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On the dynamic description of the rock failure process

W. K. NOWACKI (WARSZAWA)

AN EXAMPLE of a uni-axial state of stress is used to present an attempt of a dynamic description of failure of rocks, concretes and similar materials. A model is proposed of an elastic/viscoplastic body with softening in the range of inelastic deformations. The material is assumed to behave as a viscoplastic body of the range of stresses exceeding the elastic limit. During unloading the process follows the statical unloading curve and disturbances are propagated at infinite velocities.

Na przykładzie jednoosiowego stanu naprężenia przedstawiono próbę opisu dynamicznego zniszczenia skał, betonów itp. Zaproponowano model ciała sprężysto/lepkoplastycznego z osłabieniem w strefie deformacji niesprężystych. Przyjęto, że w zakresie naprężeń przekraczających granicę sprężystości materiał zachowuje się jak lepkoplastyczny. W odciążeniu natomiast proces odbywa się wzdłuż krzywej odciążenia — zaburzenia rozprzestrzeniają się nieskończenie szybko.

На примере одноосного напряженного состояния представлена попытка описания динамического разрушения скал, бетонов и т. п. Предложена модель упруго-вязкопластического тела с ослаблением в зоне неупругих деформаций. Принято, что в интервале напряжений, превышающих предел упругости, материал ведется как вязкопластический. В разгрузке же процесс проходит вдоль статической кривой разгрузки — воз мущения распространяются с бесконечной скоростью.

1. Introduction

THE PAPER represents an attempt to describe the failure process of rocks subject to dynamic compression in the case of one-dimensional stress states. A model is proposed to simulate the behaviour of elastic/viscoplastic body with softening in the range of plastic deformation. The paper deals only with dynamic compression processes which lead to the degradation of the material.

Let us assume the material exibits purely elastic properties in the range of stresses below the elastic limit and behaves as a viscoplastic material above that limit. During unloading the process follows the statical unloading curve. In the process of elastic and viscoplastic straining, the behaviour of the material is described by a set of partial differential equations of the hyperbolic type so that the disturbances are propagated at finite velocities; during unloading, however, the process is governed by parabolic equations and the disturbances propagate at infinite speeds.

Section 2 presents the proposed constitutive equations which describe fairly well the properties of rocks, concretes and similar materials. The remaining sections present the solution of the problem of wave propagation in a semi-infinite rod made of a material exhibiting the degradation phenomena during unloading. The paper terminates with a numerical example which illustrates the theoretical considerations.

2. Model of the body with softening

Let us assume the following properties of the body with softening (in a one-dimensional stress state):

1) Within the range of stresses smaller than a certain limiting value σ^0 the material is elastic and its deformation is reversible.

2) Under dynamic loading the material is capable of carrying stresses greater than σ^0.

3) A statical softening curve $\sigma_{st} = f(\varepsilon)$ is assumed (Fig. 1) in the form of a monotonously decreasing function of strain: $df/d\varepsilon \leqslant 0$ for $\varepsilon^0 \leqslant \varepsilon \leqslant \varepsilon^*$ — gradual degradation

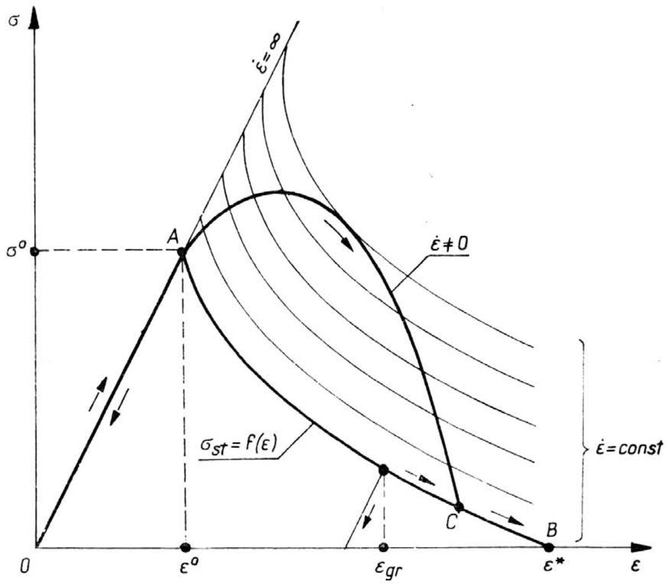

FIG. 1. Model of the material with softening.

follows; consequently, with $\varepsilon \geqslant \varepsilon^*$ one obtains $\sigma_{st} \equiv 0$ what results in complete degradation of the medium.

4) In the range of strains greater than ε the material exhibits viscous properties (at $\sigma > \sigma_{st}$).

5) For strains greater than ε^0 unloading follows the statical softening curve $\sigma_{st} = f(\varepsilon)$.

The model proposed may be used to describe the total degradation only; in order to describe also a partial degradation of the material, one should assume the existence of a certain limiting value of strain ε_{lim} below which elastic unloading would be possible.

The constitutive equations for such a medium have the following form:

in the range of elastic strains

$$\dot{\varepsilon} = \frac{1}{E_0}\dot{\sigma} \quad \text{for} \quad \varepsilon \leqslant \varepsilon^0, \tag{2.1}$$

in the range of inelastic strains

$$\dot{\varepsilon} = \frac{1}{E_0}\dot{\sigma} + \gamma\langle\Phi[\sigma - f(\varepsilon)]\rangle \operatorname{sign}\sigma \quad \text{for} \quad \varepsilon > \varepsilon^0 \text{ and } \sigma > f(\varepsilon) \tag{2.2}$$

during unloading

$$\sigma - f(\varepsilon) = 0 \quad \text{for} \quad \varepsilon > \varepsilon^0 \text{ and } \sigma \leqslant f(\varepsilon) \tag{2.3}$$

with the following notations: E_0 — Young's modulus, γ — viscosity constant, $\Phi[F]$ — nonlinear function of argument $F = \sigma - f(\varepsilon)$ symbol $\langle\ \rangle$ denotes

$$\langle \Phi[F] \rangle = \begin{cases} \Phi[F] & \text{if} \quad F > 0, \\ 0 & \text{if} \quad F \leqslant 0. \end{cases} \tag{2.4}$$

The form of function $\Phi[F]$ must be determined from the experiments performed in simple shear tests, under various strain rates.

The physical equations (2.2) resemble the equations of elastic/viscoplastic media proposed by L. E. MALVERN [1] for viscoplastic behaviour of metals but, in our case, the function $f(\varepsilon)$ is the material softening curve. Under sudden loading of the body ($\dot{\varepsilon} = \infty$) the material remains elastic. For constant strain rates the relation (2.2) represents the curves parallel to the statical material softening curve (Fig. 1). Degradation of the material at $\varepsilon > \varepsilon^0$ proceeds along the statical curve CB. Such a model may be used to describe, with good accuracy, the dynamic behaviour of rocks. In Fig. 2 are presented the results

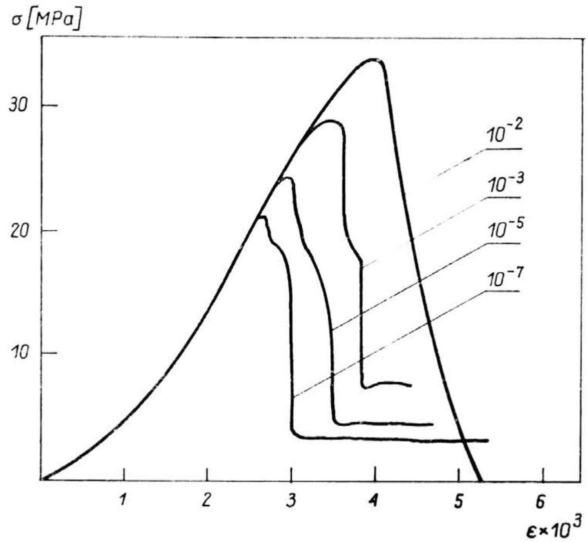

FIG. 2. Compression curves for the volcanic tuff specimens at various strain rates.

obtained by S. S. PENG and E. R. PODNIESK [2] for simple compression test of volcanic stuff specimens; this material exhibits considerable relaxation properties.

On the basis of the extensive experimental literature of the problem of dynamic behaviour of rocks (cf., e.g. [3, 4]), it may be stated that rocks exhibit considerable relaxation effects also in the range of elastic strains. The corresponding constitutive equation may be written in the form

$$\dot{\varepsilon} = g(\sigma, \varepsilon)\dot{\sigma} + \langle \Psi(\sigma, \varepsilon) \rangle \operatorname{sign} \sigma, \tag{2.5}$$

where the function $g(\sigma, \varepsilon)$ represents the instantaneous response of the material, and $\Psi(\sigma, \varepsilon)$ is the retarded response connected with the rheological properties of the material. In considering the wave propagation phenomena, the analysis will be confined to Eqs. (2.1) and (2.2).

3. Wave propagation in a semi-infinite rod

Let us present the solution of the problem of propagation of waves in a semi-infinite rod, the end $x = 0$ of the rod being subject to dynamic loading.

The equation of motion and the small deformation relation have the form

$$\sigma_{,x} - \varrho v_{,t} = 0, \qquad \varepsilon_{,t} = v_{,x}, \tag{3.1}$$

Here $v = \partial u/\partial t$ is the particle velocity of the medium.

Assume the homogeneous initial conditions

$$\sigma(x, 0) = v(x, 0) = \varepsilon(x, 0) \equiv 0 \qquad \forall x > 0. \tag{3.2}$$

and the boundary condition

$$\sigma(0, t) = -p(t), \qquad \forall t \geqslant 0, \qquad p(t) > 0, \qquad \frac{dp}{dt} \leqslant 0. \tag{3.3}$$

The system of Eqs. (3.1) together with the constitutive relations (2.1) or (2.2) leads to a set of differential equations of the hyperbolic type. Characteristics of this set of equations are the straight lines

$$x = \pm a_0 t + \text{const}, \qquad x = \text{const}, \tag{3.4}$$

where $a_0 = \sqrt{E_0/\varrho}$ is the longitudinal wave speed in the rod. The wave propagation speeds in the elastic and viscoplastic strain ranges are identical.

The following relations should be fulfilled along the characteristics (3.4):

along the line $x = \pm a_0 t + \text{const}$

$$d\sigma \pm \varrho a_0 dv + \varrho a_0^2 \langle \Phi[F] \rangle \operatorname{sign} \sigma \, dt = 0, \tag{3.5}$$

along the line $x = \text{const}$

$$d\varepsilon - \frac{1}{E_0} d\sigma - \gamma \langle \Phi[F] \rangle \operatorname{sign} \sigma \, dt = 0. \tag{3.6}$$

3.1. Solution at the strong discontinuity wave front $x = a_0 t$

Under the initial-boundary conditions assumed above, a strong discontinuity wave $x = a_0 t$ will be propagated in the rod. Let us assume that $p(0) > |\sigma^0|$. The medium in front of the wave $x = a_0 t$ is undisturbed. At the wave front the dynamic and kinematic compatibility conditions must be satisfied. For $x = a_0 t$

$$|[\sigma]| = -\varrho a_0 |[v]|, \qquad |[v]| = -a_0 |[\varepsilon]|, \tag{3.7}$$

where the symbol $|[f]|$ denotes the jump of f across the strong discontinuity wave front. In addition, $\sigma_0 = v_0 = \varepsilon_0 = 0$, the subscripts denoting the solutions in the corresponding regions of the phase plane (x, t), Fig. 3.

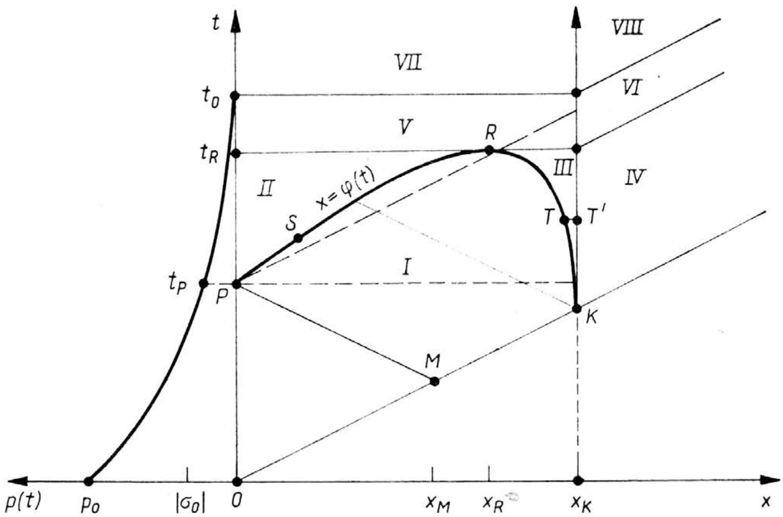

FIG. 3. Graph of wave propagation in a rod at the phase plane (x, t).

The relation holding at the positive characteristic (3.5) and the compatibility relations (3.7) yield the equation

$$2d\sigma_1 + \varrho a_0 \left\langle \Phi\left[\sigma_1 - f\left(\frac{\sigma_1}{E_0}\right)\right]\right\rangle \operatorname{sign}\sigma_1\, dx = 0 \quad \text{for} \quad x = a_0 t. \tag{3.8}$$

After integration we obtain

$$\sigma_1(x) = C - \int_0^x \Psi[\xi, \sigma_1(\xi)]\, d\xi, \tag{3.9}$$

where

$$\Psi[x, \sigma_1(x)] = \frac{1}{2}\varrho a_0^2 \left\langle \Phi\left[\sigma_1(x) - f\left(\frac{\sigma_1(x)}{E_0}\right)\right]\right\rangle \operatorname{sign}\sigma_1. \tag{3.10}$$

From the boundary condition (3.3) it follows that

$$C = -p(0). \tag{3.11}$$

The integrand of the relation (3.10) is a bounded function and satisfies the Lipschitz condition, hence the consecutive approximation method is convergent.

From the estimates we obtain

$$|\sigma_1| = p(0) + \frac{M}{R}(e^{Rh} - 1), \tag{3.12}$$

where M — upper bound of the integrand, R — value of the derivative for the parameters corresponding to the upper bound of the integrand and h denotes the smaller number of the pair

$$\frac{1}{2}x_K, \quad \left|\frac{p_0-|\sigma^0|}{2M}\right|.$$

The value of x_K is found from the condition $\sigma_1(x_K) = \sigma^0$. The velocity and strain at the wave $x = a_0 t$ is then found from the conditions (3.7).

Degradation of the material in the interval $0 \leqslant x \leqslant x_K$ will take place if the stress decreases with time (see the boundary condition (3.3)).

For $x > x_K$ the strong discontinuity wave is elastic, and at its front we obtain

$$\boldsymbol{\alpha}_1(x) = \boldsymbol{\alpha}^0, \quad v_1(x) = -\frac{1}{\varrho a_0}\boldsymbol{\alpha}^0, \quad \varepsilon_1(x) = \frac{1}{\varrho a_0^2}\sigma_1(x). \tag{3.13}$$

3.2. Solution in Region I (viscoplastic strains)

Once the solution at the wave front $x = a_0 t$ is known under the given boundary condition (3.3) expressed in terms of stresses (the boundary condition may also be of the kinematic type), the solution in Region I may be found, for instance, by the method of characteristics (see, e.g., [5]).

Region I is bounded from above by the unloading wave. It constitutes the locus of such points of the phase plane for which, at a fixed cross-section $x(0 \leqslant x \leqslant x_K)$, the following conditions is fulfilled:

$$\Phi[\sigma_1(x, t)-f(\varepsilon_1(x, t))]|_{x=\text{const}} = 0. \tag{3.14}$$

At the end $x = 0$ of the rod Eq. (3.6) may be integrated (along the vertical characteristic) since the function $\sigma_1(t) = -p(t)$ is known from the boundary condition (3.3). We obtain

$$\varepsilon_1(0, t) = -\frac{1}{E_0}p(t)-\gamma\int_0^t \Phi\lfloor p(\tau)-f(\varepsilon_1(0, \tau))\rfloor d\tau + C_1 \tag{3.15}$$

the integration constant being determined from the initial condition $\sigma_1(0, 0) = \frac{1}{E_0}\varepsilon_1(0, 0) = -p(0)$,

$$C_1 = -\frac{1}{E_0}p(0). \tag{3.16}$$

Equation (3.15) may be solved, like Eq. (3.9), by the method of iterations.

Point P (Fig. 3), from which the unloading wave will start to propagate, is determined from the condition

$$\Phi[p(t_p)-f(\varepsilon_1(0, t_p))] = 0. \tag{3.17}$$

The assumed model of the body suggests that the stress at P is smaller than $|\sigma^0|$,

$$|\sigma_1(0, t_p)| = p(t_p) < |\sigma^0|. \tag{3.18}$$

The unloading wave (its equation will be denoted by $x = \varphi(t)$) connects the points P and K of the phase plane. Stress at its front is a function of x and varies between $p(t_p) < |\sigma^0|$ (for $x = 0$) and $|\sigma\ |$ (for $x = x_K$).

3.3. Determination of the unloading wave $x = \varphi(t)$

The equation $x = \varrho(t)$ of the wave of unloading will be determined from the solution in Region I, account being taken of the solutions in the unloading regions (II and III) and the elastic region (IV).

Let us assume that the unloading process is quasi-static and follows the unloading curve CB (Fig. 1). During the inelastic unloading the following set of equations is satisfied:

$$\sigma - f(\varepsilon) = 0, \quad \sigma_{,x} = 0, \quad \varepsilon_{,t} = v_{,x}, \tag{3.19}$$

The inertial term of Eq. $(3.1)_1$ has been disregarded here. The set of equations (3.19) is of the parabolic type and the disturbances are propagated at infinite velocities.

From Eq. $(3.19)_2$ it follows that

$$\sigma(x, t) = C(t) \quad \forall t \geqslant \varphi^{-1}(x). \tag{3.20}$$

Stresses in the unloading region II are functions of time only, so that

$$\sigma_2(t) = -p(t) \quad \forall\, 0 \leqslant x \leqslant x_R, \quad t \geqslant \sigma\varphi^{-1}(x). \tag{3.21}$$

The unloading wave $x = \varrho(t)$ is assumed to be a weak discontinuity wave; at its front the stresses and velocities must be continuous,

$$\sigma_1[\varphi(t), t] = \sigma_2[\varphi(t), t], \quad v_1[\varphi(t), t] = v_2[\varphi(t), t] \quad \forall\, 0 \leqslant x \leqslant x_R \tag{3.22}$$

and

$$\sigma_1[\varphi(t), t] = \sigma_3[\varphi(t), t], \quad v_1[\varphi(t), t] = v_3[\varphi(t), t] \quad \forall\, x_R \leqslant x \leqslant x_K. \tag{3.23}$$

The initial speed of the unloading wave must be positive,

$$\varphi'(0) > 0. \tag{3.24}$$

In the opposite case, that is if $\varphi'(0) \leqslant 0$, the wave could be determined from the solution in Region I. It may be proved that the wave would then be a strong discontinuity wave, and at its front the kinematic compatibility condition

$$|[v]| = \varphi'(t)|[\varepsilon]|. \tag{3.25}$$

would not be satisfied.

In Region II and III, as it follows from Egs. $(3.19)_{1,2}$ the strains are functions of time only.

$$\varepsilon(x, t) = f^{-1}[\sigma(t)]. \tag{3.26}$$

Integration of Eq. $(3.19)_3$ with respect to x yields the formulae for the particle velocities in the unloading regions,

$$v(x, t) = \frac{d\varepsilon}{dt} x + C(t), \tag{3.27}$$

or

$$v(x, t) = \frac{df^{-1}[\sigma(t)]}{dt} x + C(t). \tag{3.28}$$

The integration constant is determined at the unloading wave front $x = \varrho(t)$,

$$C(t) = v_1[\varphi(t), t] - \frac{df^{-1}[\sigma(t)]}{dt}\varphi(t). \tag{3.29}$$

In Region I, inside the triangle OMP (Fig. 3), Picard's problem is now solved. In the remaining part of the region, Darboux's problem must be solved; it should be noted that for $0 \geqslant x \geqslant x_R$, in view of the infinite velocity of load propagation in the unloading zone (parabolic equation), one may observe the effect of the boundary $x = 0$, while for $x_R \geqslant \geqslant x \geqslant x_K$ — the effect of the other boundary $x = x_K$. The solution is constructed together with the determination of the unloading wave. For $0 \leqslant x \leqslant x_R$, at an arbitrary point S of the unloading $x = \varphi(t)$, in view of Eqs. (3.21) and (3.22), we obtain

$$\sigma_1[\varphi(t_S), t_S] = \sigma_2[\varphi(t_S), t_S] = -p(t_S). \tag{3.30}$$

From the condition $(3.5)_1$ satisfied at the negative characteristic and the condition (3.6) fulfilled at the vertical characteristic, we obtain $\sigma_1[(t_S), t_S]$ and $\varepsilon_1[\varphi(t_S), t_S]$. The position of the wave is found from the condition $\sigma_1[\varphi(t_S), t_S] = f[\varepsilon_1(\varphi(t_S), t_S)]$. Let us now determine $v_1[\varphi(t_S), t_S]$. The particle velocity in Region II is found on the basis of Eqs. (3.28) and (3.29).

$$v_2(x, t) = \frac{df^{-1}[-p(t)]}{dt}(x - \varphi(t)) + v_1[\varphi(t), t]. \tag{3.31}$$

For $x_R \leqslant x \leqslant x_K$ the solution in Region I and the wave $x = \varphi(t)$ are determined from the relations in Regions III and IV. At an arbitraty point T of the wave $x = \varphi(t)$ (Region I), the relations between $\sigma_1[\varphi(t_T), t_T]$, $v_1[\varphi(t_T), t_T]$ and $\varepsilon_1[\varphi(t_T), t_T]$ are obtained. At the segment TT' $\sigma_3(t_T) = \sigma_3(t_{T'}) = \sigma_4(x_K, t_{T'}) = \sigma_1[\varphi(t_T), t_T]$. From the conditions satisfied along the negative characteristic in Region IV and on the basis of Eq. $(3.23)_2$, we obtain $\sigma_4(x_K, t_{T'}) = -\varrho a_0 v_4(x_K, t_T) = -\varrho a_0 v_3(x_K, t_{T'})$. Determination of the integration constant $C(t)$ in Eq. (3.28) for $x = x_K$ yields

$$v_3(x, t_T) = \frac{df^{-1}[\sigma_1(\varphi(t_T), t_T)]}{dt}(x - x_K) - \frac{1}{\varrho a_0}\sigma_1[\varphi(t_T), t_T]. \tag{3.32}$$

For $x = \varphi(t_T)$ we obtain the formula

$$v_1[\varphi(t_T), t_T] = \frac{df^{-1}[\sigma_1(\varphi(t_T), t_T)]}{dt}(\varphi(t_T) - x_K) - \frac{1}{\varrho a_0}\sigma_1[\varphi(t_T), t_T], \tag{3.33}$$

which enables the determination of $x = \varphi(t_T)$ with the additional condition

$$\sigma_1[\varphi(t_T), t_T] = f[\varepsilon_1(\varphi(t_T), t_T)].$$

Let us now present the solutions in the remaining regions of the phase plane (Fig. 3).

3.4. Regions V and VI

Solutions in both regions are derived simultaneously, the conditions of continuity of velocities and stresses at the interface being taken into account,

$$\begin{aligned} &\sigma_5(x_K, t) = \sigma_6(x_K, t) = -p(t) \\ &v_5(x_K, t) = v_6(x_K, t) = -\frac{1}{\varrho a_0}\sigma_5(x_K, t) = \frac{1}{\varrho a_0}p(t). \end{aligned} \tag{3.34}$$

Here the relation $d\sigma_6 + \varrho a_0 dv_6 = 0$ holding true at the negative characteristic in Region VI (elastic) has been used. The stress and strain in Region V are functions of the time variable only,

$$\sigma_5(x, t) = -p(t), \quad \varepsilon_5(x, t) = f^{-1}[-p(t)], \tag{3.35}$$

From Eq. (3.27) it follows that in Region V

$$v_5(x, t) = \frac{df^{-1}[-p(t)]}{dt}(x - x_K) + p(t), \tag{3.36}$$

the integration constant being determined at the boundary $x = x_K$. For $t = t_0$ we obtain $\sigma_5(x, t_0) = 0$, $\varepsilon_5(x, t_0) = \varepsilon^*$.

From the conditions satisfied at the positive characteristics and under the boundary condition $(3.34)_1$ we obtain in Region VI

$$\begin{aligned} \sigma_6(x, t) &= -p\left(t - \frac{x - x_K}{a_0}\right), \\ v_6(x, t) &= \frac{1}{\varrho a_0} p\left(t - \frac{x - x_K}{a_0}\right), \\ \varepsilon_6(x, t) &= -\frac{1}{E_0} p\left(t - \frac{x - x_K}{a_0}\right). \end{aligned} \tag{3.37}$$

3.5. Regions VII and VIII

In view of the boundary condition $p(t) \equiv 0 \ \forall t > t_0$

$$\begin{aligned} \sigma_7(x, t) &= 0, \quad \varepsilon_7(x, t) = 0, \quad v_7(x, t) = 0, \\ \sigma_8(x, t) &= 0, \quad \varepsilon_8(x, t) \to \infty, \quad v_8(x, t) = 0. \end{aligned} \tag{3.38}$$

It is seen from this solution that the material in the segment $0 \leqslant x \leqslant x_K$ is degradated, while for $x \geqslant x_K$ an elastic wave will be propagated, carrying the stresses

$$p_1(t) = \begin{cases} -\sigma_3(x_K, t) & \text{for} \quad t_K \leqslant t \leqslant t_R, \\ p(t) & \text{for} \quad t \geqslant t_R. \end{cases} \tag{3.39}$$

4. Example

Let us assume the simplified model of the medium (Fig. 4) characterized by the linear softening function. The following dimensionless variables are introduced:

$$X = \frac{x}{l_0}, \quad T = \frac{a_0 t}{l_0}, \quad S = \frac{\sigma}{\sigma_0}, \quad P = \frac{p}{\sigma_0}, \quad V = \frac{\varrho a_0}{\sigma_0} v, \quad A = \frac{\sigma_0}{E_0}. \tag{4.1}$$

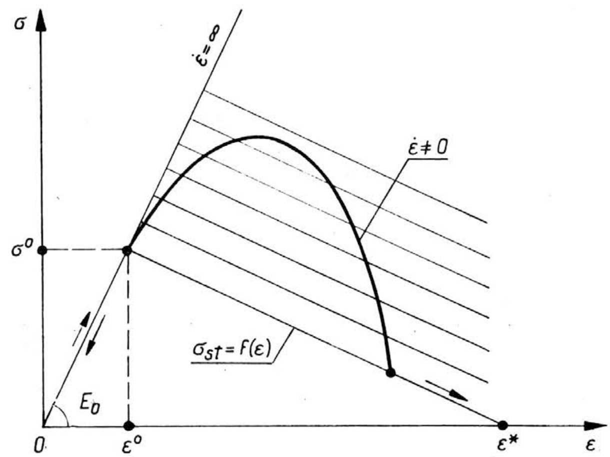

FIG. 4. Model of a material with linear softening function.

The boundary condition for $X = 0$ is assumed in the form

$$P(T) = P_0\left(1-\frac{T}{T_0}\right) \quad \text{for} \quad 0 \leqslant T \leqslant T_0, \quad P(T) = 0 \quad \text{for} \quad T \geqslant T_0, \tag{4.2}$$

where $P_0 > 1$, and the parameters take the following numerical values:

$$\begin{gathered} l_0 = 1m, \quad P_0 = 2, \quad A = 5\cdot 10^{-4}, \quad \gamma = 1, 2, \ldots, 15 \ [\mathrm{s}^{-1}], \\ \sigma_0 = 2{,}45\cdot 10^4 \ \mathrm{Pa}, \quad \varepsilon^* = 10\varepsilon_0, \quad t_0 = 10^{-3} \ \mathrm{s}. \end{gathered} \tag{4.3}$$

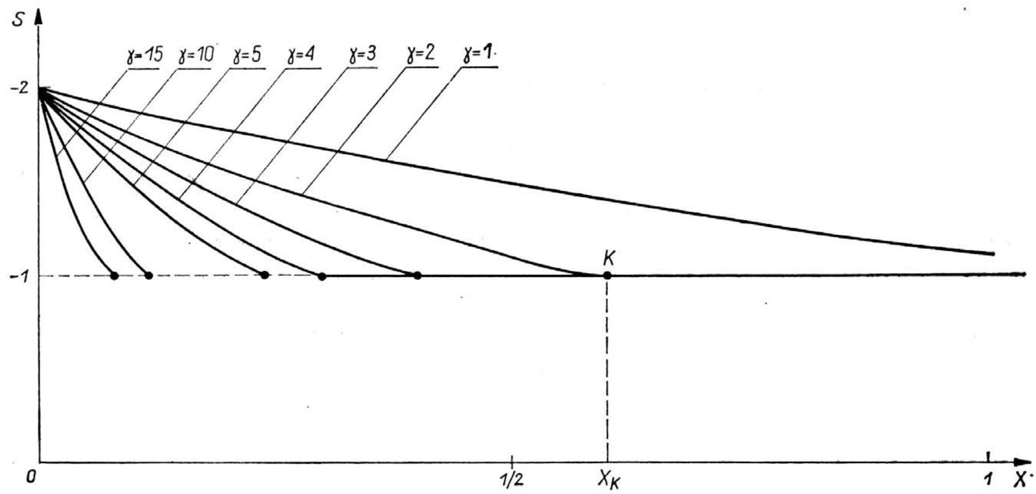

FIG. 5. Stress at the strong discontinuity wave $X = T$, ($\gamma = 1, 2, \ldots, 15$ [s^{-1}]).

Figure 5 presents the variation of stress at the strong discontinuity wave front $x = a_0 t$ calculated from Eq. (3.9) under the boundary condition $P(0) = 2$. The stress decreases with increasing ∇ to reach the elastic limit value $= 1$; for instance, at $\gamma = 2$ the elastic limit is reached at point K. In the interval $0 \leqslant X \leqslant X_K$ (with $\gamma = 2$) the medium is degradated.

Figure 6 presents the variation of strain at the boundary $X = 0$ of the rod as a function of time for three values of the viscosity coefficient $\gamma_i = 2, 3, 4$ [s^{-1}] under the given boundary condition (4.2). $T_P^{(i)}$ denotes the time instant at which the unloading wave will start to propagate from the boundary of the rod. For times greater than $T_P^{(i)}$ the strain increases linearly (due to the linear softening function assumed) to reach the value ε^*. At time T_0 complete degradation of the material at the boundary takes place.

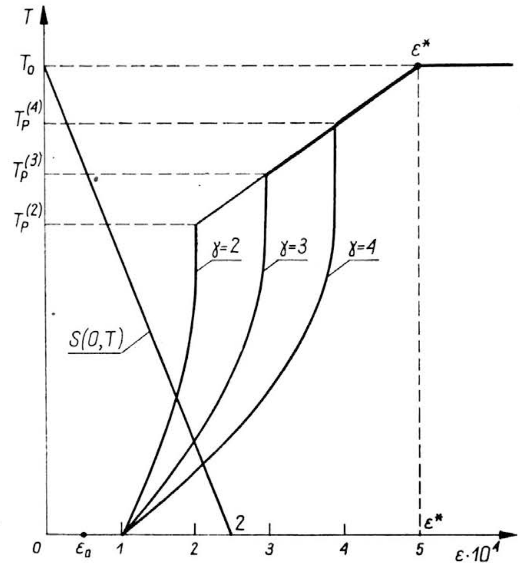

FIG. 6. Time-dependent strain at the boundary of the rod, under the given boundary condition $S(0, T)$, ($\gamma = 2, 3, 4$, [s^{-1}]).

Figure 7 presents the variation of stresses and strains in time at several cross-sections of the rod $X^{(i)}$, $i = 0, \ldots, 5$. In the interval $0 \leqslant X \leqslant X^{(3)} = X_K$ the material is subject to complete degradation, and the strains increase to infinity. For $X \geqslant X_K$ an elastic wave is propagated along the rod, and at its front the stress equals σ_0 and the strain is equal to ε_0.

5. Conclusion

The solution presented in the paper applies only to the case of wave propagation in a semi-infinite rod. It would be interesting to tacke the problem of a finite rod and waves reflecting from its free or fixed ends; of considerable interest would also be the experimental determination (at uniaxial stress state) of the length of the rock material degradation zone and comparison of the results with those derived in this paper. Such experiments may easily be performed, for instance by means of the Hopkinson bar system.

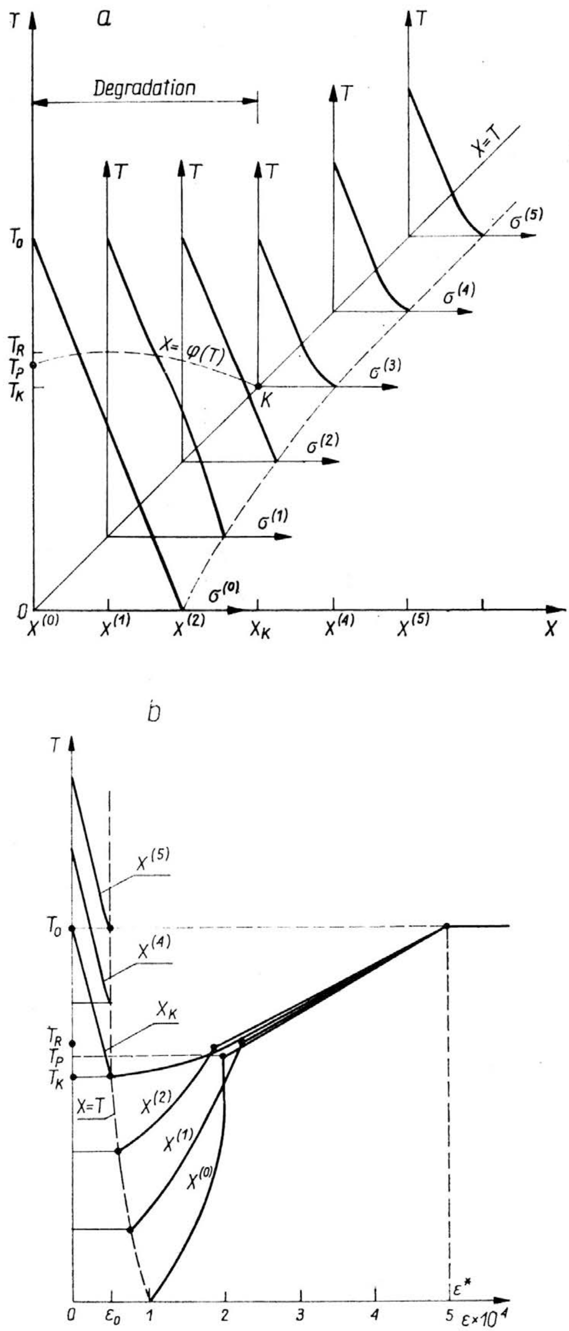

FIG. 7. Variation of a) stress and b) strain as a function of time at consecutive cross-sections of the rod.

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