functions, the anisotropic one and the creep one, only the relation of dependence of the deformation intensity on time can be used. This relation was obtained from results of tests of materials at tension in different directions φ because it is similar to the relation of deformation intensity $\varepsilon_j(t)$ determined in a plane stress state. This observation simplifies substantially experimental studies.

Since the material functions as described in the present paper for linear materials take the value of 1, it is purposeful to assume Eq. (3.13) which at constant stress describes linear viscoelastic deformations; then, nonlinear deformation are described by Eq. (3.14) by means of material functions. Such a description has a sound physical foundation.

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TECHNICAL UNIVERSITY OF WARSAW.

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