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Unsteady supersonic radial gas expansion from a rapidly started source

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THE PROBLEM of supersonic radial gas expansion to a low pressure medium from a rapidly started source is considered. The present study is mainly concerned with the initial expansion stage of an unsteady gas flow. For this stage, using a simple approximated solution, the regularities of motion of discontinuity surfaces have been found which determine the structure of the flow region. A numerical solution has been obtained for the problem of viscous heat-conducting gases.

Rozważany jest problem naddźwiękowej promieniowej ekspansji gazu do ośrodka o niższym ciśnieniu od nagle pojawiającego się źródła. Niniejsze studium dotyczy głównie etapu ekspansji niestacjonarnego przepływu gazu. Na tym etapie, stosując proste rozwiązanie przybliżone, znaleziono zasady ruchu powierzchni nieciągłości, które określają strukturę obszaru płynięcia. Otrzymano rozwiązanie numeryczne dla zagadnienia przepływu gazu lepkiego przewodzącego ciepło.

Рассматривается проблема сверхзвукового радиального расширения газа в среду с более низким давлением от внезапно появляющегося источника. Настоящее исследование касается главным образом этапа расширения нестационарного течения газа. На этом этапе, применяя простое приближенное решение, найдены регулярности движения поверхности разрыва, которые определяют структуру области течения. Получено численное решение для задачи течения вязкого, теплопроводного газа.

Nomenclature

- r radial distance,
- t time,
- T temperature,
- p pressure,
- ϱ density,
- u gas velocity,
- $\varkappa$ ratio of heat capacities,
- M Mach number,
- x distance downstream from nozzle exit,
- V_α velocity of α surface motion,
- $m_{\alpha,\beta}$ gas mass in the range (r_α, r_β),
- c sound velocity,
- u viscosity.

Indices

- $\nu = 1, 2$ cylindrical and spherical symmetry,
- dimensional value,
- $-(+)$ left (right) side of discontinuity surface,

∞ ambient space conditions,
0 stagnation condition,
* sonic velocity surface,
1 source surface.

STATIONARY supersonic flows of heated gases in nozzles and free jets past underexpanded nozzles are widely used in laboratory practice for physical and aerodynamic investigations. In this case gas heating in a gasdynamic source pre-chamber is made using a pulse electric charge or shock compression [1-3]. Under these conditions, gas flow from a pre-chamber to a diverging nozzle section or to free space is started by an abrupt pre-chamber pressure increase. In a critical nozzle section a stationary flow regime is set and further gas flow development in the diverging nozzle section or free space occurs with a stationary gas source in the critical nozzle section having the parameters p_*, T_*, u_*, which correspond to those stagnations in the pre-chamber p_0 and T_0. Provided that p_0 and T_0 remain constant for a sufficiently long period of time, the required nozzle or jet stationary flow is set. In a real case [1-3] the time during which they can be considered constant is small. Hence it is important to know the way of stationary flow formation, particularly in the central flow region which is usually used for experiments.

Gas flow in the central region of jets flowing from an axisymmetrical or slotted nozzle with a heavily underexpanded stream as well as those in axisymmetrical or slotted nozzles with a straight contour (provided that the nozzle wall friction is neglected) are close to the radial one (with a sperical or cylindrical symmetry) [1-4]. Thus, as a result of idealizing the starting problem we turn to the problem of radial (unsteady) gas expansion from a rapidly started stationary source.

For the case of gas expansion to vacuum (as applied to nozzle flow) this problem was solved in [1]. In reference [4] (as applied to jet flows) flow asymptotics is studied at $t \to \infty$ for gas expansion to flooded space. In references [1, 4] the consideration was made in terms of the ideal fluid theory.

1. Formulation of problem

Assume that in an infinite volume of rest gas with the known parameters p_∞, T_∞ there is a spherical or cylindrical surface with the radius r_1. It is necessary to determine the gas flow development in the limit (r_1, ∞) with time if at a certain moment $t = t_1$ gas parameters on the surface $r = r_1$ jump to the given stationary value $p_1, T_1, u_1 > 0$, i.e. the stationary source is rapidly started.

2. General flow picture

Let the process of flow development be considered at radial gas (ideal fluid) expansion into flooded space from a rapidly started supersonic stationary source; herewith the results of references [1-4] will partially be used. A qualitative flow picture for $\nu = 1$ and $\nu = 2$ is analogous.

Figure 1 shows the trajectories of discontinuity surfaces which determine the structure of the flow region. The letters i, e, s are given for the interface separating the source and ambient gases, the shock waves in ambient and source gases, respectively.

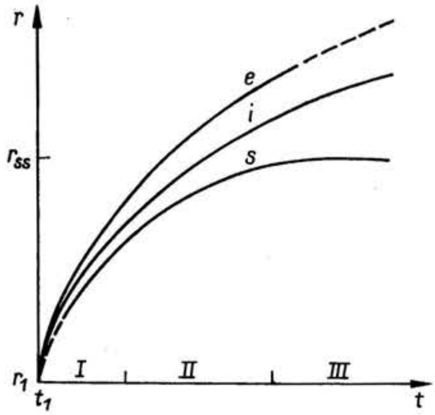

FIG. 1.

At the initial moment of time $t = t_1$ the interface motion velocity $V = u_1$. The source gas acts on the ambient one similarly as an expanding cylindric or spherical piston and, since according to the problem conditions $u_1 > 0$ is instantaneously achieved, at the moment t_1 the compression wave in the ambient gas will be centered. Hence it should be considered that the ambient gas compression from the very beginning of its flowing will occur under the action of a shock wave (e).

With rising r_1 the mass of ambient gas displaced and set in motion by a piston and the ambient pressure counteraction increase, while the driving pulse of a source per unit time remains constant. Hence the surface motion i is decelerated with time, thus hindering free gas expansion from the source and resulting in its accumulation in front of the surface i and compression. At the initial period the process of gas compression will be isentropic, since at $t = t_1$ velocities of the surface i motion and its "flowing" gas are equal, and the difference in these velocities continuously grows with time. In time an isentropic compression wave transforms to a shock one whose trajectory is shown by a solid curve s; the dashed portion of this curve corresponds to the stage of isentropic compression. Gas flow in the region (r_1, r_s), under a solid curve, is not disturbed and occurs as if the gas flowed into vacuum.

At $t \to \infty$ $r_i, r_e \to \infty$. In this case $V_i \to 0$ and the wave is degenerated to a weak disturbance. At the same time the pressure in the range (r_s, r_i) becomes close to that of the ambient gas p_∞ as a result of which the shock wave tends to a certain position $r_s = r_{ss} \sim r_1 (p_{01}/p_\infty)^{1/\nu}$[4].

Herewith the following nomenclature will be used (see Fig. 1): I — initial expansion stage $r_e, r_s, r_i \ll r_{ss}$, II — intermediate expansion stage, III — final expansion stage ($t \to \infty$; $r_i \to \infty$; $r_e \to \infty$; $r_s \to r_{ss}$).

3. Basic regularities of discontinuity surfaces motion at the initial gas expansion stage to flooded space

For simplicity in terms of the ideal fluid theory consider gas expansion from a rapidly started hypersonic stationary source ($M_1^2 \gg 1$). The conditions in flooded space will be taken practicable: $\varrho_\infty \ll \varrho_1$, $T_\infty \ll T_{01}$. In the limiting case of gas expansion to vacuum the problem has a simple solution: gas (in particular, its front edge, i.e. the interface) moves almost at a constant rate $u \simeq u_{\max} \simeq u_1$, and the distributions p, ϱ and T in the range (r_1, r_i) are close to those at stationary expansion [1].

Let the interface motion be determined. Using the concepts of the thin compressed layer theory [6], we will assume [5] that at the initial expansion stage the gas mass concentrated in a compressed layer (r_s, r_e) moves at an average velocity equal to $V_i = (dr_i)/(dt)$. Since, according to the problem conditions $M_1^2 \gg 1$ and the thickness of compressed layer is small, the influence of pressure due to the current tubes widening on the gas flow can be neglected. In addition, the counterpressure p_∞ may also be neglected, since for sufficiently small T_∞ (or, to be more precise, for $C^2 \leqslant V_i^2$) the flowing gas deceleration will occur mainly as a result of the motion of the displaced ambient gas. Taking this into consideration the pulse conservation equation will be written in the form

$$\frac{d}{dt'}\left[(m'_{s,i}+m'_{i,e})\frac{dr'_i}{dt'}+2\nu\pi\int_{r'_1}^{r'_s}\varrho' u' r'^{\nu} dr'\right] = 2\nu\pi\varrho'_1 u'^2_1 r'^{\nu}_1, \tag{3.1}$$

where a dash is given for a dimensional variable and $m'_{s,i}$ and $m'_{i,e}$ stand for the gas mass in the regions (r_s, r_i) and (r_i, r_e), respectively.

It is evident that $m'_{i,e}$ is approximately equal to the mass of the displaced ambient gas. In particular, for $r'_i \gg r'_1$

$$m'_{i,e} = \frac{2\nu}{\nu+1}\pi\varrho_\infty r'^{\nu+1}_i. \tag{3.2}$$

To determine the value $m'_{i,s}$ let an equation of the following gas mass conservation be used. At $t \gg t'_1$ it approximately equals

$$m'_{s,i} = 2\nu\pi\varrho'_1 u'_1 r'^{\nu}_1 t' - 2\nu\pi\int_{r'_1}^{r'_s}\varrho' r'^{\nu} dr'. \tag{3.3}$$

In the range (r_1, r_s) gas flows as if it expanded into vacuum. In particular, when $M_1^2 \gg 1$ we have $u' \simeq u'_1$ and

$$\varrho' r'^{\nu} = \varrho'_1 r'^{\nu}_1 \tag{3.4}$$

and this will be used to calculate integrals entering into Eqs. (3.1) and (3.3). In addition, having replaced the upper limits of integration in the above integrals for a close value r_i, after substituting Eqs. (3.2) and (3.3) into Eq. (3.1) and double integration, the law of interface motion will be obtained:

$$(r_i - t)^2 = b r_i^{\nu+2} + C_1 t + C_2, \quad b \equiv \frac{2}{(\nu+1)(\nu+2)}\varrho_\infty, \tag{3.5}$$

where $r_i = r'_i/r'_1$; $t = t'u'_1/r'_1$; $\varrho_\infty = \varrho'_\infty/\varrho'_1$. At r_i, $t = 0(1)$ from Eq. (3.5) and its associated differential first order expression for integration constants the evaluation C_1, $C_2 \ll 0(\varrho_\infty)$ follows. Hence, for r_i, $t \gg 1$ we finally have

$$t = r_i + \sqrt{b}\, r_i^{\frac{\nu+2}{2}}. \tag{3.6}$$

Note that the solution method used [5] is a simplified version of the method based on the parameter expansion into a series by the powers of $\varepsilon = (\varkappa+1)/(\varkappa-1)$[6] and conforms to its zero approximation. In this case the solution obtained (3.6) accurately coincides with an exact one (for $t \gg 1$, $r_i \gg 1$).

It should also be noted that to define the interface motion law we have used only the equations of pulse conservation and continuity and have not used that of energy. This implies that in a zero approximation of the thin compressed layer theory this layer motion does not depend on its energy transfer processes. In particular, it is independent of whether heat transfer from the compressed layer (or into it) takes place or not. Heat transfer to or from may be due to such processes as radiation, condensation or, for example, relaxation of the molecule internal freedom degrees, whose influence on gas flow in a compressed layer is similar to heat transfer to (or from) gases. The influence of flow of non-adiabatic character in a compressed layer in terms of the thin layer theory [6] is considered in [7].

At $r_i \ll b^{-1/\nu}$ Eq. (3.6) gives $r_i = t$, which is the solution for gas expansion to vacuum [1]. The effect of flooded space begins to manifest itself at $r_i = 0(b^{-1/\nu})$. Consider the solution obtained at $r_i \gg 0(b^{-1/\nu})$ where

$$r_i = b^{-\frac{1}{\nu+2}} t^{\frac{2}{\nu+2}}. \tag{3.7}$$

Coming back in Eq. (3.7) to the dimensional variables

$$r'_i = \left[\frac{(\nu+1)(\nu+2)}{2}\, \frac{\varrho'_1 u_1'^2 r_1'^\nu}{\varrho'_\infty}\right]^{\frac{1}{\nu+2}} t'^{\frac{2}{\nu+2}} \tag{3.8}$$

one can see that the interface motion for each $\nu = 1, 2$ is determined only by two values: ϱ'_∞ and the source-pulse per unit time $I'_1 = 2\pi\nu\varrho'_1 u_1'^2 r_1'^\nu$. If the pressure p'_∞ is neglected, then, among the parameters determining the gas state in the ambient space only one is unequal to zero, i.e. the dimensional value ϱ'_∞. Hence the gas flow process itself in the region (r_i, ∞) is determined only by two dimensional values I'_1 and ϱ'_∞. Since their dimensions are independent, in the region (r_i, ∞) there is as similar solution of the problem dependent on ν, $\varkappa$ and the only variable [8]:

$$\lambda = \left(\frac{I'_1}{\varrho'_\infty}\right)^{-\frac{1}{2+\nu}} r' t'^{-\frac{2}{\nu+2}}. \tag{3.9}$$

Discontinuity surfaces are ascribed to certain fixed values λ.

This solution reduces to the known solution of the problem on gas displacement by a cylindrical or spherical piston moving according to the power law $r'_1 = C' t'^{n+1}$ (3.8). In particular, we have

$$r_e(t) = \frac{\lambda_e}{\lambda_i} r_i(t), \qquad \frac{\lambda_e}{\lambda_i} = a(n, \varkappa, \nu), \tag{3.10}$$

where $n = -\nu/(\nu+2)$. For $\varkappa = 1.4$ the numerical values $a = a(n, \varkappa, \nu)$ are given in [9].

To evaluate the value $r_e(t)$ the formula (3.10) may be used at $r_i \leqslant 0(b^{-1/\nu})$ as well, since the value a is weakly dependent on the n exponent (which cannot be said about the parameters' distribution itself in the region (r_i, r_e)). Thus, e.g. for $\varkappa = 1.4$ at the maximum variation of n for Eq. (3.6) (from 0 to $-\nu/(\nu+2)$) the value a for $\nu = 1$ and $\nu = 2$ changes to the extent of 5% about a certain $a_{\text{aver}} = 1.13$ [9].

Let the approximate law of shock wave motion be found. Assuming that in the region (r_s, r_i) $\varrho' \simeq \varrho'_{s-}(\varkappa+1)/(\varkappa-1)$ from Eq. (3.3) and using Eq. (3.4) we obtain

$$\frac{\varkappa+1}{\varkappa-1}\,\frac{r_i^{\nu+1}-r_s^{\nu+1}}{r_s^{\nu}} = (\nu+1)(t-r_s),$$

where $t = t'u'_1/r'_1$, $r_\alpha = r'_\alpha/r'_1$. Or with the account of $r_i - r_s \ll r_i$

$$r_s = \frac{\varkappa+1}{2}\left(r_i - \frac{\varkappa-1}{\varkappa+1}t\right). \tag{3.11}$$

The results of the above consideration make it possible to explain some regularities of the discontinuity surface motion at a flow of shock-heated gas from a slit (the analog $\nu = 1$) and an annular slot ($\nu = 2$) [2, 3].

First the following should be noted. The condition $r_i \gg b^{-1/\nu}$ is equivalent to the condition implying that the mass of the displaced ambient gas $m'_{i,e}(\sim \varrho'_\infty r_i'^{\nu+1}(t))$ is much above that of the gas supplied by the source, $m_{1,i}(\sim \varrho'_1 u'_1 r'_1(t'))$. This provides a physical foundation of the fact that at $r'_i \gg r'_1$ and $t' \gg t'_1$ the law of interface motion $r'_i = r'_i(t)$ is determined only by the values ν, I'_1 and ϱ'_∞.

It is reasonable to suggest that in the case of gas expansion from the source with $M_1 = 1$, as is the case in the experiments [2, 3] at the initial expansion stage where $m'_{i,e} \gg m'_{1,i}$ and $r'_i \gg r'_1$, the motion of interface as before will be determined by two dimensional values $I'_1 = 2\nu\pi(\varrho'_1 u_1'^2 + p'_1) r_1'^{\nu}$ and ϱ'_∞. Here then the considerations which have led to Eqs. (3.9)–(3.10), are valid.

Reference [2] presents laws of motion of the interface and shock wave in an ambient gas experimentally determined and given as $x_\alpha = C_\alpha t^{n_\alpha}$ dependences. The analysis of experimental conditions shows that practically for the whole stage of expansion the condition $m'_{i,e} \gg m'_{1,i}$ has been met. The table presents a comparison between the values of n exponents and theoretical ones.

For $\varkappa = 1.4$ numerical values of a from Eq. (3.10) are known [9]. Hence the value x_e/x_i can be compared with a theoretical one. For a flow of N_2 ($\varkappa = 1.4$) from a slit where $n_e = n_i \simeq 0.67$ we have $x_e/x_i = 1.19$ and $r_e/r_i = 1.16$.

The formulas (3.6), (3.10) and (3.11) determining the time laws of discontinuity surface motion i, e, s at the initial expansion stage make it possible to introduce new variables

$$\tau = \left(\frac{\varrho'_\infty}{\varrho'_1}\right)^{1/\nu} \frac{t'u'_1}{r'_1}, \quad \xi_\alpha = \left(\frac{\varrho'_\infty}{\varrho'_1}\right)^{1/\nu} \frac{r'_\alpha}{r'_1}, \quad \alpha = i, e, s.$$

Herewith the motion of the above surfaces for various $\varrho'_\infty/\varrho'_1$ will be given by unified dependences. The treatment of data on the interface motion law using these variables points to the actual generalization of these data [3]. Ibidem a comparison is given between the law of interface motion and approximate solution (3.6) which gives an adequate agreement over a wide range of τ ($10^{-2} \lesssim \tau \lesssim 10^2$).

4. Numerical solution of problem for viscous heat conducting gas

Unsteady radial flow of a viscous heat conducting compressible perfect gas is described by the following system of equations [10]:

$$\varrho \frac{\partial u}{\partial t} + \varrho u \frac{\partial u}{\partial r} + \frac{1}{\varkappa M_1^2} \frac{\partial p}{\partial r} = \frac{1}{\mathrm{Re}_1} \left\{ \frac{4}{3} \frac{\partial}{\partial r} \left[\frac{\mu}{r^\nu} \frac{\partial}{\partial r} (r^\nu u) - 2\nu \frac{u}{r} \frac{\partial \mu}{\partial r} \right] \right\}, \tag{4.1}$$

$$\varrho \frac{\partial T}{\partial t} + \varrho u \frac{\partial T}{\partial r} - \frac{\varkappa - 1}{\varkappa} \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right) = \frac{1}{\mathrm{Re}_1} \left\{ \frac{1}{\sigma} \frac{1}{r^\nu} \frac{\partial}{\partial r} \left(r^\nu \mu \frac{\partial T}{\partial r} \right) + \frac{4}{3} (\varkappa - 1) M_1^2 \mu \left[\left(\frac{\partial u}{\partial r} \right)^2 - \nu \frac{u}{r} \frac{\partial u}{\partial r} + \left(\frac{u}{r} \right)^2 \right] \right\}, \tag{4.2}$$

$$\frac{\partial \varrho}{\partial t} + \frac{1}{r^\nu} \frac{\partial}{\partial r} (r^\nu \varrho u) = 0, \tag{4.3}$$

$$p = \varrho T, \tag{4.4}$$

where all values are related to their associated at $r' = r_1'$; $t = t' u_1'/r_1'$; $\mathrm{Re}_1 = \varrho_1' u_1' r_1' / \mu_1'$; $M_1 = u_1'/c_1'$; the Prandtl number σ and the heat capacity ratio $\varkappa$ are assumed to be constant.

Proceeding from a general formulation of the problem and taking the account of the character of the equation system (4.1)–(4.4), the boundary conditions can be written in the following form:

$$\begin{array}{llllll} \text{at } t = 1 & u = 1, & T = 1, & \varrho = 1 & \text{for } r = 1, \\ & u = 0, & T = T_\infty, & \varrho = \varrho_\infty & \text{for } 1 < r \leqslant \infty; \\ \text{at } t > 1 & u = 1, & T = 1, & \varrho = 1 & \text{for } r = 1, \\ & u = 0, & T = T_\infty & & \text{for } r = \infty. \end{array} \tag{4.5}$$

The system (4.1)–(4.5) has been solved numerically using a finite-difference approximation given in [11].

Figure 2 as an example illustrating the development of viscous heat conducting gas flow, presents the flow parameters' distribution for gas expansion from a rapidly started cylindrical source ($\nu = 1$) with $M_1 = M_* = 1$ to a medium with $\varrho_\infty = 0.12$, $T_\infty = T_{0*} = 1.2$. Here $\varkappa = 7/5$, $\sigma = 3/4$, $\mu = T$.

Zero marks the distribution of those parameters which correspond to the moment $t_0 = 1$. These distributions representing smooth discontinuities were set in the calculations as initial ones. The indices 1–13 mark the moments with respect to time $t_k = 1 + 0.32 \cdot 2^{k-1}$. Solid and dashed lines stand for $\mathrm{Re}_* = 25$ and $\mathrm{Re}_* = 200$, respectively. Arrows with indices s, i, e point to the positions (varying within 5–7%) of the corresponding discontinuity surfaces for the moment $t_6 = 11.28$.

A general flow picture conforms to that given in Point 2. The shock wave e is formed practically immediately after the beginning of gas flowing. Thus at the moment t_2 the difference in the value $T/\varrho^{\varkappa-1}$ "before" and "after" wave is as large as 30% (for $\mathrm{Re}_* = 200$). The process of gas compression in front of the internal "piston" surface during a certain

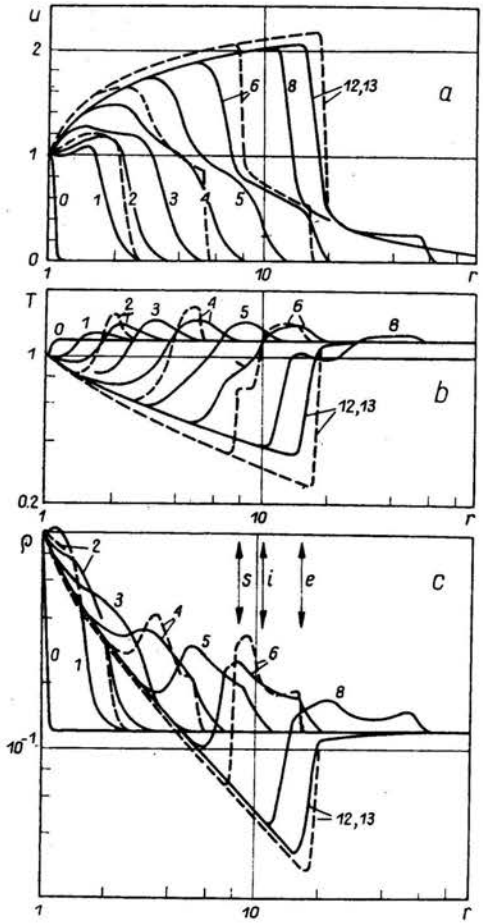

FIG. 2.

period of time remains isentropic. Thus, for the same Re_* at the moment t_4 the behaviour of ϱ and T over the range from $r = 1$ to the point with the maximum value ϱ obeys the law $T = \varrho^{\varkappa-1}$, accurate to 5%, as well. At the moment t_6 a shock wave s has already been formed.

Gas flow in the range $(1, r_s)$ is the same as in the case of its expansion to vacuum. And since in this case ϱ_∞ and T_∞ are sufficiently high, the interface moves slower than the boundary of the stationary flow at gas expansion to vacuum [1]. Hence gas flowing in the region $(1, r_s)$ is stationary. It possesses all the properties of the stationary radial flow at gas expansion to vacuum [12]. It should be noted that here the influence of Re_*, $\varkappa$, σ and the dependence $\mu = \mu(T)$ for $\nu = 1$ and $\nu = 2$ manifests itself in a similar way.

The behaviour of flow parameters in the region (r_i, r_e) at the initial moment of expansion (up to $t = 0(t_6)$) qualitatively conforms to that in the piston problem [9].

At $t \to \infty$ $r_e, r_i \to \infty$ and $r_s \to r_{ss}$. In this case as a result of the shock wave e degeneration to a weak disturbance $T_{i+} \to T_\infty$. At the same time T_{i-} tends to the flowing gas stagnation temperature T_{0*} here equal to T_∞. Hence $T_{i+} \to T_{i-}$. In addition, since on the contact surface $u_{i+} = u_{i-}$, $p_{i+} = p_{i-}$, the parameters' drop on it at $t \to \infty$ disappears and the flow goes to a stationary regime.

It should be noted that a decrease in the total flow continuity (a decrease in Re_*), resulting in a considerable change of the parameters over the total flow region, rather slightly influences the instantaneous positions of shock waves and interface.

A quantitative pattern of flows for the case of spherical ($\nu = 2$) expansion of a viscous heat conducting gas to flooded space with $T_\infty = T_{0*}$ is in good agreement with the one above. A significant difference between the cases $\nu = 1$ and $\nu = 2$ is registered at $T_\infty \neq T_{0*}$ and mainly refers to the final expansion stage. This problem requires a special consideration and will not be discussed here.

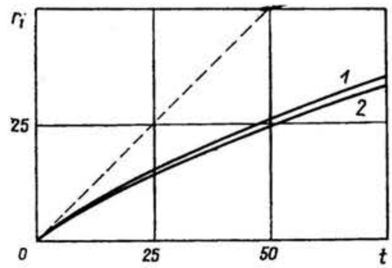

FIG. 3.

Figure 3 presents a comparison between the dependences $r_i = r_i\,(t)$ for the initial moment of gas expansion at $\nu = 1$, this was found using the approximate consideration of Point 3 (Formula (3.6), Curve *1*) and according to the results of numerical calculation

Table 1.

		Experiment			Theory
		$\varkappa = 9/7$	$\varkappa = 7/5$	$\varkappa = 5/3$	$\varkappa = 9/7;7/5;5/3$
$\nu = 1$	n_i	0.68	0.71	0.65	0.67
	n_e	0.71	0.71	0.69	
$\nu = 2$	n_i	—	0.54	0.52	0.5
	n_e	—	0.66	0.63	

(Curve *2*). Here $\varkappa = 7/5$; $M_1 = 5$; $\varrho'_\infty/\varrho'_1 = 0.123$; $T'/T'_1 = 0.145$. Since at $\varkappa = 7/5 > 1$ a compressed layer is not infinitely thin and $r_e \neq r_i$, to achieve a better accuracy of the approximate solution the value $r_e = ar_i$ (3.10) was introduced into Eq. (3.2) instead of r_i which, in a physical sense, corresponds to $m'_{i,e}$. This led to a new value in Eq. (3.6): $b = 2a^{\nu+1}/[(\nu+1)(\nu+2)]$, $a = 1.13$. The additional conditions for a numerical calculation are: $\sigma = 3/4$; $\mu = T$; $\mathrm{Re}_1 = 400$. As an illustrative example a dashed line is given for the dependence $r_i = t$, corresponding to the expansion to vacuum.

The approximate dependences $r_e(t)$ and $r_s(t)$ determined by the formulae (3.6) and (3.10) with $a = 1.13$, Eq. (3.11), are in agreement with those obtained from the results of numerical calculation accurate to 5—10%.

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