HEMIVARIATIONAL INEQUALITIES MODELING DYNAMIC CONTACT PROBLEMS IN VISCOELASTICITY

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In the paper we study a mathematical model of the dynamic process of frictional contact between a deformable body and a foundation. The unknown variables of the system are displacement vector field and stress tensor field defined on the set $\Omega \subset \mathbb{R}^d$, $d = 1, 2, 3$, which the body occupies. The body under consideration is assumed to be viscoelastic with a linear elasticity operator and a nonlinear viscosity operator. The contact is modeled with a general normal damped response condition. The quasistatic and dynamic contact problems for viscoelastic bodies have been recently investigated in many contributions, see e.g. Han and Sofonea [4], Jarusek [5], Kuttler [6], Rochdi et al. [9] and the literature therein. In this paper we consider two additional phenomena connected with the contact process.

The first of them is adhesive interaction between the body and the foundation. We refer to Frémond [2, 3] in order to introduce a surface internal bonding field having values between zero and one, which describes the fractional density of active bonds on the contact surface. An evolution of the bonding field is governed by an ordinary differential equation. In particular we consider adhesive viscoelastic bilateral contact. The main feature of this model is the fact that during the process there is no gap between the body and the foundation. From the mathematical point of view the bilateral contact condition is very convenient since it leads to a linear subspace of admissible displacements.

The second phenomenon is a wear of the material. To model the wear of the contacting surfaces we introduce (following Section 3.2 of [10]) the wear function, which measures the depth, in the normal direction, of the removed material. We treat the problem with a simplified version of the Archard law which is a rate condition for wear production. This law allows to eliminate the unknown variable, the wear function, from the model. In this manner the problem decouples and we are led to a variational formulation involving only the displacement field.

In both cases the dependence of the normal and tangential stress on the normal and tangential displacement is supposed to have nonmonotone character of the subdifferential form. Therefore, a convex analysis approach to the problem is not possible. We are lead to a mathematical model, called a hemivariational inequality, which involves the Clarke subdifferential of a locally Lipschitz functional. For instance, we formulate the system coupled with a differential equation and an evolution hemivariational inequality obtained as a variational formulation of a hyperbolic equation. The problem is following: find the displacement field u and the bonding field β such that

$$(1)\qquad \begin{cases} \langle u''(t) + A(t, u'(t)) + Bu(t) - f(t), v\rangle_{V^*\times V} + \displaystyle\int_{\Gamma_C} j^0(x, t, \beta(x,t), \gamma u(t); \gamma v)\, d\sigma(x) \geq 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for all } v \in V, \text{ a.e. } t \in (0,T) \\ u(0) = u_0, \quad u'(0) = u_1 \\ \beta'(t) = F(t, u(t), \beta(t)) \quad \text{on} \quad \Gamma_C \times (0,T) \\ \beta(0) = \beta_0 \quad \text{on} \quad \Gamma_C, \end{cases}$$

where $A\colon (0,T) \times V \to V^*$ is a nonlinear damping operator, $B\colon V \to V^*$ is a linear elasticity operator, V denotes a subspace of the Sobolev space $H^1(\Omega; \mathbb{R}^d)$, V^* is its dual, $j^0(x,t,\cdot)$ is Clarke directional derivative of a locally Lipschitz function $j(x,t,\cdot)\colon \mathbb{R}^d \to \mathbb{R}$, $f \in L^2(0,T;V^*)$, γ stands for a trace operator and Γ_C is the part of the boundary of the set Ω on which the contact take place. The function $\beta\colon \Gamma_C \times (0,T) \to [0,1]$ measures an intensity of adhesive bonds and the function β_0 denotes the initial bonding field. The function F is prescribed.

The main result of the paper is to provide the existence of a weak solution to the adhesive frictional contact problem and to the wear contact one, respectively. It is attained by embedding the problems into a class of second order evolution inclusions and by applying a surjectivity result for multivalued operators. The novelty of the model is to consider the coupling between the viscoelastic properties of the material with the adhesive properties on the contact surface and nonmonotone possibly multivalued boundary conditions. The work is completed with a few model examples of subdifferential boundary conditions which include the functions of d.c. type (difference of convex functions) being useful in modeling of nonmonotone sawtooth contact and friction laws. These examples illustrate the applicability of our results.

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