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Global solution of the initial value problem for the discrete Boltzmann equation

H. CABANNES (PARIS)

In the discrete kinetic theory the initial value problem has a local solution. When the local solution is bounded by a number which depends only on the initial values, the solution exists globally. The first global existence theorem of this type has been obtained by Nishida and Mimura for a Broadwell gas (three-dimensional model with six velocities) when four of the six densities are equal. In the following work a similar theorem is proved for a more complex model: three-dimensional model with 14 velocities, obtained by joining the center of a cube first to the center of each face, then to each vertex. The theorem is proved first when the initial densities are small, then, following a method by Crandall and Tartar, when the initial densities are bounded. As a starting point certain properties of the local solution are shown to be satisfied.

W dyskretnej teorii kinetycznej problem początkowy ma rozwiązanie lokalne. Gdy rozwiązanie lokalne jest ograniczone przez liczbę, która zależy tylko od wartości początkowych, istnieje rozwiązanie globalne. Pierwsze tego typu twierdzenie o istnieniu otrzymali Nishida i Mimura dla gazu Broadwella (trójwymiarowy model z sześcioma prędkościami) w przypadku, gdy cztery z sześciu gęstości są sobie równe. W niniejszej pracy zostało udowodnione podobne twierdzenie dla bardziej złożonego modelu — modelu trójwymiarowego z czternastoma prędkościami, otrzymanego z połączenia środka sześcianu najpierw ze środkiem każdej ścianki a następnie z każdym wierzchołkiem. Twierdzenie udowodniono najpierw dla przypadku małych gęstości początkowych, a następnie posługując się metodą Crandalla i Tartara — dla ograniczonych prędkości początkowych. Jako punkt wyjścia pokazano, że niektóre własności rozwiązania lokalnego są spełnione.

В дискретной кинетической теории начальная задача имеет локальное решение. Когда локальное решение ограничено числом, которое зависит только от начальных значений тогда существует глобальное решение. Первую этого типа теорему существования получили Нишида и Мимура для газа Броудвелла (трехмерная модель с шестью скоростями) в случае, когда четыре из шести плотностей равны друг другу. В настоящей работе доказана аналогичная теорема для более сложной модели — трехмерной модели с четырнадцатью скоростями, полученными из соединения центра куба сначала с центром каждой стенки, а затем с каждой вершиной. Теорема доказана сначала для случая малых начальных плотностей, а затем, послуживаясь методом Крандалла и Тартара, для ограниченных начальных скоростей. Как исходная точка показано, что некоторые свойства локального решения удовлетворены.

1. Introduction

The discretization of the velocity space in the kinetic theory of gases allows the replacement of the Boltzmann equation, an integro-differential equation, by a system of semilinear partial differential equations [1]. For those equations, called kinetic equations, the initial value problem has a local solution when the initial values are bounded and differentiable; the local solution possesses the main properties listed in Sect. 2. Among the models with discrete repartition of velocities, one of the simplest is the Broadwell model [2] for which the velocities are obtained by joining the center of a cube at the origin of the velocity space to the centers of the faces. Using this model, and assuming a one-dimensional motion parallel to one of the velocities and equality of the densities of the four velocities orthogonal to that direction, NISHIDA and MIMURA [3] have proved the global existence of the solution of the initial value problem, provided the initial values are small in a certain sense. For a similar model TARTAR and CRANDALL [4] have proved the global existence of the solution when the initial values are no longer small, but periodic. The method of Nishida and Mimura, a proof of the global existence, consists in proving that the local solution is bounded by a constant which depends only on the initial values. The bound is obtained by the integration of conservation equations over triangles, each having an edge which corresponds to the axis $t = 0$, the other edges being characteristics of the system of kinetic equations. The purpose of the present work is to extend the proof and conclusions first of NISHIDA and MIMURA, then of TARTAR and CRANDALL, to more complex model. The model considered is a three-dimensional model with 14 velocities obtained by joining the center of a cube at the origin of the velocity space to the centers of the faces and to the vertices [5]. For this model the generalization is possible, because the components of the velocities in the direction of motion are smaller in number than the number of conservation equations. Section 2 is devoted to a summary of the properties of the local solution. The subsequent sections are concerned with the global existence theorem when the initial densities given for $x \in R$ are successively "small", periodic and bounded.

2. Properties of the local solution

The general evolution equations of a gas with a discrete repartition of velocities appear in the form

$$\frac{\partial N_i}{\partial t} + \mathbf{u}_i \cdot \nabla N_i = \frac{1}{2} \sum_{jkl} A_{ij}^{kl}(N_k N_l - N_i N_j) \qquad (i = 1, 2, \ldots, p). \tag{2.1}$$

The unknown functions $N_i(\mathbf{x}, t)$ denote the densities of different velocities $\mathbf{u}_i$, represented by p constant vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p$. The coefficients A_{ij}^{kl}, the transition probabilities, are positive constants (or zero). $\mathbf{x}$ is the position vector, with components x, y, z in a Cartesian rectangular system $Oxyz$; t is the time. The Cauchy problem consists in finding a solution of a system of Eq. (2.1) which, at the initial time, is equal to given values

$$N_i(\mathbf{x}, 0) = N_{0i}(\mathbf{x}) \qquad (i = 1, 2, \ldots, p). \tag{2.2}$$

THEOREM 1. *If the functions $N_{0i}(\mathbf{x})$ are continuous and differentiable, there exists a positive number δ_0 such that, in the interval $0 < t \leqslant \delta_0$, the problem* (2.1), (2.2) *has only one solution.*

This theorem, classical in analysis, assures the existence and uniqueness of a local solution, the properties of which can be studied by a method of successive approximations. We can, for example, put

$$\begin{gathered}\frac{\partial N_i^{n+1}}{\partial t} + \mathbf{u}_i \cdot \nabla N_i^{n+1} + \lambda N_i^{n+1} = \lambda N_i^n + \frac{1}{2} \sum_{jkl} A_{ij}^{kl}(N_k^n N_l^n - N_i^n N_j^n), \\ N_i^{n+1}(\mathbf{x}, 0) = N_{0i}(\mathbf{x});\end{gathered} \tag{2.3}$$

$$N_i^1(\mathbf{x}, t) = N(\mathbf{x}). \tag{2.4}$$

We deduce from Eqs. (2.3)

$$N_i^{n+1}(\mathbf{x}, t) = e^{-\lambda t}N_{0i}(\mathbf{x}-\mathbf{u}_i t)+\int_0^t h_i^n(\mathbf{x}-\mathbf{u}_i s, t-s)e^{-\lambda s}ds, \tag{2.5}$$

where h_i^n is the right-hand side of Eq. (2.3). Considering in the four-dimensional space the point $A(\mathbf{x}_A, t_A)$ and the points $B_i(\mathbf{x}_A-\mathbf{u}_i t_A, 0)$, we denote by $\mathscr{D}_A$ the smallest convex domain of the hyperplane $t = 0$ containing all the points B_i. From the formula (2.5) we deduce

THEOREM 2. *The values of the functions $N_i(\mathbf{x}, t)$ at the point A depend only on the initial values $N_{0i}(\mathbf{x})$ in the domain $\mathscr{D}_A$.*

THEOREM 3. *If the initial densities satisfy the inequalities $0 \leqslant N_{0i}(x) \leqslant K_0$, the solution of the problem* (2.1), (2.2) *satisfy the inequalities $N_i(\mathbf{x}, t) \geqslant 0$ for all $\mathbf{x} \in \mathrm{R}^3$ and $0 < t \leqslant \delta_0$.*

THEOREM 4. *If the initial densities are independent of one of the space variables, $\frac{\partial}{\partial y} N_{i0}(\mathbf{x}) = 0$, the solution of the problem* (2.1), (2.2) *satisfies the relations*

$$\frac{\partial}{\partial y} N_i(\mathbf{x}, t) = 0 \quad \textit{for all} \quad \mathbf{x} \in \mathrm{R}^3 \quad \textit{and} \quad 0 < t \leqslant \delta_0.$$

THEOREM 5. *If the initial densities $N_{0i}(\mathbf{x})$ are periodic functions with the period $\boldsymbol{\pi}$, the solution $N_i(\mathbf{x}, t)$ of the problem* (2.1), (2.2) *is, for $\mathbf{x} \in \mathrm{R}^3$, $0 < t \leqslant \delta_0$, periodic in $\mathbf{x}$ with the period $\boldsymbol{\pi}$.*

The proofs of Theorem 2, 4 and 5 are trivial. Theorem 3 is proved by choosing for λ in Eq. (2.3) a large enough positive constant. The solution of the problem (2.1), (2.2) can be majorized by the solution M_i of the associated problem:

$$\frac{\partial M_i}{\partial t}+\mathbf{u}_i\cdot\nabla M_i = \frac{1}{2}\sum_{jkl} A_{ij}^{kl} M_k M_l, \tag{2.1'}$$

$$M_i(\mathbf{x}, 0) = K_0 \tag{2.2'}$$

and the solution of this new problem is again majorized by the solution $L_i(t)$ of the problem

$$\frac{dL_i}{dt} = A(L_1+L_2+\ldots+L_p)^2, \quad A = \frac{p^2}{2}\sup A_{ij}^{kl}; \tag{2.1''}$$

$$L_i(0) = K_0. \tag{2.2''}$$

We have therefore

$$N_i(x, t) \leqslant L_i(t) = \frac{K_0}{1-AK_0 t}. \tag{2.6}$$

THEOREM 6. *If the initial values $N_{0i}(\mathbf{x})$ are continuous and differentiable functions satisfying the inequalities $0 \leqslant N_i(\mathbf{x}) \leqslant K_0$, then the unique solution of the problem* (2.1), (2.2) *exists for $\mathbf{x} \in \mathrm{R}^3$ and $0 < t \leqslant \delta_0 = \frac{1}{AK_0}$, where $A = \frac{p^2}{2}\sup A_{ij}^{kl}$.*

For certain particular models it is possible to show that in the domain $\mathbf{x} \in \mathrm{R}^3$ and $0 < t \leqslant \delta_0$ and under certain conditions for the initial values the (local) solution satisfies the inequalities $0 \leqslant N_i(\mathbf{x}, t) \leqslant K$, where the constant K depends only on the initial values.

We can consider the instant $t = \delta_0$ as initial and repeat the argument; so the solution exists for $\delta_0 < t \leqslant \delta_0+\delta$, with $\delta = 1/AK$; and for $t = t_1 = \delta_0+\delta$, we always have $0 \leqslant N_i(\mathbf{x}, t) \leqslant K$. This proves the global existence of the solution. We will show in the next section the existence of a number K for a three-dimensional model with 14 velocities.

3. Global solution for "small" initial values

The model considered is obtained by joining the center of a cube to the vertices and the centers of faces. The velocities are denoted by $\mathbf{u}_i(i = 1, \dots, 8)$ and $\mathbf{v}_j(j = 1, \dots, 6)$, and their components in the directions Ox, Oy, Oz are

$$\mathbf{u}_1 = c(-1, 1, 1), \quad \mathbf{u}_2 = c(1, 1, 1), \quad \mathbf{u}_3 = c(-1, 1, 1), \quad \mathbf{u}_4 = c(1, -1, 1),$$
$$\mathbf{v}_1 = c(1, 0, 0), \quad \mathbf{v}_2 = c(0, 1, 0), \quad \mathbf{v}_3 = c(0, 0, 1)$$

and

$$\mathbf{u}_{9-i} = -\mathbf{u}_i \quad (i = 1, 2, 3, 4), \quad \mathbf{v}_{j+3} = -\mathbf{v}_j \quad (j = 1, 2, 3)$$

the moduli being given by $|\mathbf{v}_j| = c$, $|\mathbf{u}_i| = c\sqrt{3}$. The number density of molecules with the velocity $\mathbf{u}_i$ is denoted by N_i, that of molecules with velocity $\mathbf{v}_j$ by M_j.

To write the kinetic equations we designate by $\mathbf{u}_{i1}$, $\mathbf{u}_{i2}$, $\mathbf{u}_{i3}$ the velocities associated with the velocity $\mathbf{u}_i$ so that $\mathbf{u}_{i1}-\mathbf{u}_i$, $\mathbf{u}_{i2}-\mathbf{u}_i$, $\mathbf{u}_{i3}-\mathbf{u}_i$ are parallel to the coordinate' axis. Then we put

$$\begin{aligned} \mathbf{u}_{i4} &= \mathbf{u}_{i2}+\mathbf{u}_{i3}-\mathbf{u}_i, & 2\mathbf{v}_{i1} &= \mathbf{u}_i+\mathbf{u}_{i1}, \\ \mathbf{u}_{i5} &= \mathbf{u}_{i3}+\mathbf{u}_{i1}-\mathbf{u}_i, & 2\mathbf{v}_{i2} &= \mathbf{u}_i+\mathbf{u}_{i2}, \\ \mathbf{u}_{i6} &= \mathbf{u}_{i1}+\mathbf{u}_{i2}-\mathbf{u}_i, & 2\mathbf{v}_{i3} &= \mathbf{u}_i+\mathbf{u}_{i3}. \end{aligned} \tag{3.1}$$

The kinetic equations, are then (for details see [5]):

$$\frac{\partial N_i}{\partial t}+\mathbf{u}_i\cdot\nabla N_i = \frac{\sqrt{3}}{2}cS\sum_{s=1}^{4}(N_sN_{9-s}-N_iN_{9-i}) \tag{3.2}$$
$$+\sqrt{2}cS\left\{N_{i2}N_{i3}+N_{i3}N_{i1}+N_{i1}N_{i2}-N_i\sum_{\alpha=4}^{6}N_{i\alpha}\right\}+\frac{\sqrt{6}}{2}cS\sum_{\alpha=1}^{3}(N_{i\alpha}M_{3+i_\alpha}-N_iM_{i_\alpha}),$$

$$\frac{\partial M_j}{\partial t}+\mathbf{v}_j\cdot\nabla_j M = \frac{2}{3}cS\sum_{r=1}^{3}(M_{j+r}M_{j+r+3}-M_jM_{j+3}) \tag{3.3}$$
$$+\frac{\sqrt{6}}{2}cS\sum_{s=1}^{4}(M_{j+3}N_{j+1+2s}-M_jN_{j+2s}).$$

There are 8 equations, (3.2) $i = 1, \dots, 8$ and 6 equations, (3.3), $(j = 1, \dots, 6)$. When the initial densities are independent of y and z, which we assume, the densities are independent of y and z. Therefore we look for the solution $N_i(x, t)$, $M_j(x, t)$ of the kinetic equations which satisfy the following initial conditions:

$$\begin{aligned} N_i(x, 0) &= N_{0i}(x) \quad (i = 1, \dots, 8), \\ M_j(x, 0) &= M_{0j}(x) \quad (j = 1, \dots, 6). \end{aligned} \tag{3.4}$$

We assume that the initial densities are differentiable and satisfy the following conditions in which K_0 and α_0 are two positive constants:

$$0 \leqslant N_{0i}(x) \leqslant K_0, \quad 0 \leqslant M_{0j}(x) \leqslant K_0. \tag{3.5}$$

$$\int_{-\infty}^{\infty} \Big\{ \sum_{i=1}^{8} N_{0i}(\xi) + \sum_{j=1}^{6} M_{0j}(\xi) \Big\} S d\xi = \alpha_0. \tag{3.6}$$

THEOREM 7. *When the conditions* (3.5) *and* (3.6) *are satisfied, and when α_0 is less than* 3/4, *the solution of the initial-value problem* (3.2), (3.3), (3.4) *exists for all x and all $t > 0$.*

To prove this theorem by the method of Nishida and Mimura, we consider the sums of the densities of the velocities having the same components on the x axis, i.e. we put

$$\begin{aligned} \mathscr{A}_1(x,t) &= N_2+N_4+N_6+N_8+M_1, \\ \mathscr{A}_2(x,t) &= N_1+N_3+N_5+N_7+M_4, \\ 2\mathscr{A}_3(x,t) &= M_2+M_3+M_5+M_6. \end{aligned} \tag{3.7}$$

We write $\xi_1 = c > 0$, $\xi_2 = -c$, $\xi_3 = 0$; from Eqs. (3.2)–(3.3) we deduce the three equations

$$\frac{\partial \mathscr{A}_i}{\partial t} + \xi_i \frac{\partial \mathscr{A}_i}{\partial x} = f_i(x,t) \quad (i = 1, 2, 3), \tag{3.8}$$

$$f_1 = f_2 = -f_3 = \frac{2}{3} cS(M_2M_5 + M_3M_5M_6 - 2M_1M_4). \tag{3.9}$$

Equations (3.8) can be integrated in the form

$$\mathscr{A}_i(x,t) = \mathscr{A}_i(x-\xi_i t, 0) + \int_0^t f_i(x-\xi_i s, t-s)ds \quad (i = 1, 2, 3). \tag{3.10}$$

The functions $\mathscr{A}_i(x-\xi_i t, 0)$ are bounded by $5K_0$. And in the domain $x \in \mathrm{R}$, $0 < t \leqslant \delta_0$, the densities being positive, the integrals in the second term of Eqs. $(3.10)_1$, $(3.10)_2$ and $(3.10)_3$ are bounded by

$$\frac{2}{3} KcS \int_0^t (M_2+M_3)(x-\xi_i s, t-s)ds \quad (i = 1, 2), \tag*{$(3.11)_i$}$$

$$\frac{4}{3} KcS \int_0^t M_1(x, t-s)ds, \tag*{$(3.11)_3$}$$

K being a bound of the densities in the domain. It is then possible to majorize the last integrals by integrating the conservation equations

$$\frac{\partial}{\partial t}(\mathscr{A}_i + \mathscr{A}_3) + \xi_i \frac{\partial \mathscr{A}_i}{\partial x} = 0 \quad (i = 1, 2) \tag*{$(3.12)_i$}$$

over the triangles AA_iA_3 of the x, t plane. The points A and A_i have as coordinates (x, t) and $(x-\xi_i t, 0)$, respectively. The Stokes theorem gives for $i = 1$,

$$\int^t \mathscr{A}_1(x, t-s)cds + \int_0^t \mathscr{A}_3(x-cs, t-s)cds = \int_{x-ct}^{x} \{\mathscr{A}_1(\xi, 0) + \mathscr{A}_3(\xi, 0)\} d\xi \leqslant \frac{\alpha_0}{S} \tag{3.13}$$

and a similar formula for $i = 2$. We deduce that the integrals $(3.11)_i$ $(i = 1, 2, 3)$ are bounded by $\frac{4}{3}\alpha_0 K$, and

$$K \leqslant \sup \mathscr{A}_i(x, t) \leqslant 5 K_0 + \frac{4}{3}\alpha_0 K \tag{3.14}$$

or, for $\alpha_0 < \frac{4}{3}$

$$K \leqslant \frac{5 K_0}{1 - \frac{4}{3}\alpha_0} \tag{3.15}$$

which proves the global existence of the solution of the problem (3.2), (3.3), (3.4).

4. Global existence for large initial-values

The global existence theorem proved in the former section assumes that the initial mass in a tube of the cross-section S is sufficiently small. For a plane regular model with 4 velocities, Crandall and Tartar, using the H-theorem, have been able to drop this assumption when the initial densities $N_{0i}(x)$ are periodic functions [4]. The demonstration of Crandall and Tartar is valid for all models for which the results of the previous section are valid: existence of a bound of the local solution.

The initial densities being independent of y and z, the densities $N_i(x, t)$ will also only be periodic in x; we will designate the period by π. We have therefore to solve the following problem:

$$\frac{\partial N_i}{\partial t} + u_i \frac{\partial N_i}{\partial x} = \frac{1}{2}\sum_{jkl} A_{ij}^{kl}(N_k N_l - N_i N_j) \qquad (i = 1, 2, \ldots, p), \tag{4.1}$$

$$N_i(x, 0) = N_{0i}(x) \tag{4.2}$$

with

$$0 \leqslant N_{0i}(x) \leqslant K_0, \qquad \text{and} \qquad N_{0i}(x+\pi) = N_{0i}(x).$$

By multiplying the two members of Eq. (4.1) by $1 + \log N_i$ and by adding the equations obtained for all values of i, we obtain

$$\sum_{i=1}^{p}\left(\frac{\partial N_i}{\partial t} + u_i \frac{\partial N_i}{\partial x}\right)(N_i \log N_i) = \frac{1}{2}\sum_{jkl} A_{ij}^{kl} \log\frac{N_i N_j}{N_k N_l}(N_k N_l - N_i N_j). \tag{4.3}$$

The right-hand side is negative or zero. Thus the first member will also be negative, as will be its integral, say, over a period

$$\frac{d}{dt}\int_0^{\pi}\left(\sum_{i=1}^{r} N_i \log\frac{N_i}{K_0}\right) dx \leqslant 0; \tag{4.4}$$

as a consequence,

$$I(t) = \sum_{i=1}^{p} \int_{0}^{\pi} N_i(x, t) \log \frac{N_i(x, t)}{K_0} dx \leqslant I(0) \leqslant 0. \tag{4.5}$$

From this inequality we deduce, for $2cT < \pi$,

$$\sum_{i=1}^{p} \int_{x-cT}^{x+cT} N_i(x, t) dx \leqslant \frac{4\pi p K_0}{1 - \log \frac{2cT}{\pi}}; \tag{4.6}$$

the detail of the calculation are given in the reference [6].

Returning to the 14-velocity model, we now denote by $N_i(x,t)$ the densities (i vary from 1 to 14), and we consider the functions $\tilde{N}_i(x, t)$ which satisfy the following conditions:

$$\begin{aligned} &\tilde{N}_i(x, t_1) = N_i(x, t_1) \quad \text{for} \quad X - cT \leqslant x \leqslant X + cT, \\ &\tilde{N}_i(x, t_1) = 0 \qquad\qquad \text{for} \quad |X - x| > cT. \end{aligned} \tag{4.7}$$

The functions $\tilde{N}_i(x, t_1)$ satisfy, for all t_1 positive, the condition

$$\int_{-\infty}^{\infty} \left\{ \sum_{i=1}^{14} N_i(\xi, t_1) \right\} S d\xi < \alpha_1 = \frac{56\pi S K_0}{1 - \log \frac{2cT}{\pi}}. \tag{4.8}$$

If we choose

$$\frac{2cT}{\pi} \leqslant \exp\left\{1 - \frac{224}{3} \pi S K_0\right\}, \tag{4.9}$$

the first member of the inequality (4.8) is less than $\frac{3}{4}$, and we can apply:

THEOREM 7. *Therefore, the functions $\tilde{N}_i(x, t)$ exist for all values of $x \in \mathsf{R}$ and $t > t_1$. In the triangle with the vertices $(X, t_1 + T)$, $(X \pm cT, t_1)$ the solution $\tilde{N}_i(x, t)$ coincides with the solution of the kinetic equations which takes the values $N_i(x, t_1)$ for $t = t_1$; as X is arbitrary, this proves the existence of the solution for $t_1 < t \leqslant t_1 + T$. The inequality* (4.8) *is still valid for $t_2 = t_1 + T$, hence existence also holds for $t_1 + T < t \leqslant t_1 + 2T$; the argument can be repeated, and as t_1 is arbitrary and can be chosen less than δ_0, global existence follows.*

It is now easy to pass from the periodic case to the general case where the initial densities satisfy only the conditions $0 \leqslant N_{0i}(x) \leqslant K_0$. We can define new initial values $\tilde{N}_{0i}(x)$ periodic, of period π, with continuous derivatives, and satisfying the conditions $\tilde{N}_0(x) = N_{0i}(x)$ for $|x - X| \leqslant cT$, with $2cT < \pi$. The corresponding solution $\tilde{N}_i(x, t)$ exists globally and coincides with the solution corresponding to the initial densities $N_{i0}(x)$ in the triangle (X, T), $(X \pm cT, 0)$ of the x, t plane. As X is arbitrary, the solution $N_i(x, t)$ exists for $x \in \mathsf{R}$, $0 < t < T$; and as T is arbitrary the existence is global. Hence we have the following general conclusion:

THEOREM 8. *If the initial densities are continuous, differentiable, positive and bounded, the solution of the initial value problem, for the* 14*-velocity model, exists globally.*

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LABORATOIRE DE MÉCANIQUE THÉORIQUE
UNIVERSITÉ PIERRE ET MARIE CURIE, PARIS, FRANCE.

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