I.

§ 1.

1. $a \supset a$ [Pp.]
2. $a \supset aa$ [Pp.]
* 3. $a = b \,.=.\, a \supset b \,.\, b \supset a$ [Def.]
* 4. $a = a$ [P1 . ⊃ . P4]
* 5. $ab \supset a$ [Pp.]
* 6. $a = aa$ [P2 . $\begin{pmatrix} a \\ b \end{pmatrix}$ P5 . ⊃ . P6]
7. $ab \supset ba$ [Pp.]
* 8. $ab = ba$ [P7 . $\begin{pmatrix} b, a \\ a, b \end{pmatrix}$ P7 . ⊃ . P8]
9. $abc \supset acb$ [Pp.]
*10. $abc = acb$ [P9 . ⊃ . P10]
*11. $a \supset b \,.\supset.\, ac \supset bc$ [Pp.]
*12. $a \,.\, a \supset b \,.\supset.\, b$ [Pp.]
*13. $a \supset b \,.\, b \supset c \,.\supset.\, a \supset c$ [Pp.]
14. $abc \supset bac$ [P7 . $\begin{pmatrix} ab, ba \\ a, b \end{pmatrix}$ P11 . P12 . ⊃ . P14]
14' $abc = bac$
15. $ab \supset b$ [P7 . $\begin{pmatrix} b, a \\ a, b \end{pmatrix}$ P5 . P13 . ⊃ . P15]
16. $a = b \,.\supset.\, a \supset b$ [P3 . P5 . ⊃ . P11]
17. $a = b \,.\supset.\, b \supset a$ [P3 . P15 . ⊃ . P17]
18. $a \supset b \,.\, b \supset a \,.\supset.\, a = b$ [P3 . P1 . ⊃ . P18]
19. $a = b \,.\supset.\, b = a$ [P3 . P7 . ⊃ . P19]
*20. $a = b \,.=.\, b = a$ [P19 . $\begin{pmatrix} b, a \\ a, b \end{pmatrix}$ P19 . = . P20]
21. $a = b \,.\, b \supset c \,.\supset.\, a \supset c$ [Hp. $\supset .\, a \supset b \,.\, b \supset c \,.\supset.\, a \supset c$]
22. $a \supset b \,.\, b = c \,.\supset.\, a \supset c$ [Idem]

*23. $a = b \,.\, b = c \,.\, \supset .\, a = c$

$[\mathrm{Hp.} \supset .\, a \supset b \,.\, b \supset c \,.\, c \supset b \,.\, b \supset a \,.\, \supset .\, a \supset c \,.\, c \supset a \,.\, \supset .\ \mathrm{Ts.}]$

(α) $a \supset b \,.\, b \supset c \,.\, c \supset d \,.\, \supset .\, a \supset c \,.\, c \supset d$ $[\mathrm{P13} \,.\, \mathrm{P11} \,.\, \supset .\, (\alpha)]$

*24. $a \supset b \,.\, b \supset c \,.\, c \supset d \,.\, \supset .\, a \supset d$ $[(\alpha) \,.\, \mathrm{P13} \,.\, \supset .\, \mathrm{P24}]$

25. $a = b \,.\, b = c \,.\, c = d \,.\, \supset .\, a = d .$ $[\mathrm{Hp.\ P23} \,.\, \supset .\, a = c \,.\, c = d \,.\, \supset .\ \mathrm{Ts.}]$

26. $b \supset .\, a \supset ab$, [Pp.]

26′. $b \supset .\, a = ab$.

(α) $c \,.\, \supset .\, a \supset ac$ $\left[\binom{c}{b} \mathrm{P26} \,.\, \supset .\, (\alpha)\right]$

(β) $c \,.\, ac \supset bc \,.\, \supset .\, a \supset ac \,.\, ac \supset bc$ $[(\alpha) \,.\, \mathrm{P11} \,.\, \supset .\, (\beta)]$

(γ) $c \,.\, ac \supset bc \,.\, \supset .\, a \supset bc$ $[(\beta) \supset (\gamma)]$

(δ) $a \supset b \,.\, c \,.\, \supset .\, ac \supset bc \,.\, c$ $[\mathrm{P11} \,.\, \supset .\, (\delta)]$

(ε) $a \supset b \,.\, c \,.\, \supset .\, c \,.\, ac \supset bc$ $[(\delta) \,.\, \mathrm{P7} \,.\, \supset .\, (\varepsilon)]$

27. $a \supset b \,.\, c \,.\, \supset .\, a \supset bc$ $[(\varepsilon)\,(\gamma) \,.\, \supset .\, \mathrm{P27}]$

28. $a \supset b \,.\, \supset .\, ca \supset cb$ $[\mathrm{Hp.\ P11} \,.\, \mathrm{P7} \,.\, \supset .\, ac \supset bc \,.\, ca \supset ac \,.\, bc \supset cb \,.\, \supset .\ \mathrm{Ts.}]$

29. $a \supset b \,.\, \supset .\, a \supset ab$ $[\mathrm{Hp.} \binom{a}{c} \mathrm{P28} \,.\, \mathrm{P2} \,.\, \supset .\ \mathrm{Ts.}]$

*30. $a \supset b \,.\, c \supset d \,.\, \supset .\, ac \supset bd$ $[\mathrm{Hp.} \supset . ac \supset bc . c \supset d . \supset . ac \supset bc . bc \supset bd . \supset .\ \mathrm{Ts.}]$

*31. $a = b \,.\, \supset .\, ac = bc$ $[\mathrm{Hp.} \supset . a \supset b . b \supset a . \supset . ac \supset bc . bc \supset ac . \supset .\ \mathrm{Ts.}]$

*32. $a = b \,.\, c = d \,.\, \supset .\, ac = bd$ $[\mathrm{Hp.} \supset .\, ac = bc \,.\, bc = bd \,.\, \supset .\ \mathrm{Ts.}]$

(α) $a \supset b \,.\, \supset .\, a = ab$ $[\mathrm{Hp.\ P29} \,.\, \mathrm{P5} \,.\, \supset .\ \mathrm{Ts.}]$

(β) $a = ab \,.\, \supset .\, a \supset b$ $[\mathrm{Hp.\ P15} \,.\, \supset .\ \mathrm{Ts.}]$

*33. $a \supset b \,.\, = .\, a = ab$ $[(\alpha)\,(\beta) = \mathrm{P33}]$

34. $a \supset b \,.\, a \supset c \,.\, \supset .\, a \supset bc$ $[\mathrm{Hp.\ P30} \,.\, \supset .\, aa \supset bc \,.\, \mathrm{P2} \,.\, \supset .\ \mathrm{Ts.}]$

35. $a \supset bc \,.\, \supset .\, a \supset b \,.\, a \supset c$ $[\mathrm{P5} \,.\, \mathrm{P15} \,.\, \mathrm{P34} : \supset .\, \mathrm{P35}]$

*36. $a \supset bc \,.\, = .\, a \supset b \,.\, a \supset c$ $[\mathrm{P34} \,.\, \mathrm{P35} \,.\, = .\, \mathrm{P36}]$

37. $a \supset .\, b \supset c : \supset .\, ab \supset c$ $[\mathrm{Hp.} \supset :: ab \supset : b \supset c \,.\, b \therefore \mathrm{P12} :: \supset .\ \mathrm{Ts.}]$

38. $ab \supset c \,.\, \supset : a \supset .\, b \supset c$ $[\mathrm{Hp.\ P26} \,.\, \supset \therefore a \supset .\, b \supset ab : ab \supset c :$

$\mathrm{P27} \therefore \supset \therefore a \supset : b \supset ab \,.\, ab \supset c \therefore \mathrm{P13} \therefore \supset .\ \mathrm{Ts.}]$

*39. $a \supset .\, b \supset c : = .\, ab \supset c$ $[\mathrm{P37} \,.\, \mathrm{P38} \,.\, = .\, \mathrm{P39}]$

40. $b \supset c \,.\, \supset : a \supset b \,.\, \supset .\, a \supset c$ $[\mathrm{P13} \,.\, \mathrm{P38} \,.\, \supset .\, \mathrm{P40}]$

41. $a \supset b \,.\, \supset : b \supset c \,.\, \supset .\, a \supset c$ $[\mathrm{P13} \,.\, \mathrm{P38} \,.\, \supset .\, \mathrm{P41}]$

42. $a \supset .\, b \supset a$ $\left[\binom{a}{c} \mathrm{P38} \supset \mathrm{P42}\right]$

43. $a \supset : a \supset b \,.\, \supset b$ $[\mathrm{P38} \,.\, \mathrm{P12} \,.\, \supset .\, \mathrm{P43}]$

44. $ab \supset c \,.\, ac \supset b : = : a \supset .\, b = c$ $[\mathrm{Hp.} = (a \supset . b \supset c)(a \supset . c \supset b) = \mathrm{Ts.}]$

$ac. =: a \supset . b = c$

45. $a \supset .\, bc = bd : = : ab \supset .\, c = d$ $[\mathrm{P44} \supset \mathrm{P45}]$

46. $a \supset . b = cd : \supset : ac \supset . b = d$.

47. $abc \supset . bd = ce : = : abc \supset . d = e$.

48. $a \supset b . ab \supset c . \supset . a \supset c$

49. $ab \supset c . ac \supset d . \supset . ab \supset cd$

§ 2.

1. $a \supset b . \supset . -b \supset -a$ [Pp.]

2. $a = b . \supset . -a = -b$ $\left[\mathrm{P1} . \begin{pmatrix} b, a \\ a, b \end{pmatrix} \mathrm{P1} : \supset . \mathrm{P2}\right]$

* 3. $-(-a) = a$ [Pp.]

* 4. $a \supset b . = . -b \supset -a$ $\left[\mathrm{P1} . \begin{pmatrix} -b, -a \\ a, b \end{pmatrix} \mathrm{P1} : \supset . \mathrm{P4}\right]$

* 5. $a = b . = . -a = -b$ $[\mathrm{P2} \supset \mathrm{P5}]$

6. $a \cup b = -[(-a)(-b)]$ [Def.]

* 7. $-(a \cup b) = (-a)(-b)$ $[\mathrm{P6} \supset \mathrm{P7}]$

* 8. $-(ab) = (-a) \cup (-b)$ $\left[\begin{pmatrix} -a, -b \\ a, b \end{pmatrix} \mathrm{P6} = \mathrm{P8}\right]$

* 9. $a \cup b = b \cup a$ $[a \cup b = -[(-a)(-b)] = -[(-b)(-a)] = b \cup a]$

* 9'. $a \cup b \cup c = a \cup c \cup b$

*10. $a \cup a = a$ $[a \cup a = -[(-a)(-a)] = -(-a) = a]$

*11. $a \supset b . \supset . a \cup c \supset b \cup c$ $[\mathrm{Hp.} \supset . -b \supset -a . \supset . -b -c \supset -a -c . \supset . \mathrm{Ts.}]$

*12. $a = b . \supset . a \cup c = b \cup c$ $[\mathrm{P11} \supset \mathrm{P12}]$

*13. $a \supset b . c \supset d . \supset . a \cup c \supset b \cup d$ $[\mathrm{P11} \supset \mathrm{P13}]$

*14. $a = b . c = d . \supset . a \cup c = b \cup d$ $[\mathrm{P12} \supset \mathrm{P14}]$

*15. $a \supset a \cup b$ $[\S 1\ \mathrm{P5} \supset . -a -b \supset -a . \supset . \mathrm{Ts.}]$

*16. $a \supset b . = . b = a \cup b$ $[\S 1\ \mathrm{P33} \supset \mathrm{P16}]$

*17. $a \supset c . b \supset c . = . a \cup b \supset c$ $[\S 1\ \mathrm{P36} \supset \mathrm{P17}]$

18. $a \cup ab = a$ $[\S 1\ \mathrm{P5} \supset : ab \supset a . \mathrm{P16} : \supset . a \cup ab = a]$

19. $a(a \cup b) = a$ $[\mathrm{P15} . \S 1\ \mathrm{P33} . \supset . \mathrm{P19}]$

20. $ac \cup bc \supset (a \cup b)c$ $[\mathrm{P15} . \supset : ac \supset (a \cup b)c . bc \supset (a \cup b)c . \mathrm{P17} : \supset . \mathrm{Ts.}]$

(α) $c \supset . a \supset ac$ $[\S 1\ \mathrm{P26} \supset (\alpha)]$

(β) $c \supset . b \supset bc$ $[\S 1\ \mathrm{P26} \supset (\beta)]$

(γ) $c \supset . a \supset ac . b \supset bc$ $[(\alpha)(\beta) . \S 1\ \mathrm{P34} \supset (\gamma)]$

(δ) $c \supset . a \cup b \supset ac \cup bc$ $[(\gamma)\ \mathrm{P13} \supset (\delta)]$

21. $(a \cup b)c \supset ac \cup bc$ $[(\delta) . \S 1\ \mathrm{P39} \supset . \mathrm{P21}]$

*22. $(a \cup b)c = ac \cup bc$ $[\mathrm{P22} = . \mathrm{P20} . \mathrm{P21}]$

23. $(a \cup c)(b \cup c) = ab \cup c$ $[(a \cup c)(b \cup c) = ab \cup ac \cup bc \cup c = ab \cup c]$

*24. $ab \supset c . = . a -c \supset -b .$
$[ab \supset c : = : a \supset . b \supset c : = : a \supset . -c \supset -b : = : a -c \supset -b]$

*25. $ab \supset c . = . a \supset c \cup -b$ 25' $a \supset b \cup c . = . a -b \supset c$ $\left[\begin{pmatrix} -b \\ b \end{pmatrix} \mathrm{P25}\right.$

*26. $ab \supset c \cup d . = . a -c \supset -b \cup d$

27. $ac \supset bc . a \cup c \supset b \cup c . \supset . a \supset b$
28. $ac = bc . a \cup c = b \cup c . \supset . a = b$
29. $a = b . = . a \cup b \supset ab$
30. $(a \cup b)(b \cup c)(c \cup a) = ab \cup bc \cup ca$
31. $ab \supset cd . b \cup c \supset a \cup d . \supset . b \supset d$
32. $(a \cup x)(b \cup -x) = a - x \cup bx$
33. $(ax \cup b - x)(a'x \cup b' - x) = aa'x \cup bb' - x$
34. $-(ax \cup b - x) = (-a)x \cup (-b)(-x)$
35. $a \cup b = a \cup (-a)b$
36. $a \supset c . \cup . b \supset c . \supset . ab \supset c$
37. $c \supset a . \cup . c \supset b . \supset . c \supset a \cup b$

28'. $ac \supset b . a \supset b \cup c . \supset . a \supset b$ Peirce (v. Schröder, Alg. d. Log. I, p. 363)

38. $a - a \supset b$ [P38 = P15]
(α) $a - a \supset b - b$ [(α) = P32 = (α)]
39. $a - a = b - b$ [(α)(β)(α) = P39]

§ 3.

* 1. ~~...~~ $\Lambda = a - a$ (C. Burali-Forti, Logica mat. p. 49) [Pp.]
1'. $\mathrm{V} = -\Lambda$ [Def.]
1''. $a \cup -a = \mathrm{V}$
* 2. $a\Lambda = \Lambda$ [$a\Lambda = aa - a = a - a = \Lambda$]
2'. $a \cup \mathrm{V} = \mathrm{V}$
* 3. $\Lambda \supset a$ [$\Lambda \supset a - a \supset a$]
3'. $a \supset \mathrm{V}$
* 4. $a \cup \Lambda = a$ [P3 . §2 . P16 : ɔ . P4]
4'. $a \mathrm{V} = a$
5. $a \cup (b - b) = a$ [P1 . P4 . ɔ . P5]
6. $ab \cup a - b = a$ [P5 ɔ P6]
7. $a \supset \Lambda . = . a = \Lambda$ [P3 ɔ P7]
7'. $\mathrm{V} \supset a . = . \mathrm{V} = a$
(α) $a \supset b . \supset . a - b = \Lambda$ [Hp. ɔ: $a - b \supset b - b$. P1 :ɔ: $a - b \supset \Lambda$. P7 :ɔ. Ts.]
(β) $a - b = \Lambda . \supset . a \supset b$ [Hp. P6 : ɔ : $a = ab$. §2 P33 : ɔ . Ts]
* 8. $a \supset b . = . a - b = \Lambda$ [(α) (β) = P8]
8'. $a \supset b . = . b \cup -a = \mathrm{V}$
8''. $ab = \Lambda . = . a \supset -b$.
* 9. $a \cup b = \Lambda . = . a = \Lambda . b = \Lambda$ [P7 . §2 . P17 : ɔ . P9]
10. $a \cup b - = \Lambda . = . a - = \Lambda . \cup . b - = \Lambda$. [P9 = P10]
11. $a = \Lambda . \cup . b = \Lambda . \supset . ab = \Lambda$ [P2 ɔ P11]
12. $ab - = \Lambda . \supset . a - = \Lambda . b - = \Lambda$ [P11 = P12]
13. $a \supset b . b = \Lambda . \supset . a = \Lambda$. [P7 ɔ P13]

9'. $a = b . = . a - b \cup b - a = \Lambda$ (Schröder, Bd. I, p. 309)

14. $a \supset b . a -= \Lambda . \supset . b -= \Lambda .$ [P13 = P14]
15. $ax \cup b - x = \Lambda . = . b \supset x \supset - a$
16. $ax \cup b - x = \Lambda . \supset . ab = \Lambda$
17. $ax \cup b - x -= \Lambda . \supset . a \cup b -= \Lambda .$
18. $ax \cup b - x = \Lambda . px \cup q - x -= \Lambda . \supset . ab = \Lambda . p - a \cup q - b -= \Lambda$
19. $x \supset a . y \supset b . ab = \Lambda . \supset . xy = \Lambda$
20. $xy -= \Lambda . x \supset a . y \supset b . \supset . ab -= \Lambda$ [P19 = P20]
21. $ab = \Lambda . x \cup y = a \cup b . x \supset a . y \supset b : \supset : a \supset x . b \supset y . xy = \Lambda$
22. $x\supset a.y\supset b.z\supset c.x \cup y \cup z = a \cup b \cup c.ab=\Lambda.ac=\Lambda.bc=\Lambda.\supset.a=x.b=y.c=z$
23. $ab=ac=bc=\Lambda.a \cup b \cup c = x \cup y \cup z:\supset.\therefore x\supset a.y\supset b.z\supset c:=:x=a.y=b.z=c.$
24. $a \circ b = a - b \cup b - a$ [Def.]
25. $a \circ \Lambda = a .$
26. $a \circ a = \Lambda$
27. $a \circ b = b \circ a$
28. $(a \circ b) \circ c = \cancel{(a \circ b) \circ c}$ $a \circ (b \circ c)$
29. $-(a \circ b) = (-a) \circ b = a \circ (-b)$
30. $a = b \circ c . = . b = a \circ c . = . c = a \circ b$
31. $a \supset b . bc = \Lambda . \supset . ac = \Lambda$

§ 4.

1. $a, b \varepsilon K . \supset \therefore a \supset b . = : x \varepsilon a . \supset_x . x \varepsilon b$ [Def.]
2. » $. \supset : a = b . = . a \supset b . b \supset a$ [Def.]
3. » $. \supset . a \cap b = \overline{x \varepsilon}(x \varepsilon a . x \varepsilon b)$ [Def.]
3'. » $. \supset . ab = a \cap b$
4. » $. \supset . a \cup b = \overline{x \varepsilon}(x \varepsilon a . \cup . x \varepsilon b)$ [Def.]
5. $a \varepsilon K . \supset . -a = \overline{x \varepsilon}(x - \varepsilon a)$ [Def.]
6. » $. \supset \therefore a = \Lambda . = : x \varepsilon a . =_x \Lambda$ [Def.]
7. $u \varepsilon K . a \varepsilon u . \supset . a = a$
8. » $. a, b \varepsilon u . \supset : a = b . = . b = a .$
9. » $. a, b, c \varepsilon u . \supset : a = b . b = c . \supset . a = c .$
10. » $. a = b . b \varepsilon u . \supset . a \varepsilon u .$

$a, b \varepsilon K . \supset .\cdot$
.11. $x \varepsilon a . a \supset b . \supset . x \varepsilon b$ [P1 ɔ P11]
12. $x \varepsilon a . \supset . a -= \Lambda$
13. $x \varepsilon \iota y . = x = y$ [Def]
13'. $\iota y = \overline{x \varepsilon}(x = y)$

§ 5.

$a,\ b,\ c,\ d\ \varepsilon\ \mathrm{K} \,.\supset:$

1. $f\ \varepsilon\ b|a \,.=:\, x\ \varepsilon\ a \,.\supset_x.\, fx\ \varepsilon\ b\,.$ [Def.]
2. » $.\, x,\ y\ \varepsilon\ a \,.\, x = y \,.\supset.\, fx = fy\,.$
3. » $.\supset.\, fa = \overline{y\ \varepsilon}\,[x\ \varepsilon\ a \,.\, fx = y : -=_x \Lambda]\,.$ [Def.]
4. » $.\, c \supset a \,.\supset.\, f\ \varepsilon\ b|c\,.$
5. » $.\, c \supset a \,.\supset.\, fc \supset fa\,.$
6. » $.\supset.\, f\Lambda = \Lambda\,.$
7. » $.\, b \supset c \,.\supset.\, f\ \varepsilon\ c|a\,.$
8. $f\ \varepsilon\ c|a \,.\, f\ \varepsilon\ c|b \,.\supset.\, f\ \varepsilon\ c|(a \cup b)\,.$
9. » » $.\supset.\, f(a \cup b) = fa \cup fb\,.$
10. » » $.\supset.\, f(a \cap b) \supset (fa) \cap (fb)\,.$
11. $f,\ g\ \varepsilon\ b|a \,.\supset\therefore f = g \,.=:\, x\ \varepsilon\ a \,.\supset_x.\, fx = gx\,.$ [Def.]
12. $f\ \varepsilon\ b|a \,.\, g\ \varepsilon\ c|b \,.\, x\ \varepsilon\ a \,.\supset.\, (gf)x = g(fx)\,.$ [Def.]
13. » » $.\supset.\, gf\ \varepsilon\ c|a\,.$
14. » » $.\, h\ \varepsilon\ d|c \,.\supset.\, h(gf) = (hg)f = hgf\,.$

$s\ \varepsilon\ \mathrm{K} \,.\supset:$

15. $f\ \varepsilon\ s|s \,.\, x\ \varepsilon\ s \,.\supset.\, f^1x = fx\,.$ [Def.]
16. » » $.\, m\ \varepsilon\ \mathrm{N} \,.\supset.\, f^{m+1}x = ff^mx\,.$
17. » $.\, m\ \varepsilon\ \mathrm{N} \,.\supset.\, f^m\ \varepsilon\ s|s\,.$
18. » $.\, m,\ n\ \varepsilon\ \mathrm{N} \,.\supset.\, f^mf^n = f^{m+n}\,.$
19. $f,\ g\ \varepsilon\ s|s \,.\, fg = gf \,.\, m,\ n\ \varepsilon\ \mathrm{N} \,.\supset.\, f^mg^n = g^nf^m\,.$
20. » » $.\, m\ \varepsilon\ \mathrm{N} \,.\supset.\, (fg)^m = f^mg^m\,.$
21. $a,\ b\ \varepsilon\ \mathrm{K} \,.\, f\ \varepsilon\ b|a \,.\, y\ \varepsilon\ b \,.\supset.\, \overline{f}y = \overline{x\ \varepsilon}\,(fx = y)\,.$ [Def.]

21'. » » » $.\supset:\, x\ \varepsilon\ \overline{f}y \,.=.\, fx = y\,.$

22. » $.\supset:\, f\ \varepsilon\ (b|a)\,\mathrm{sim} \,.=.\, f\ \varepsilon\ b|a \,.\, \overline{f}\ \varepsilon\ a|b\,.$ [Def.]
23. $f\ \varepsilon\ (s|s)\,\mathrm{sim} \,.\, x\ \varepsilon\ s \,.\supset.\, \overline{f}fx = x \,.\, f\overline{f}x = x \,.\, \overline{\overline{f}} = f\,.$
24. » $.\, a,\ b\ \varepsilon\ \mathrm{K}s \,.\supset:\, a \supset b \,.=.\, fa \supset fb\,.$
25. » » $.\supset:\, a = b \,.=.\, fa = fb\,.$
26. » » $.\supset:\, f(a \cap b) = (fa) \cap (fb)\,.$
27. » $.\, m\ \varepsilon\ \mathrm{N} \,.\supset.\, \overline{f^m} = \overline{f}^m\,.$
28. $f,\ g\ \varepsilon\ (s|s)\,\mathrm{sim} \,.\supset.\, \overline{gf} = \overline{f}\,\overline{g}\,.$
29. » $.\, fg = gf \,.\supset.\, \overline{f}g = g\overline{f} \,.\, \overline{f}\,\overline{g} = \overline{g}\,\overline{f}\,.$

30. $f \varepsilon s|s . x \varepsilon s . \supset . f^0 x = x$. [Def.]

31. $f \varepsilon (s|s)\,\mathrm{sim} . m \varepsilon \mathrm{N} . \supset . f^{-m} = \bar{f}^m$. [Def.]

32. $f \varepsilon s|s . m, n \varepsilon \mathrm{N} . \supset . (f^m)^n = f^{mn}$.

33. $f \varepsilon (s|s)\,\mathrm{sim} . m, n \varepsilon \mathrm{n} . \supset . (f^m)^n = f^{mn}$.

34. $a, b \varepsilon K . f \varepsilon b \mathrm{f} a . \supset :: f \varepsilon \mathrm{Sim} . = : x, y \varepsilon a . x -= y . \supset_{x,y} fx -$ [De

35 " $f \varepsilon (b \mathrm{f} a)\,\mathrm{sim} . \supset . f \varepsilon (b \mathrm{f} a)\,\mathrm{Sim}$

36 " $f \varepsilon (b \mathrm{f} a)\,\mathrm{Sim} . \supset . f \varepsilon (fa\, \mathrm{f} a)\,\mathrm{Sim}$

37. $s \varepsilon K . \mathrm{num}\, s \varepsilon N . \supset : f \varepsilon (\underline{s} \mathrm{f} s)\,\mathrm{sim} . = . f \varepsilon (s \mathrm{f} s)\,\mathrm{Sim}$

38. $a, b \varepsilon K . f \varepsilon (b \mathrm{f} a)\,\mathrm{sim} . \supset . \bar{f} \varepsilon (a \mathrm{f} b)\,\mathrm{sim}$.

39. $a, b, c \varepsilon K . f \varepsilon (b \mathrm{f} a)\,\mathrm{sim} . g \varepsilon (c \mathrm{f} b)\,\mathrm{sim} . \supset . g f \varepsilon (c \mathrm{f}$