Electro-magneto-thermo-elasticity of extrinsic semiconductors Classical irreversible thermodynamic approach

B. MARUSZEWSKI (POZNAŃ)

THE PAPER aims at investigating, in the irreversible thermodynamic way, an extrinsic thermo-elastic semiconductor interacting with an electromagnetic field. Thanks to the above considerations, it is possible to describe a vast majority of nonequilibrium processes occurring inside the semiconductor.

Praca dotyczy opisu sprzężonych oddziaływań między polami sprężystym, termicznym, dyfuzji nośników ładunku i elektromagnetycznym w niesamoistnym półprzewodniku, na podstawie klasycznej termodynamiki procesów nierównowagowych. Uwzględniając powyższe rozważania, możliwy staje się opis większości nierównowagowych zjawisk, jakie mogą zachodzić w ośrodku półprzewodzącym.

Работа касается описания сопряженных взаимодействий между упругим, термическим, диффузии носителей заряда и электромагнитным полями в несобственном полупроводнике, опираясь на классическую термодинамику неравновесных процессов. На основе вышеупомянутых рассуждений становится воз можным описание большинства неравновесных явлений, какие могут иметь место в полупроводящей среде.

Introduction

TECHNOLOGICAL progress in the last decades is visible mainly in semiconductor physics and technology. Basic and applied investigations developed parallelly allowing to discover new and unexpected possibilities of technical applications.

In a way, a semiconductor can be considered as an insulator. At low temperature the valency band of a pure semiconducting material is completely filled by electrons. Since neither the completely full band nor obviously the empty band allow to carry any charges over, the conductivity of a medium is equal to zero. The difference between a metal and a semiconductor is of a qualitative kind: in a semiconductor there is an energetic break between the valency band and the conduction band; in a metal there isn't one; the bands cross each other. However, between a semiconductor and an insulator is only a quantitative difference which is connected with the width of the energetic break. In the semiconductor that break is small and a great number of electrons pass to the conduction band by thermal forcing at temperatures below melting point; in the insulator that break is greater than in the semiconductor and the number of thermally-excited electrons is so small that one can omit it.

However, charge carriers do not have to be excited necessarily up the forbidden band to the conductivity band (like in intrinsic semiconductors). Some impurities dissolved inside a mother substance introduce such energetic levels which are situated within the energetic break (extrinsic semiconductor). There are donor impurities (*n*-type) and acceptor

(p-type) according to whether at low temperatures near absolute zero an impurity level includes an electron or is deprived of it (includes a hole). If, for instance, that level includes the electron, its excitation into the conductivity band is easier than excitation of the valency electron because the donor level lies nearer to the bottom of the conductivity band. The ability of a material to be a semiconductor is determined by the possibility of solving of response impurities.

The present paper is devoted to the irreveisible thermodynamic description of nonequilibrium phenomena in semiconductors. As one knows, each phenomenon which can be physically observed is always connected with deviation of the system from equilibrium state as, for instance: electric current flow, thermoelectric power generation, heat flow, thermogalvanomagnetic, piezoelectric, piezomagnetic and thermoelastic phenomena, diffusion, and the like. The most important nonequilibrium processes in semiconductors are such which are connected with deviations of charge carrier concentrations from equilibrium values. It is a specific feature of a semiconductor that one can change the values of the above concentrations very strongly (contrary to metals). Moreover, such processes are usually sensitive in the presence of every kind of defects in semiconducting crystals (impurities, for instance) and the influence of mechanical and nonmechanical fields (electromagnetic or thermal).

A part of the above mentioned problems in deformable semiconductors has been considered by many authors. The most complete description of the above processes was made by H. D. de Lorenzi and H. F. Tiersten [1], M. G. Ancona and H. F. Tiersten [2], [3], H. F. Tiersten [4] (ibidem a comprehensive bibliography list of subject matter references), M. G. Ancona [5] and B. Maruszewski [6, 7]. However, none of the above mentioned references includes either the complex description of reciprocal interactions of the mentioned physical fields or the complete description of the mentioned nonequilibrium processes in extrinsic semiconductors. The aim of this paper is to fill this gap.

Let us consider an elastic, polarizable, magnetizable, extrinsic, homogeneous and isotropic semiconductor. The choice of the medium is due to the possibility of simultaneous investigations of correlations of the elastic, temperature, electromagnetic and charge carrier diffusion fields in it as well as to the fact that such a material collects properties of dielectrics, magnetics, electrics, conductors and semiconductors. Distributions of the above fields, i.e. u_i — elastic displacements, E_i — electric field intensity, B_i — magnetic induction, n — negative charge concentration (electron mass density), p — positive charge concentration (hole mass density), T — temperature, can be found as solutions of initial-boundary-value problems which are founded on field equations set up from the universal laws of mechanics and thermodynamics (balances), electrodynamics and depend on a kind of medium constitutive relations.

1. Balances of mass, momentum and energy

We assume the monophase and unary medium of mass density ϱ. Mass of the charge carriers is omitted in comparison to mass of atoms which build our medium and we do not take account of impurities diffusion both charged and neutral because the relaxation

time of that process is uncomparably longer than relaxation times of the charge diffusion and relaxation in their drift forced by applied fields, (conductivity). Moreover, we admit that amplitudes of applied fields will not make the whole energy of impurity greater than its activation energy during mass diffusion processes. For such a body balances of mass, momentum and energy are as follows [8–10]:

(1.1)
$$\dot{\varrho} + \varrho v_{i,i} = 0,$$

(1.2)
$$\varrho \dot{v}_i - \tau_{ji,j} + \mathscr{E}_k P_{k,i} - \left(\frac{\partial \mathbf{P}}{\partial t} \times \mathbf{B} \right)_i + B_k \bar{M}_{k,i}$$
$$- \varrho [p + \bar{p} - p_0 - (n + \bar{n} - \bar{n}_0)] \mathscr{E}_i - [(\mathbf{j}_n + \mathbf{j}_p) \times \mathbf{B}]_i - f_i = 0,$$

(1.3)
$$\varrho \frac{d}{dt} \left(\frac{v^2}{2} + U \right) + \frac{\partial U_e}{\partial t} = \varrho r + f_i v_i + [\tau_{ji} v_i - q_j - (\mathbf{E} \times \mathbf{H})_j]_{,j}.$$

In the expressions (1.1)–(1.3) v_i denotes barycentric velocity, τ_{ji} — electromagnetodiffusive stress tensor, $\mathscr{E}_k = E_k + (\mathbf{v} \times \mathbf{B})_k$ — objective electric field intensity, P_k — polarization vector, $\bar{M}_k$ — magnetization vector defined by $(2.2)_{2,3,4}$, p, p_0, n, n_0 — nonequilibrium and equilibrium mass densities of electrons and holes, $\bar{p}$, $\bar{n}$ — mass densities of impurities charges, $\mathbf{j}_n$, $\mathbf{j}_p$ — electron and hole fluxes, f_i — volume forces, U — internal energy density, U_e — electromagnetic energy, r — heat source distribution and q_i — heat flux. A superimposed dot denotes the total material derivative. Moreover, it is assumed that the medium considered is magnetically saturated, i.e.,

(1.4)
$$\bar{M}_k \bar{M}_k = \bar{M}^2 = \text{const}, \quad \bar{M}_{,i} = 0, \quad \bar{M}_k \bar{M}_{k,i} = 0,$$
$$\dot{\bar{M}} = 0, \quad \dot{\bar{M}}_k \bar{M}_k = 0.$$

2. Electromagnetic field equations

The electromagnetic field in a moving semiconductor defined above is described by Maxwell's equations [6, 12, 13]

(2.1)
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{D} = \varrho [p + \bar{p} - p_0 - (n + \bar{n} - n_0)],$$

$$\nabla \times \mathbf{H} = \mathbf{j}' + \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0,$$

and the expressions

(2.2)
$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \bar{\mathbf{M}}, \quad \bar{\mathbf{M}} = \mathbf{M} + \mathbf{M}^{eq}, \quad \mathbf{M}^{eq} = \mathbf{P} \times \mathbf{v},$$

$$\mathbf{j} = \mathbf{j}_n + \mathbf{j}_p, \quad \overset{*}{\underline{\mathscr{M}}} = \frac{\gamma}{\mu_0} \underline{\mathscr{M}} \times \mathbf{B}, \quad \bar{\mathbf{M}} = \varrho \underline{\mathscr{M}}, \quad \mathbf{P} = \varrho \mathscr{P},$$

$$\dot{u}_i = v_i, \quad w_{ik} = \frac{1}{2} (v_{i,k} - v_{k,i}).$$

The total current $\mathbf{j}'$ is defined as follows:

(2.3)
$$\mathbf{j}' = \mathbf{j} + \varrho [p + \bar{p} - p_0 - (n + \bar{n} - n_0)] \mathbf{v}.$$

In the above relations ε_0 and μ_0 denote the dielectric constant of free space and permeability of free space respectively, γ — gyromagnetic ratio, j_i — objective current flow vector and superimposed star — so-called Jaumann's objective time derivative [11]. Because barycentric velocity v_i is very small in comparison to velocities of electrons and holes, we assume that practically $\mathbf{j}_n^- = 0$ and $\mathbf{j}_p^- = 0$ [1]. Utilization of Eqs. $(2.1)_{2,3}$, $(2.2)_5$, (2.3) and the above approximation allows us to write very important charge conservation laws of electrons, holes and impurities:

$$\varrho\,\frac{\partial p}{\partial t}+\nabla\cdot\mathbf{j}_p+\varrho\mathbf{v}\cdot\nabla p = g_p^+,$$

$$\varrho\,\frac{\partial n}{\partial t}-\nabla\cdot\mathbf{j}_n+\varrho\mathbf{v}\cdot\nabla n = g_n^+,$$

(2.4)

$$\varrho\,\frac{\partial \overline{p}}{\partial t}+\varrho\mathbf{v}\cdot\nabla\overline{p} = \overline{g}_p^+,$$

$$\varrho\,\frac{\partial \overline{n}}{\partial t}+\varrho\mathbf{v}\cdot\nabla\overline{n} = \overline{g}_n^+,$$

(2.5)
$$g_n^+ + g_p^+ + \overline{g}_n^+ + \overline{g}_p^+ = 0.$$

In the above mentioned and in all the further investigations we assume that the electron and hole fields do not influence each other. The functions g_n^+, g_p^+, $\overline{g}_n^+$ and $\overline{g}_p^+$ describe generation and recombination of electrons and holes and charge production of donors and acceptors respectively. Equations $(2.4)_{3,4}$, however, need some additional comment [14]. Let, in a considered medium which is built on atoms of the IVth group (classical semiconductor — Ge or Si), one of the basic atoms be exchanged with an atom of the Vth group (with P, for instance) (doping of impurities). Then four of its valency electrons participate in a covalent bond with the neighbouring atoms of the base, and the bond energy of the fifth valency electron with impurity is ε^2 times lower from the other bonds (ε — relative dielectric constant of the base). Therefore an impurity can be ionized very easily but does not participate in electric current flow. A similar situation takes place when as the impurity we take an atom of the IIIrd group (As, for instance). Then one of the bonds with the base is empty. The energy which is needed to transfer the electron than the atom of the base to the impurity which grows the negative ion, is much lower from the energy needed to make the electron free. Since the negative charge is localized within the impurity, it does not participate in conductivity, either. Taking into account the above considerations, we admit that concentrations of impurity charges $\overline{p}$ and $\overline{n}$ are practically constant in time. Hence [11]

(2.6)
$$\dot{\overline{p}} = \dot{\overline{n}} = 0$$

and Eqs. $(2.4)_{3,4}$ pass into

(2.7)
$$\overline{g}_n^+ = \overline{g}_p^+ = 0$$

or the impurity charges can only vary by a convection of the impurity field during deformation. In the particular case of compensated semiconductors, when $\overline{p} = \overline{n}$, Eqs. $(2.4)_{3,4}$ vanish.

3. Constitutive relations (1)

In classical irreversible thermodynamics, a state of a body is described by thermo-dynamic potentials — functions of the state of independent variables belonging to a set of state parameters. Here, as the function of state, basing on the previous considerations and assumptions, we choose the internal energy taking into account the case of infinitesimal deformations about a natural undeformed configuration [11]

(3.1) $$U = U(\varepsilon_{ij}, \mathscr{P}_i, \overline{\mathscr{M}}_i, n, p, S, \varepsilon_{kl,i}, n_{,i}, p_{,i}, T_{,i})$$

which is a function of parameters: ε_{ij} — elastic strain tensor, $\mathscr{P}_i, \overline{\mathscr{M}}_i, n, p, S$ — entropy density and gradients $\varepsilon_{ij,k}, n_{,i}, p_{,i}, T_{,i}$. The elastic strain tensor is taken in the following approximation:

(3.2) $$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

where u_i denotes the displacement vector. Since it is very difficult to measure the magnitudes $\mathscr{P}_i, \overline{\mathscr{M}}_i$ or S in the direct way, we take instead of internal energy (3.1) the free energy with the help of Legendre's transformation as

(3.3) $$F = U - \mathscr{P}_i \mathscr{E}_i - \overline{\mathscr{M}}_i B_i - TS,$$

(3.4) $$F = F(C),$$

where

$$C = \{\varepsilon_{ij}, \mathscr{E}_i, B_i, n, p, T, \varepsilon_{kl,i}, n_{,i}, p_{,i}, T_{,i}\}.$$

Now we should define the electromagneto-thermodiffusive stress tensor τ_{ji} in balances (1.2) and (1.3) in conditions of magnetic saturation

(3.5) $$\tau_{jl} = \sigma_{ij} + (\mathscr{E}_k P_k + B_k \overline{M}_k)\delta_{ij},$$

where the thermodiffusive stress tensor σ_{ij} is defined as follows:

(3.6) $$\sigma_{ij} = \varrho \frac{\partial F}{\partial \varepsilon_{ij}}.$$

Finally we have to specify the remaining unknowns in Eqs. (1.2), (1.3), (2.2), (2.4)$_{1,2}$, (3.1) and (3.5) i.e. $\sigma_{ij}, \overline{P}_i, M_i, S, g_p^+, g_n^+, q_i, j_{ni}, j_{pi}$. We find the first six of them with the help of constitutive relations:

(3.7) $$\begin{aligned} \sigma_{ij} &= \sigma_{ij}(C), \\ P_i &= P_i(C), \\ \overline{M}_i &= \overline{M}_i(C), \\ S &= S(C), \\ g_p^+ &= g_p^+(C), \\ g_n^+ &= g_n^+(C) \end{aligned}$$

and definitions

(3.7′) $$P_i = -\varrho \frac{\partial F}{\partial \mathscr{E}_i}, \qquad \overline{M}_i = -\varrho \frac{\partial F}{\partial B_i},$$

then the last three fluxes are found from the kinetic relations presented in Sect. 6.

All the magnitudes which construct the constitutive relations (3.8), both tensorial dependent σ_{ij} and independent ε_{ij}, both vector dependent P_i, $\overline{M}_i$ and independent $\mathscr{E}_i$, B_i, $T_{,i}$, $\varepsilon_{kl,i}$, $n_{,i}$, $p_{,i}$ and both scalar dependent S, g_p^+, g_n^+ and independent n, p, T are objective [11, 15, 16].

Taking now into account the previous considerations, we can specify a system of equations necessary to obtain field variables of interest to us. That system is built on Eqs. (1.2), (1.3) and (2.1).

4. Entropy inequality

The balances (1.1)–(1.3) do not include sufficient restrictions on the physical processes occurring inside the considered semiconductor (according to their reality) and on the constitutive relations. In order to fill the hole above, we introduce the entropy principle in the following way:

in each body there exists additive scalar function — entropy the balance of which is written in the form

(4.1)
$$\varrho\dot{S}+\Phi_{i,i} = \sigma+s$$

and is true for each thermodynamical process. Φ_i *denotes the objective entropy flux,* s — *entropy supply which, in linear approximation* [17], *has the well-known form*

(4.2)
$$s = \frac{\varrho r}{T},$$

then σ *which denotes entropy production is assumed as nonnegative*

(4.3)
$$\sigma \geqslant 0.$$

Utilization of the above definition leads us to the following entropy inequality:

(4.4)
$$\varrho\dot{S}+\Phi_{i,i}-\frac{\varrho r}{T} \geqslant 0,$$

which is just the missing expression that was mentioned at the beginning of this section. According to the assumptions connected with the properties of the considered medium and the possible processes occurring inside it, we take the entropy flux Φ_i in the form [9, 15]

(4.5)
$$\Phi_i = \frac{q_i+\mu_n j_{ni}-\mu_p j_{pi}+(\mathscr{E}\times\overline{\mathscr{M}})_i}{T},$$

where the chemical potentials of electrons and holes μ_n, μ_p are specified from the definitions

(4.6)
$$\mu_n = \frac{\partial F}{\partial n}, \qquad \mu_p = \frac{\partial F}{\partial p}$$

as constitutive relations

(4.7)
$$\mu_n = \mu_n(C), \qquad \mu_p = \mu_p(C),$$

which, basing on the principle of equipresence, are the functions of the same variables like in the relation (3.4). Obviously, according to the above principle, entropy flux also depends on the same variables

$$(4.8) \qquad \Phi_i = \Phi_i(C)$$

and fluxes

$$(4.9) \qquad \begin{aligned} q_i &= q_i(C), \\ j_{ni} &= j_{ni}(C), \\ j_{pi} &= j_{pi}(C). \end{aligned}$$

In order to obtain information on restrictions in detail, which result from entropy inequality, we substitute Eq. (4.5) and balance Eq. (1.3) into the relation (4.4) in the following way:

the rate of electromagnetic energy is calculated with the help of the balance (1.1), the definition (1.4), Maxwell's equations (2.1) and the expressions (2.2)

$$(4.10) \qquad \frac{\partial U_e}{\partial t} = -(\mathbf{E} \times \mathbf{H})_{i,i} - j_i \mathscr{E}_i - \varrho[p + \bar{p} - p_0 - (n + \bar{n} - n_0)]\mathscr{E}_i v_i - (\mathbf{j} \times \mathbf{B})_i v_i$$

$$- \varrho \mathscr{E}_i \dot{\mathscr{P}}_i - \left(\frac{\partial \mathbf{P}}{\partial t} \times \mathbf{B}\right)_i v_i + \mathscr{E}_k P_{k,i} v_i + \mathscr{E}_i P_i v_{j,j} - \varrho B_i \dot{\bar{\mathscr{M}}}_i + B_i \bar{M}_i v_{j,j} + B_k \bar{M}_{k,i} v_i,$$

the rate of internal energy is calculated with the help of the expressions (3.3) and (3.4), and the definitions (1.2), (2.4), (3.2), (3.5), (3.6), (3.7') and (4.6) are utilized.

Hence the inequality considered is as follows:

$$(4.11) \qquad -\varrho\left(\frac{\partial F}{\partial T} + S\right)\dot{T} - \varrho\frac{\partial F}{\partial \varepsilon_{kl,i}}\dot{\varepsilon}_{kl,i} - \varrho\frac{\partial F}{\partial n_{,i}}\dot{n}_{,i} - \varrho\frac{\partial F}{\partial p_{,i}}\dot{p}_{,i} - \varrho\frac{\partial F}{\partial T_{,i}}\dot{T}_{,i}$$

$$+ (j_{ni} + j_{pi})\mathscr{E}_i - \frac{1}{T}q_i T_{,i} + T j_{ni}\left(\frac{\mu_n}{T}\right)_{,i} - T j_{pi}\left(\frac{\mu_p}{T}\right)_{,i} + (\mathscr{E} \times \bar{\mathbf{M}})_{i,i}$$

$$- \frac{1}{T}(\mathscr{E} \times \bar{\mathbf{M}})_i T_{,i} - \mu_n g_n^+ - \mu_p g_p^+ - (\sigma_{ki} - \bar{M}_k B_i - P_k \mathscr{E}_i)w_{ki} \geqslant 0.$$

Since the expression (4.11) is linear with respect to the variables $\dot{T}$, $\dot{T}_{,i}$, $\dot{n}_{,i}$, $\dot{p}_{,i}$, $\dot{\varepsilon}_{kl,i}$, we obtain

$$(4.12) \qquad S = -\frac{\partial F}{\partial T},$$

$$(4.13) \qquad \frac{\partial F}{\partial T_{,i}} = 0, \qquad \frac{\partial F}{\partial n_{,i}} = 0, \qquad \frac{\partial F}{\partial p_{,i}} = 0, \qquad \frac{\partial F}{\partial \varepsilon_{kl,i}} = 0,$$

claiming that free energy must be invariant under an infinitesimal rigid rotation of the body [19]:

$$(4.14) \qquad \sigma_{ki} - \sigma_{ik} = P_k \mathscr{E}_i + \bar{M}_k B_i - P_i \mathscr{E}_k - \bar{M}_i B_k,$$

which, in the case of linear constitutive equations, occurs symmetric [11] and leads to the so-called residual inequality

$$(4.15) \quad j_{ni}\left[\mathscr{E}_i + T\left(\frac{\mu_n}{T}\right)_{,i}\right] + j_{pi}\left[\mathscr{E}_i - T\left(\frac{\mu_p}{T}\right)_{,i}\right] - [q_i + (\underline{\mathscr{E}} \times \overline{\mathbf{M}})_i]\frac{T_{,i}}{T}$$

$$+ (\underline{\mathscr{E}} \times \overline{\mathbf{M}})_{i,i} - \mu_n g_n^+ - \mu_p g_p^+ \geqslant 0.$$

Relation (4.12) defines the entropy, then the relations (4.13) prove that free energy F is independent of $\varepsilon_{kl,i}, n_{,i}, p_{,i}, T_{,i}$. But from the expression (4.14) there results an equivalent way to define a symmetry of tensor σ_{ij} instead of the common way, that is, the introduction moment of the momentum balance [20].

5. Constitutive relations (2)

In the previous section were found restrictions on free energy F. From the relation (4.13) there results that

$$(5.1) \qquad\qquad F = F(\varepsilon_{ij}, \mathscr{E}_i, B_i, n, p, T).$$

Also the expressions (3.6), (3.7'), (4.6), (4.12) are the functions of $\{\varepsilon_{ij}, \mathscr{E}_i, B_i, n, p, T\}$. Basing on those and the relation (5.1) we can write Gibbs' equation in the form

$$(5.1') \qquad dF = \frac{1}{\varrho}\sigma_{ij}d\varepsilon_{ij} - \mathscr{P}_i d\mathscr{E}_i - \overline{\mathscr{M}}_i dB_i + \mu_n dn + \mu_p dp - S dT.$$

Let us consider now the behaviour of our semiconductor at states very close to equilibrium. In order to make that assumption we introduce the following functions:

$$(5.2) \qquad \theta = T - T_0, \quad \left|\frac{\theta}{T_0}\right| \ll 1, \quad N = n - n_0, \quad \left|\frac{N}{n_0}\right| \ll 1,$$

$$P = p - p_0, \quad \left|\frac{P}{p_0}\right| \ll 1.$$

To obtain the constitutive relations (3.6), (3.7'), (4.6) and (4.12) in the explicit form, we expand free energy F into Taylor's series with respect to equilibrium state confining the consideration to the quadratic terms

$$(5.3) \quad F = \frac{1}{\varrho}\sigma_{ij}^0\varepsilon_{ij} - \frac{1}{\varrho}P_i^0\mathscr{E}_i - \frac{1}{\varrho}\overline{M}_i^0 B_i + \mu_n^0 N + \mu_p^0 P - S^0\theta + \frac{\mu}{\varrho}\varepsilon_{ij}\varepsilon_{ij}$$

$$+ \frac{\lambda}{2\varrho}\varepsilon_{kk}^2 - \frac{1}{2}\frac{\chi_e}{\varrho}\mathscr{E}_i\mathscr{E}_i - \frac{1}{2}\frac{\chi_m}{\varrho}B_iB_i - \frac{\delta_n}{2n_0}N^2 - \frac{\delta_p}{2p_0}P^2 - \frac{1}{2}\frac{c}{T_0}\theta^2$$

$$- \frac{\gamma_n}{\varrho}\varepsilon_{kk}N - \frac{\gamma_p}{\varrho}\varepsilon_{kk}P - \frac{\gamma_\vartheta}{\varrho}\varepsilon_{kk}\theta - \frac{\Sigma}{\varrho}\mathscr{E}_iB_i - \alpha_n\theta N - \alpha_p\theta P,$$

where $\sigma_{ij}^0, P_i^0, \overline{M}_i^0, \mu_n^0, \mu_p^0, S$ are the equilibrium values, λ, μ Lame's constants, χ_e, χ_m — electric and magnetic susceptibilities, δ_n, δ_p — diffusive constants c — specific heat coefficient, γ_n, γ_p — elastodiffusive constants, γ_ϑ — thermoelastic constant, Σ — magneto-eectric constant and α_n, α_p — thermodiffusive constants.

Utilizing the expansions (5.3), one obtains the constitutive relations (3.6), (3.7'), (4.6) and (4.12) in the form

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma_\vartheta\theta - \gamma_n N - \gamma_p P)\,\delta_{ij} + \sigma_{ij}^0,$$

$$P_i = \chi_e \mathscr{E}_i + \Sigma B_i + P_i^0,$$

$$M_i = \Sigma\mathscr{E}_i + \chi_m B_i + M_i^0,$$

(5.4)
$$\mu_n = -\frac{\gamma_n}{\varrho}\,\varepsilon_{kk} + \frac{\delta_n}{n_0}\,N - \alpha_n\theta + \mu_n^0,$$

$$\mu_p = -\frac{\gamma_p}{\varrho}\,\varepsilon_{kk} + \frac{\delta_p}{p_0}\,P - \alpha_p\theta + \mu_p^0,$$

$$S = \frac{\gamma_\vartheta}{\varrho}\,\varepsilon_{kk} + \alpha_n N + \alpha_p P + \frac{c}{T_0}\,\theta + S^0.$$

The last two constitutive relations $(3.7)_{5,6}$ are specified by expanding the functions g_n^+ and g_p^+ into Maclaurin's series with respect to the equilibrium. Hence

(5.5)
$$g_n^+ = b_{\varepsilon n}^+\varepsilon_{kk} - \frac{\varrho}{\tau_n^+}\,N + b_{Tn}^+\theta,$$

$$g_p^+ = b_{\varepsilon p}^+\varepsilon_{kk} - \frac{\varrho}{\tau_p^+}\,P + b_{Tp}^+\theta,$$

where in the equilibrium the following equalities have to be satisfied:

(5.6)
$$g_n^{+0} = g_p^{+0} = 0$$

and were introduced below the abbreviations

(5.7)
$$\left.\frac{\partial g_n^+}{\partial\varepsilon_{kk}}\right|_0 = b_{\varepsilon n}^+, \qquad \left.\frac{\partial g_n^+}{\partial N}\right|_0 = -\frac{\varrho}{\tau_n^+}, \qquad \left.\frac{\partial g_n^+}{\partial\theta}\right|_0 = b_{Tn}^+,$$

$$\left.\frac{\partial g_p^+}{\partial\varepsilon_{kk}}\right|_0 = b_{\varepsilon p}^+, \qquad \left.\frac{\partial g_p^+}{\partial P}\right|_0 = -\frac{\varrho}{\tau_p^+}, \qquad \left.\frac{\partial g_p^+}{\partial\theta}\right|_0 = b_{Tp}.$$

τ_n^+ and τ_p^+ are the relaxation times of the charge carrier lives (recombination relaxation times) [13], [14].

6. Kinetic relations

To finish the specifications of the magnitudes which define the basic laws responsible for all the considerations presented here, we should still give exact forms of expressions for irreversible fluxes (4.9). According to the above remark, we utilize the residual inequality (4.15). The l.h.s. of it describes entropy production σ as follows:

(6.1)
$$\sigma = \sigma_1 + \sigma_2,$$

where

(6.2)
$$\sigma_1 = j_{ni}\left[\mathscr{E}_i + T\left(\frac{\mu_n}{T}\right)_{,i}\right] + j_{pi}\left[\mathscr{E}_i - T\left(\frac{\mu_p}{T}\right)_{,i}\right] - [q_i + (\mathscr{E}\times\bar{\mathbf{M}})_i]\frac{T_{,i}}{T}$$

while σ_2 describes entropy supply:

(6.3)
$$\sigma_2 = (\underline{\mathscr{E}} \times \bar{\mathbf{M}})_{i,i} - \mu_n g_n^+ - \mu_p g_p^+.$$

On the other hand, we know that we can present σ_1 in the following bilinear form:

(6.4)
$$\sigma_1 = J^a X^a,$$

where J^a denotes generalized fluxes and X^a — generalized forces. Assuming also the linear dependence between fluxes J^a and forces X^a

(6.5)
$$J^a = L^{ac} X^c,$$

we obtain

(6.6)
$$\sigma_1 = L^{ac} X^a X^c.$$

The phenomenological coefficients L^{ac} are to be constant. Basing now on Eqs. (6.2) and (6.6), we choose

(6.7)
$$\{\mathbf{J}\} = \left\{ \begin{array}{c} \mathbf{q} + (\underline{\mathscr{E}} \times \bar{\mathbf{M}}) \\ \mathbf{j}_n \\ \mathbf{j}_p \end{array} \right\} \quad \text{and} \quad \{\mathbf{X}\} = \left\{ \begin{array}{c} -\dfrac{1}{T} \nabla T \\ \underline{\mathscr{E}} + T\nabla\left(\dfrac{\mu_n}{T}\right) \\ \underline{\mathscr{E}} - T\nabla\left(\dfrac{\mu_p}{T}\right) \end{array} \right\}$$

and with [6]

(6.8)
$$\{\mathsf{L}\} = \left\{ \begin{array}{ccc} \varkappa T & \beta_n T & \beta_p T \\ \beta_n T & \varrho \xi_n n & 0 \\ \beta_p T & 0 & \varrho \xi_p p \end{array} \right\}$$

the requested expressions for irreversible fluxes are as follows

(6.9)
$$q_i + (\underline{\mathscr{E}} \times \bar{\mathbf{M}})_i = -\varkappa T_{,i} + \beta_n T\left[\mathscr{E}_i + T\left(\frac{\mu_n}{T}\right)_{,i}\right] + \beta_p T\left[\mathscr{E}_i - T\left(\frac{\mu_p}{T}\right)_{,i}\right],$$

$$j_{ni} = -\beta_n T_{,i} + \varrho \xi_n n\left[\mathscr{E}_i + T\left(\frac{\mu_n}{T}\right)_{,i}\right],$$

$$j_{pi} = -\beta_p T_{,i} + \varrho \xi_p p\left[\mathscr{E}_i - T\left(\frac{\mu_p}{T}\right)_{,i}\right].$$

Relations (6.9) for nonmagnetics ($\mathbf{M} = 0$) and in the case of the states very close to the equilibrium (5.2) pass into the relations shown in [6]. Moreover, from the matrix (6.8) it results that $L^{ac} = L^{ca}$, i.e. phenomenological coefficients satisfy Onsager's relations. In the expressions (6.9) $\varkappa$ denotes the thermal conductivity coefficient, β_n, β_p — thermo-diffusive constants and ξ_n, ξ_p — mobilities of electrons and holes.

Let us consider now, for instance, the case when the phenomenological coefficients L^{ac} are functions of the magnetic field $\mathbf{B}$. If we expand them into Taylor's series and confine them only to the linear terms with respect to $\mathbf{B}$, we can present them in the form

(6.10)
$$L^{ac}(\mathbf{B}) \equiv L_{ik}^{ac(0)} + L_{ikm}^{ac(1)} B_m,$$

where

$$L_{ikm}^{ac(1)} = \left(\frac{\partial L_{ik}}{\partial B_m}\right)_{B=0}.$$

Specyfying now terms of the expansions (6.10), the coefficient matrix is the following [7]:

$$(6.11) \quad \{\bar{L}(\mathbf{B})\} = \begin{cases} \varkappa T\delta_{ik} + \mathscr{L}T\varepsilon_{iks}B_s, & \beta_n T\delta_{ik} + \mathscr{N}_n T\varepsilon_{ikm}B_m, & \beta_p T\delta_{ik} + \mathscr{N}_p T\varepsilon_{ikm}B_m \\ \beta_n T\delta_{ik} + \mathscr{N}_n T\varepsilon_{ikm}B_m, & \varrho\xi_n n\delta_{ik} - \mathscr{R}_n \varepsilon_{iks}B_s, & 0 \\ \beta_p T\delta_{ik} + \mathscr{N}_p T\varepsilon_{ikm}B_m, & 0, & \varrho\xi_p p\delta_{ik} - \mathscr{R}_p \varepsilon_{iks}B_s \end{cases},$$

where ε_{ikm} is the alternating tensor equal to $+1$ if the subscripts are cyclic permutations of 1, 2, 3, equal 1 if the subscripts are 321, 213, 132 and otherwise equal to zero. The above matrix refers only to an isotropic medium and gives the symmetry relations for $L^{ac}(\mathbf{B})$:

$$(6.12) \qquad\qquad L_{ikm}^{ac}(\mathbf{B}) = L_{kim}^{ca}(-\mathbf{B});$$

i.e., the phenological coefficients satisfy Onsager–Casimir's relations. Moreover, the matrix (6.11) generalizes the matrix (6.8) on thermogalvanomagnetic effects, that is, besides Fouriers' and Ohm's laws, Peltier's, Seebeck's, thermodiffusive and electro-diffusive effects (the relation (6.8)) includes the so-called second rank kinetic effects, such as Hall's effect ($\mathscr{R}_n$, $\mathscr{R}_p$), Ettingshausen–Nernst's effect ($\mathscr{N}_n$, $\mathscr{N}_p$) and Righi–Leduc's effect ($\mathscr{L}$). If we utilize now the expressions (6.5), (6.7) and (6.11), take into account the approximations (5.2) and constitutive relations $(5.4)_{3-5}$, we obtain very interesting forms of irreversible fluxes:

$$(6.13) \quad q_i + q_i^{EM} = -[\varkappa + T_0(\alpha_n\beta_n - \alpha_p\beta_p)]\theta_{,i} + T_0(\beta_n + \beta_p)\mathscr{E}_i$$

$$+ \frac{1}{\varrho} T_0(\beta_n\gamma_n - \beta_p\gamma_p)u_{k,ki} + \frac{1}{n_0} T_0 \delta_n\beta_n N_{,i}$$

$$- \frac{1}{p_0} \delta_p\beta_p P_{,i} - [\mathscr{L} + T_0(\mathscr{N}_n\alpha_n - \mathscr{N}_p\alpha_p)] \varepsilon_{ikm}B_m\theta_{,k}$$

$$+ T_0(\mathscr{N}_n + \mathscr{N}_p) \varepsilon_{ikm}B_m\mathscr{E}_k + \frac{1}{\varrho} T_0(\gamma_n\mathscr{N}_n - \gamma_p\mathscr{N}_p) \varepsilon_{ikm}B_m u_{s,sk}$$

$$+ \frac{1}{n_0} T_0 \mathscr{N}_n \delta_n \varepsilon_{ikm}B_m N_{,k} - \frac{1}{p_0} T_0 \mathscr{N}_p \delta_p \varepsilon_{ikm}B_m P_{,k},$$

$$j_{ni} = -(\beta_n + \alpha_n\varrho\xi_n n_0)\theta_{,i} + \varrho\xi_n n_0 \mathscr{E}_i + \xi_n\gamma_n n_0 u_{k,ki} + \varrho\xi_n \delta_n N_{,i} - (\mathscr{N}_n - \alpha_n\mathscr{R}_n) \varepsilon_{ikm}B_m\theta_{,k}$$

$$- \mathscr{R}_n \varepsilon_{iks}B_s\mathscr{E}_k - \frac{1}{\varrho} \mathscr{R}_n\gamma_n \varepsilon_{iks}B_s u_{m,mk} - \frac{1}{n_0} \mathscr{R}_n \delta_n \varepsilon_{iks}B_s N_{,k},$$

$$j_{pi} = -(\beta_p - \alpha_p\varrho\xi_p p_0)\theta_{,i} + \varrho\xi_p p_0 \mathscr{E}_i - \xi_p\gamma_p p_0 u_{k,ki} - \varrho\xi_p \delta_p P_{,i} - (\mathscr{N}_p + \alpha_p\mathscr{R}_p) \varepsilon_{ikm}B_m\theta_{,k}$$

$$- \mathscr{R}_p \varepsilon_{iks}B_s\mathscr{E}_k + \frac{1}{\varrho} \mathscr{R}_p\gamma_p \varepsilon_{iks}B_s u_{m,mk} + \frac{1}{p_0} \mathscr{R}_p \delta_p \varepsilon_{iks}B_s P_{,k},$$

where one introduced notation:

$$q_i^{EM} = [\underline{\mathscr{E}} \times (\chi_m\mathbf{B} + \bar{\mathbf{M}}^0)]_i.$$

And so it appears that the relations (6.13) besides the effects mentioned earlier include also terms referring to the following effects:

effects referring to the linear influence of dilatation and concentration of charge carrier gradients on heat flux [6, 18],

effect referring to the rising of the heat flux by the existence of a perpendicular magnetic field and dilatation gradient [7],

photomagnetothermal effect describing the rising of the heat flux by the influence of a magnetic field perpendicular to it and concentration of charge gradients (field which results from illuminating of the semiconductor by strong absorbed light) [7],

effects describing the rising of currents of electrons and holes forced by (and in the direction of) the dilatation gradient [7],

effects responsible for the rising of an electron and hole currents perpendicularly to the applied magnetic field and dilatation gradient [7],

and photomagnetoelectric effects describing the rising of an electron and hole currents perpendicularly to the magnetic field and charge concentration gradients (field which results from illuminating of the semiconductor by strong absorbed light [14]).

Recapitulating, one should stress that basing on the theory presented above, it is possible to describe all piezoeffects, both first and second kind and all others which would result from the dependence of phenomenological coefficients on the odd parameters (separately or simultaneously).

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TECHNICAL UNIVERSITY OF POZNAŃ
INSTITUTE OF APPLIED MECHANICS.