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Multicriterial optimization of geometrical and structural properties of the basic module of a single-branch Truss-Z structure

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1. Abstract

Truss-Z (TZ) is an Extremely Modular System (EMS). Such systems allow for creation of structurally sound free-form structures, are comprised of as few types of modules as possible, and are not constrained by a regular tessellation of space. Their objective is to create spatial structures in given environments connecting given terminals without self-intersections and obstacle-intersections. In an EMS, the assembly, reconfiguration and deployment difficulty is moved towards the module, which is relatively complex and whose assembly is not intuitive. As a result, an EMS requires intensive computation for assembling its desired free-form geometrical configuration, while its advantage is the economization of construction and reconfiguration by extreme modularization and mass prefabrication.

TZ is a skeletal modular system for creating free-form pedestrian ramps and ramp networks among any number of terminals in space. TZ structures are composed of four variations of a single basic module (Truss-Z module, TZM) subjected to affine transformations (mirror reflection and rotation). The previous research on TZ focused on global discrete optimization of the spatial configuration of modules. This contribution presents the first attempt at structural optimization of the TZM for a single-branch TZ. The result is a multicriterial optimization, where the Pareto front provides the means to strike the optimal balance between the geometric and structural assessment criteria.

2. Keywords: multicriterial optimization, Truss-Z, effective stress, modular systems

3. Introduction

A stairway is the most common means of pedestrian vertical transportation used in the built environment. Elevators and escalators are relatively expensive to install and maintain, and their traffic flow capacity is much lower than that of stairs. Moreover, it is not always possible to install an elevator or escalator due to limited space. However, most people occasionally or temporarily cannot use stairs, as when riding a bicycle, pushing a baby stroller or carrying heavy luggage. For elders and people in wheelchairs, stairs form a permanent and impassable barrier. This is an important social issue, especially since the proportion of elderly people in society is higher than in the past, and some predict that this tendency will continue [1]. A comprehensive literature review for elderly pedestrians is carried out in [2].

Truss-Z (TZ) is a modular skeletal system for creating free-form ramps and ramp networks among any number of terminals in space. The concept has been introduced in [7]. The motivation for TZ in the context of human mobility, and in particular the mobility problems of elders is discussed in [4]. The underlying idea of this system is to create structurally sound provisional or permanent structures using the minimal number of types of modular elements. Further discussion on modularity vs. free-form can be found in [4].

In principle, TZ connects two points in space, called terminals. Constructing an efficient TZ can be formally expressed as a constrained discrete multicriterial optimization problem. In the previous research only the geometrical properties, such as the total number of modules (n), "geometrical simplicity" (GS) and "number of turns" (NT), have been minimized. The criteria GS and NT measure how many units do not follow a straight line, and how many continuous turns there are in the path, respectively. Common constraints have been the locations of terminals, positions and shapes of obstacles in the environment, etc. This paper presents the first attempt at structural optimization of the Truss-Z module for a single-branch TZ in a particular environment. In such a task, two qualitatively different objectives need to be taken into account:

1. The capability of the TZM to create free-form shapes;
2. The structural performance of the resulting TZ.

The former objective is quantified in terms of the ability of the considered TZ to accurately connect two terminals in a given typical environment (mismatch error) as well as in terms of required number of modules. The latter objective is quantified in terms of the largest effective stress in the worst case of all 256 unique 5-module configurations under given static load. The result is a multicriterial optimization, where the Pareto front provides the means to strike the optimal balance between the geometric and structural assessment criteria.

4. The concept of Truss-Z

In geometrical terms, all TZ structures are composed of only four variations of a single basic unit or module (R). Figure 1 shows the geometrical properties of R which have been set arbitrarily and used in theretofore research. Unit L is a mirror reflection of the unit R. By rotation, they can be assembled in two additional ways (R_2 is the rotated R, and L_2 is the rotated L), effectively giving four types of units. Some examples are shown in Figure 2. Figure 3 shows an example from a case study of retrofitting an existing concrete overpass system comprised of two bridges connected with three stairways [4]. This contribution presents a preliminary study on structural improvement of the module.

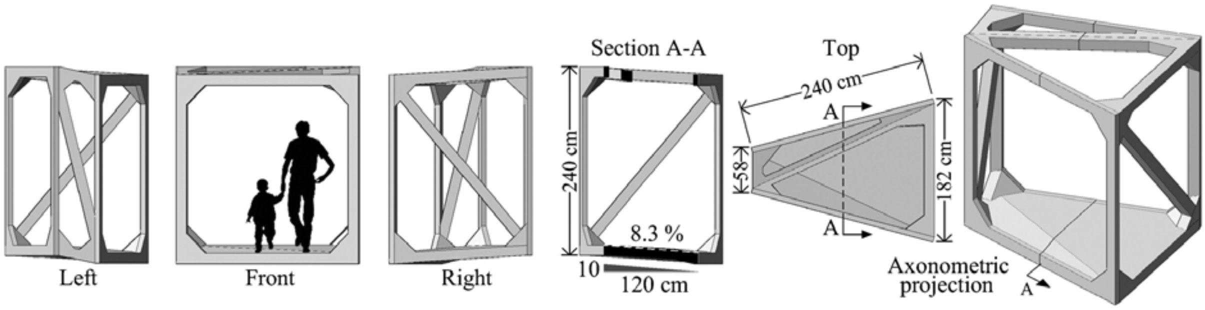

Figure 1: The original TZ basic unit (R). From the left: three orthographic views, section A-A showing the slope, top view and axonometric view.

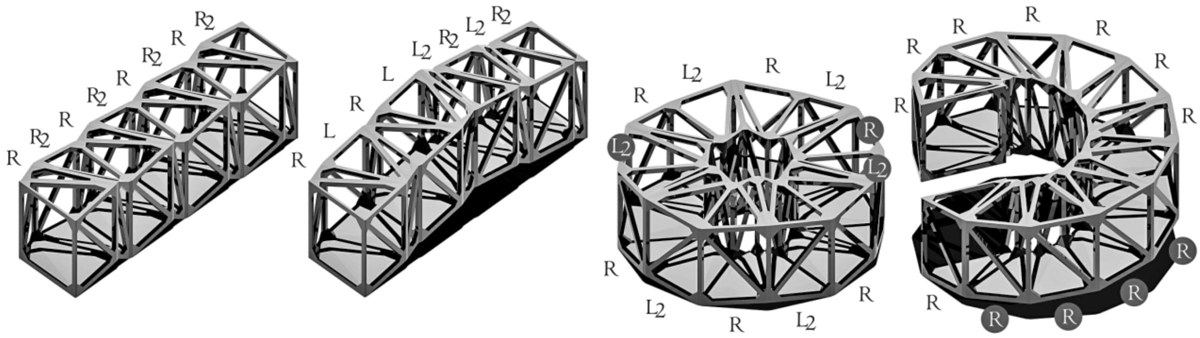

Figure 2: Some basic examples of single-branch TZ structures based on the original TZM: "straight and flat" with 8 units, "straight up & down" (8 units), a flat ring (12 units), and a spiral (12 units).

The structural rigidity of the TZ module has been demonstrated in [5], along with other topological properties such as *nullity*, degree of static indeterminacy (DSI = 0), etc. Due to the modularity of this system, it is natural to apply discrete optimization methods for creating TZ connectors and networks. Such structures can be optimized for various criteria: the minimal number of modules, the minimal number of changes in direction, and in a case of multiple branches, the minimal network distance, etc. Various deterministic and meta-heuristic methods have been successfully implemented for single TZ paths, including backtracking [6], evolution strategy [7], and evolutionary algorithms [8]. These methods produced usually good, but not ideal, solutions. A graph-theoretical exhaustive search method, which produces the best allowable, that is ideal solutions, has been described in [3].

5. Assessment criteria

The process of designing a Truss-Z module (TZM) is basically a multicriterial optimization problem [10, 11], in which two qualitatively very different classes of objectives should be taken into account: (1) the capability of TZM to create free-form shaped ramps, so that they can (flexibly enough) suit complex geometrical constraints of real construction environments, (2) the structural performance of the resulting TZ ramp. Respectively, besides the geometric parameters, a TZM needs to be defined also in terms of its structural parameters, which all have to be taken into account in the optimization process.

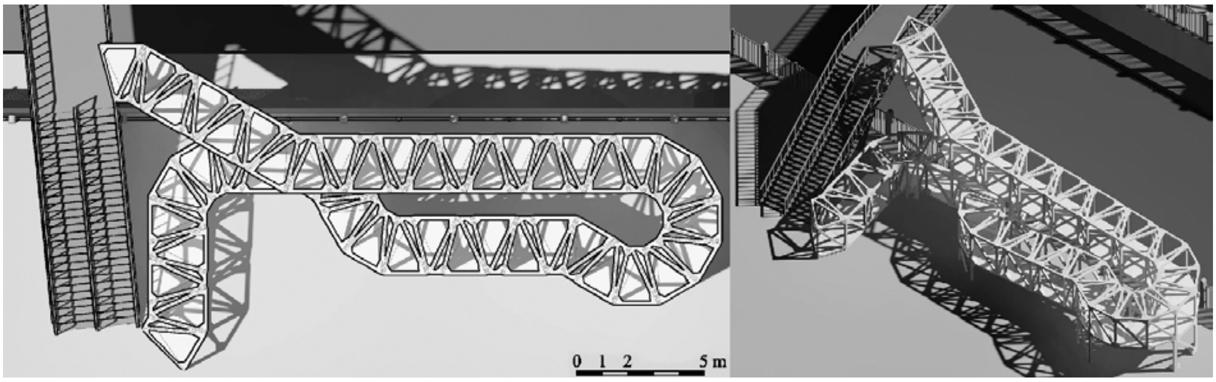

Figure 3: A visualization of a TZ designed to allow the use of wheelchairs in an existing overpass. The slope of TZ is 1:12 (8.3%). Left and right: plan and axonometric views, respectively. The existing stairway (on the left) is very steep and particularly long: 29 rises without an intermediate landing.

5.1. TZ module parameters

The *geometry* of the module is determined by the parameters: planar angle θ, width r, "slenderness" s, vertical displacement δ_Z, and height h, as shown in Figure 4. The slenderness s is the ratio between the offset from the apex d to the width r. For the three cases of $s = 0$, $0 < s < \infty$ and $s = \inf$, the corresponding projections of TZM form a triangle, a trapezoid, and a rectangle, respectively. For clarity, further in text TZMs are visualized by their center-line vectors $\mathbf{c}$. The TZ structures studied in theretofore investigations have been based on original module defined by the following parameters: $\theta_0 = 30\,\mathrm{deg}$, $r_0 = 1.242$ m, $s_0 = 0.5$, $\delta_{Z0} = 0.1$ m, and $h_0 = 2.4$ m. The vertical displacement determines the centerline inclination, which is constrained to maximal acceptable value of approximately 8%. Due to functional requirements, the height and width of the "main frames" for a 2.4 m $\times$ 2.4 m square (see Figure 4). For the purpose of optimization, only two parameters are selected: planar angle θ and the center-line length c. They are stored in the vector $\mathbf{x}$,

$$\mathbf{x} = (\theta, c)\,. \tag{1}$$

The remaining geometric parameters of the module are uniquely determined by the functional requirements:

- The dimensions of the entrance and the exit is fixed at 2.4 m $\times$ 2.4 m, which determines the parameter of slenderness s and the height h;
- The vertical displacement δ_Z is determined by keeping the center-line inclination as in the original module, that is at the constant value of approximately 8%.

Additionally, the planar angle θ is constrained by imposing the lower and upper bounds: the lower bound of 10°is assumed, while the upper bound depends on the module width r and is determined to keep the maximum floor inclination (which occurs at the shorter side of the module) below the maximum value of 25%. The specific configuration of diagonal beams shown in Figure 4 has been obtained in [9].

In *structural* terms, the module is assumed to be a 3D frame constructed of thin-walled circular hollow sections. The wall thickness of 2 mm is assumed and the material parameters correspond to steel with the density of 7800 kg/m^3, Young's modulus 205 GPa and the shear modulus of 79.3 GPa. The to-be-optimized structural parameters of the module are described by the vector $\mathbf{y}$ of the diameters of the 16 tubes that form the module,

$$\mathbf{y} = (\phi_1, \phi_2, \ldots, \phi_1 6)\,. \tag{2}$$

Besides the obvious non-negativity constraints, there is an additional single linear constraint that fixes the total mass m of the module according to its center-line length c:

$$m = \frac{c}{c_0}\,30\ \mathrm{kg}, \tag{3}$$

where $c_0 = 1.242$ m is the center-line length of the original module. The successive parameters in the vector $\mathbf{y}$ describe respectively the sections that join the following pairs of nodes of the module, as numbered in Figure 4: (1, 2), (5, 6), (1, 3), (2, 4), (5, 7), (6, 8), (3, 4), (7, 8), (1, 5), (2, 6), (3, 7), (4, 8), (2, 5), (4, 6), (3, 5), and (4, 7).

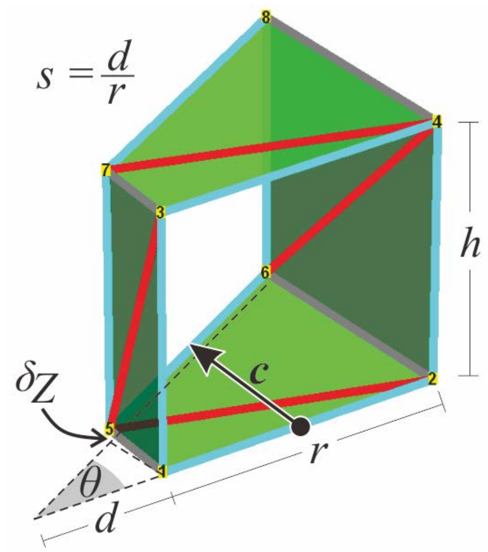

Figure 4: The geometrical parameters of TZM. The "main frames", diagonal members and center-line vector **c** are shown in cyan, red and black, respectively.

5.2. Assessment of geometrical quality

The geometrical quality of a TZM can be understood as its capability to create free-form shapes that can satisfactorily suit complex geometrical constraints of the real construction environments. In this contribution, Truss-Z paths (TZPs) are required to complete the following geometrical (functional) task:

- Create a connector from the start point sP to the end point eP.
- The last TZM must reach eP within given proximity pR. The inaccuracy of reaching eP is measured by the "reaching error" ε_R, which is defined as the distance between the end point eP and the line segment that represents the center-line vector **c**.
- TZPs must be confined within the "allowable zone" AZ.
- TZP must not collide with obstacles.
- The maximum number N_{TZM} of the modules is arbitrarily limited to 15.

Figure 5 shows an example of a typical environment, which is used here as the test environment. The figure shows also an example of two alternative TZPs created with the original TZM. Consequently, the geometrical quality of a TZM is quantified by two objective functions:

1. The reaching error ε_R.
2. The number N_{TZM} of the modules used to form the path.

Given the test environment, both functions are derived from the vector **x** of TZM geometric parameters. In the further optimization process, only the modules that are capable of forming a proper path from sP to eP are taken into account.

5.3. Assessment of structural quality

In general, the structural quality of a TZM can be understood as its ability to create free-form shaped ramps (TZs) of a relatively high structural performance. For a TZ that corresponds to a given configuration s of modules (example configurations are shown in Figure 2), the vague notion of structural performance can be quantified in terms of the maximum von Mises effective stress $\sigma_s(\mathbf{x},\mathbf{y})$ of the TZM segments that occurs under a given static load vector $\mathbf{P}(\mathbf{x})$ of the configuration s. The stress depends on the vectors **x** and **y**, which define respectively the geometry and the segment diameters of the module, as explained in Section 5.1. In this contribution, it is assumed that the load vector $\mathbf{P}(\mathbf{x})$ corresponds to the live vertical load of 6000 N/m 2, which is evenly allocated to the floor nodes of each module and then assembled into the load of the entire TZ.

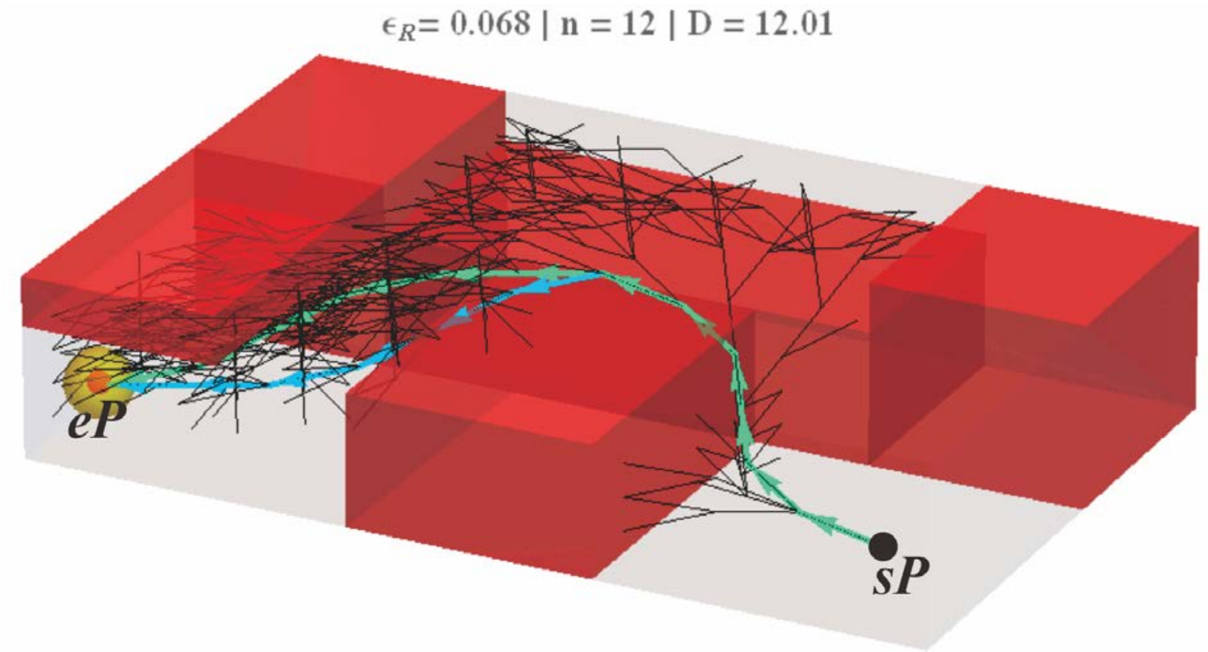

Figure 5: The environment for Truss-Z paths (TZPs). The "allowable zone" (AZ) and obstacles are shown in gray and red, respectively. All TZMs are represented by the center-line vectors **c**. Two alternative TZPs are shown in cyan and green. The start and end points sT and eT are shown as black and orange dots, respectively. All allowable center-line vectors are shown as thin black segments. The proximity pR is visualized as a yellow sphere. The Z-coordinates have been scaled by 2.4 to enhance visibility. The paths are generated for the original TZM.

The modular character of the TZ system makes it necessary to asses structurally not a single specific configuration s of the modules, but rather an entire set S of configurations that are expected to be used in real environments. In practice, a TZ ramp can be assumed to be supported not sparser than every five modules, thus the set S is assumed here to contain all possible configurations of five TZMs with fixed supports in all degrees of freedom of the entrance and exit square plane frames. In general, there are $5^4 = 1024$ such structures, but if the left–right and entrance–exit structural symmetries are accounted for, the number can be reduced to 256. Consequently, the structural performance of a TZ module is defined based on the worst possible scenario as the maximum effective stress of the segments in all modules in all possible configurations $s \in S$ under the defined static load,

$$\sigma_s(\mathbf{x},\mathbf{y}) = \max_{s\in S} \sigma_s(\mathbf{x},\mathbf{y}). \tag{4}$$

Since the maximum strain defined this way depends on the vector **y** of segment diameters, the TZ module can be optimized with respect to the vector **y** in order to minimize the maximum strain, which leads to the final form of the objective function to capture the structural quality of the TZM:

$$\sigma(\mathbf{x}) = \inf_{\mathbf{y}} \max_{s\in S} \sigma_s(\mathbf{x},\mathbf{y}), \tag{5}$$

where the infimum is defined with respect to the vector **y** subject to the constraint on the total module mass, as mentioned in Eq.(3).

6. Multicriterial optimization

As discussed in Sections 5.2 and 5.3, there are three objective functions that need to be taken into account in the optimization process of a Truss-Z module. Two of them express the geometrical quality:

1. The reaching error $\varepsilon_R(\mathbf{x})$;
2. The number $N_{TZM}(\mathbf{x})$ of the modules used to form the path;

while the third function quantifies the structural quality of a TZM:

3. The maximum effective stress $\sigma(\mathbf{x})$ for a TZM with optimized diameters of segments, with respect to all 5-unit TZ configurations.

The unavoidable result is a multicriterial optimization problem, where the Pareto front can be used to strike the optimal balance between the geometric and structural assessment criteria. Since there are two geometric criteria, two multicriterial optimizations need to be performed.

Due to the combinatorial nature of the geometric objective functions, an exhaustive search with respect to the vector $\mathbf{x}$ has been performed. First, the search space had to be constrained and discretized:

- the center-line length C has been assumed to be bound between 30% and 200% of the length c_0 of the original TZM with a step of 5%, that is

$$c \in \{0.30c_0, 0.35c_0, 0.40c_0, \ldots, 1.95c_0, 2.00c_0\}, \tag{6}$$

- the planar angle θ has been bound by 10°from below, and progressed in steps of 1°up to the upper bound defined by the 25% inclination limit of the floor, as described in Section 5.1, which can be verified to be

$$\theta \in \left\{10^\circ, 11^\circ, \ldots, \left\lfloor 75\pi^{-1} c \frac{c_0 - 4\delta_{Z0}}{c_0} \right\rfloor^\circ \right\}. \tag{7}$$

The resulting search space contained 1371 values of the vector $\mathbf{x}$. Each of the corresponding TZMs has been then used to create a path of the minimal length in the typical environment shown in Figure 5 and to compute the corresponding values of the geometric objective functions $\varepsilon_R(\mathbf{x})$ and $N_{TZM}(\mathbf{x})$. Only 63 TZMs are capable to form a path subject to the functional constraints described in Section 5.2. Finally, each of these 63 TZM have been optimized with respect to the vector $\mathbf{y}$ of segment diameters to compute the corresponding value of the structural objective function $\sigma(\mathbf{x})$, see Eq.(5). Each of these 63 cases has been assessed using the formal tool of the Pareto front.

6.1. Effective stress vs. reaching error

Figure 6 shows the scatter plot of the effective stress σ vs. the reaching error ε_R. Each point represents a single case out of the 63 allowable TZMs. The orange line marks the Pareto front of four TZMs.

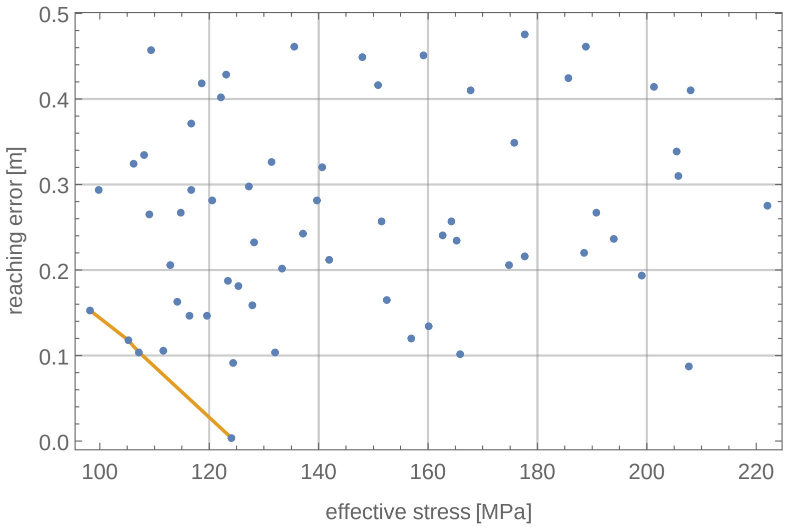

Figure 6: The scatter plot of the effective stress σ vs. the reaching error ε_R. The Pareto front contains four cases and is marked by the orange line.

6.2. Effective stress vs. the number of modules

Figure 7 shows the scatter plot of the effective stress σ vs. the number of modules N_{TZM}. Each point represents a single case out of the 63 allowable TZMs. The orange line marks the Pareto front of three TZMs.

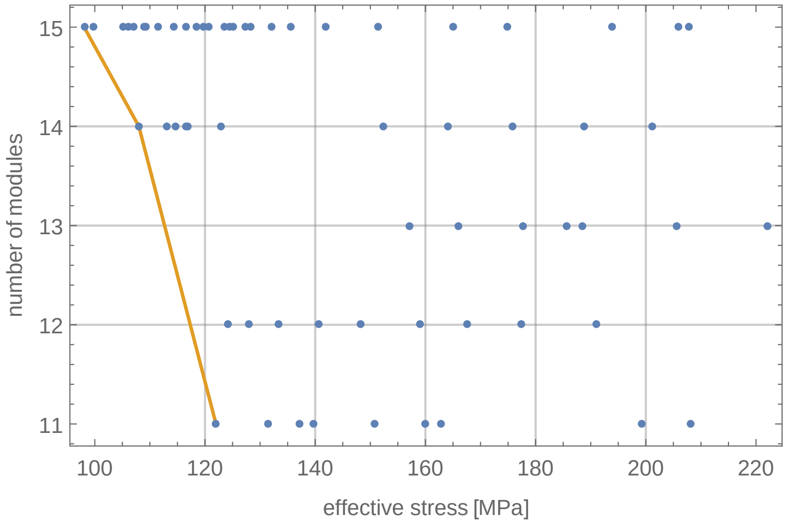

Figure 7: The scatter plot of the effective stress σ vs. the number of modules N_{TZM}. The Pareto front contains three cases and is marked by the orange line.

6.3. Optimized modules

The Pareto fronts presented in Figures 6 and 7 share the same (left-most) point. The multicriterial optimization has thus lead to six non-dominated points, which are listed in Table 1. Figure 8 shows the six corresponding paths in the considered environment, where each module is represented by its center-line vector. The corresponding six full TZ ramps are shown in Figure 9, where each segment is drawn using its (scaled) optimized diameter.

Table 1: The six non-dominated Pareto points obtained from the multicriterial optimization.

No.	angle θ	relative center-line length c/c_0	effective stress [MPa]	reaching error [m]	no. of TZMs
1	18°	0.85	98.1	0.153	15
2	22°	0.90	105.1	0.118	15
3	23°	0.90	107.1	0.104	15
4	31°	1.00	124.1	0.003	12
5	24°	0.95	108.0	0.334	14
6	30°	1.00	122.0	0.402	11

7. Conclusions

This contribution presents a preliminary study on structural improvement of the Truss-Z module. Three optimality measures have been defined; two of them are geometrical in nature and represent the ability of the module to create free-form ramps, while the third criterion is structural in nature and defined as the maximum effective stress under a given static load. An exhaustive search has been performed, and a total of 63 geometrically-allowable candidate solutions have been found. Six of them are non-dominated Pareto points with respect to the structural criterion and at least one of the geometrical criteria. In future work, a similar procedure of multicriterial optimization will be applied with modified objective functions that take into account buckling and the lateral stiffness of the resulting structures.

8. Acknowledgements

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9. References

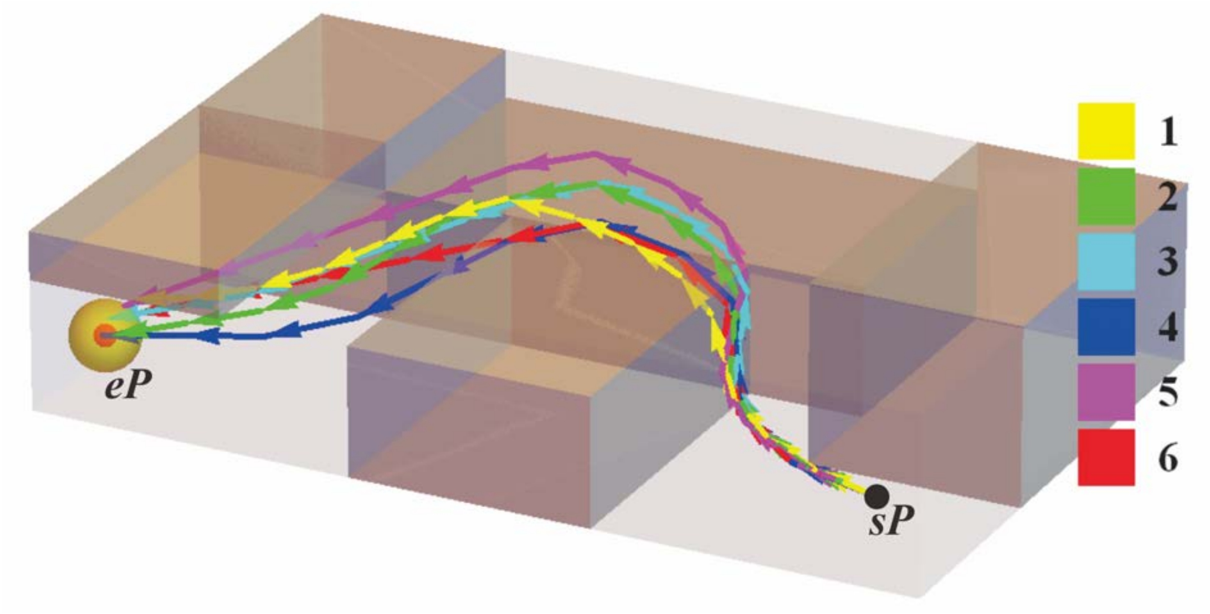

Figure 8: The TZ paths in the considered environment that correspond to the six obtained Pareto-optimal modules. Each module is represented by its center-line vector. The colors and numbers correspond to Table 1.

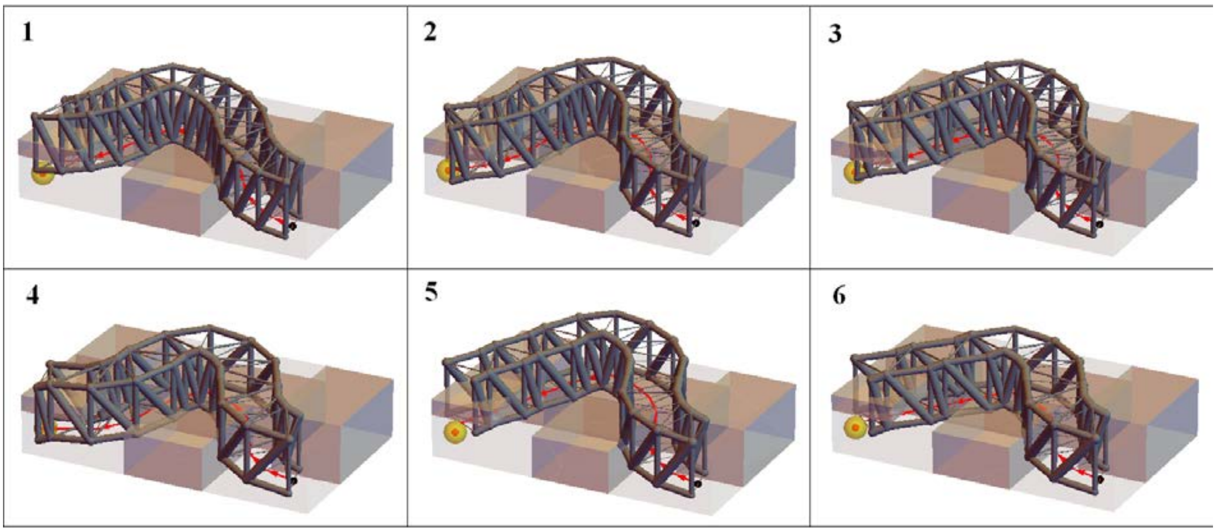

Figure 9: The TZ ramps that correspond to the six paths shown schematically in Figure 8. Each segment is drawn using its optimized diameter. For better legibility, the diameters are scaled by factor of 5. The corresponding center-line vectors are shown in red.

[1] M.E. Pollack, Intelligent Technology for an Aging Population: The Use of AI to Assist Elders with Cognitive Impairment, *Artificial Intelligence Magazine*, 26 (2), 9–24, 2005.

[2] G. Dunbar and C. Holland, Road Safety Research Report No. 37. Older Pedestrians: A Critical Review of the Literature. *Department of Transport*: London, England, 2004.

[3] M. Zawidzki, Creating organic 3-dimensional structures for pedestrian traffic with reconfigurable modular "Truss-Z" system, *International Journal of Design & Nature and Ecodynamics* 8(1), 61–87, 2013.

[4] M. Zawidzki, Retrofitting of pedestrian overpass by Truss-Z modular systems using graph-theory approach, *Advances in Engineering Software*, 81, 41–49, 2015.

[5] M. Zawidzki, K. Nishinari, Modular Truss-Z system for self-supporting skeletal free-form pedestrian networks, *Advances in Engineering Software*, 47 (1), 147–159, 2012.

[6] M. Zawidzki, "Tiling of a Path with Trapezoids in a Constrained Environment with Backtracking Algorithm", Wolfram Demonstrations Project [published: 20.09.11].

[7] M. Zawidzki and K. Tateyama, Application of Evolution Strategy for Minimization of the Number of Modules in a Truss Branch Created with the Truss-Z System, in Y. Tsompanakis, B.H.V. Topping (Eds.), Proceedings of the *Second International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering*, Civil-Comp Press, Stirlingshire, UK, Paper 9, doi:10.4203/ccp.97.9, 2011.

[8] M. Zawidzki and K. Nishinari, Application of evolutionary algorithms for optimum layout of Truss-Z linkage in an environment with obstacles, *Advances in Engineering Software*, 65, pp. 43–59, 2013.

[9] M. Zawidzki and Ł. Jankowski, Structural Optimization of a Five-Unit Single-Branch Truss-Z Modular Structure, to appear in Proceedings of the *8th ECCOMAS Thematic Conference on Smart Structures and Materials* (SMaRT 2017), June 5–8, 2017, Madrid, Spain.

[10] M. Ehrgott, Multicriteria Optimization, 2nd ed., Springer-Verlag, Berlin, 2005.

[11] R.T. Marler and J.S. Arora, Survey of multi-objective optimization methods for engineering, *Structural and Multidisciplinary Optimization*, 26 (6), 369–395, 2004.