

## A NOVEL METHOD FOR STATIC ANALYSIS OF THIN CURVED SHELLS WITH VARIABLE THICKNESS

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Stiffness method (displacement based formulation) is a widely applicable method for analysis of structures such as beams, plates, shells and etc. Analyzing structures with variable thickness by using stiffness method, the structure is discretized into straight elements with constant mechanical properties and a constant thickness which is an average of thickness at the element's nodes. Increasing number of elements, a better approximation is obtained but on the other hand, dimensions of stiffness and mass matrices increase so that more time and memory would be required. However, this method is the proper conventional method which can be found in technical literature. In general, application of the displacement based formulations lead to violation of one of the three basic equilibrium equations due to assumption of displacement fields in that method.

Force methods (flexibility method) can be a proper substitute for stiffness method as they absolutely satisfy the three basic equilibrium equations at all points of the element. In the present paper, unit load method which is considered as force method has been applied for derivation of stiffness matrix. However, application of force method is generally more complicated than stiffness method so that a combination of these methods (stiffness and force methods) can bring in accuracy, simplicity and time and memory saving.

In this paper, a new method for static analysis of thin curved shells with variable thickness is presented. The basis of the method is employing shape functions of non-prismatic curved beams which are obtained by utilizing unit load method. This new method can be applied to any kind of shells with any variable thickness function and boundary conditions. This method could be easily extended to complex analysis of structures such as non-linear analysis in both geometric and material non-linearity cases. In fact, this method is the logical development of finite element calculations and employs mathematical computations and numerical integrals.

Comparing the results obtained by this method and those obtained by ordinary methods obviously proves its excellence in both convergence and economy. In static analysis of spherical and cylindrical shells, indeed, the results computed by using few elements with this formulation are completely comparable with the results obtained by using numerous elements with ordinary formulations. It is worth to note that this new formulation can be applied in any standard displacement based finite elements programs and algorithms.

The authors have started a new studying on extension of this method to dynamic analysis of shells with variable thickness and good results have been gained, so far.

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