FREE VIBRATION ANALYSIS OF STIFFENED PLATES BY THE BOUNDARY ELEMENT METHOD

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1. Introduction

Natural frequencies, which depend on geometry, material properties and support conditions of bodies, characterize the dynamical properties of structures. For structures of complex geometry and made of different materials, natural frequencies are usually calculated by the finite element method (FEM) or the boundary element method (BEM). The methods can be coupled in order to exploit their advantages and solve the problem more efficiently.

The free vibration analysis of homogeneous plates by the dual reciprocity BEM (DRBEM) was presented for the first time by Nardini and Brebbia [2]. Albuquerque et al. [1] applied the method for the free vibration analysis of anisotropic plates. Górski and Fedeliński [3] used the DRBEM for determination of natural frequencies and mode shapes of non-homogeneous plates.

In the present paper, the formulation and application of the coupled DRBEM and FEM in the free vibration analysis of stiffened plates is presented. The generalized algebraic eigenvalue problem is transformed into the standard one and solved. One numerical example is presented and frequencies for the reinforced cantilever plate, computed by the present method and the FEM, are compared.

2. Formulation of the eigenvalue problem

Consider a plate, occupying the domain Ω^{I} and enclosed by the outer boundaries Γ_1^{I} and Γ_2^{I}, reinforced by a stiffener, occupying the domain Ω^{II}, as shown in Fig.1. The Γ_1^{I} and Γ_2^{I} are parts of the boundary, where displacement and traction boundary conditions are prescribed, respectively. The boundary connecting the plate and stiffener (the interface) is $\Gamma_3^{I}\equiv\Gamma_3^{II}\equiv\Gamma_3^{I,II}$. The translational degrees of freedom of the plate and stiffener (beam) at the $\Gamma_3^{I,II}$ are coupled and the rotational ones of the beam at the Γ_2^{II} are free. The subscripts 1, 2, 3 correspond to the fixed, free and common boundary and the superscripts I, II correspond to the plate and beam domain, respectively.

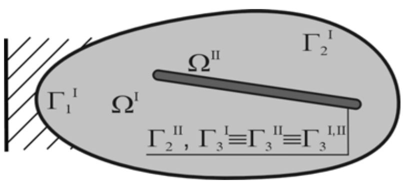

Figure 1. A plate reinforced by a stiffener.

The materials of the plate in plane stress or strain and the stiffener are linear elastic, isotropic and homogeneous. The plate is modeled by the DRBEM and the stiffener by the FEM. The numerical solution is obtained after discretization of the body into the curved boundary and straight beam finite elements. One boundary element along the $\Gamma_3^{I,II}$ is connected with two finite elements. The ideal connection at the nodes of the plate and stiffener is assumed.

The detailed transformations and the resulting matrices are presented in the full length paper. The final algebraic eigenvalue problem for the body in Fig.1 has the following form

$$\begin{bmatrix} \bar{\mathbf{H}}_{22}^{\mathbf{I}} & 0 & \bar{\mathbf{H}}_{23}^{\mathbf{I}} & -\bar{\mathbf{G}}_{23}^{\mathbf{I}} \\ \bar{\mathbf{H}}_{32}^{\mathbf{I}} & 0 & \bar{\mathbf{H}}_{33}^{\mathbf{I}} & -\bar{\mathbf{G}}_{33}^{\mathbf{I}} \\ 0 & \mathbf{K}_{22}^{\mathbf{II}} & \mathbf{K}_{23}^{\mathbf{II}} & \mathbf{T}_{23}^{\mathbf{II}} \\ 0 & \mathbf{K}_{32}^{\mathbf{II}} & \mathbf{K}_{33}^{\mathbf{II}} & \mathbf{T}_{33}^{\mathbf{II}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_2^{\mathbf{I}} \\ \mathbf{u}_2^{\mathbf{II}} \\ \mathbf{u}_3^{\mathbf{I,II}} \\ \mathbf{t}_3^{\mathbf{I,II}} \end{bmatrix} = \omega^2 \begin{bmatrix} \bar{\mathbf{M}}_{22}^{\mathbf{I}} & 0 & \bar{\mathbf{M}}_{23}^{\mathbf{I}} & 0 \\ \bar{\mathbf{M}}_{32}^{\mathbf{I}} & 0 & \bar{\mathbf{M}}_{33}^{\mathbf{I}} & 0 \\ 0 & \mathbf{M}_{22}^{\mathbf{II}} & \mathbf{M}_{23}^{\mathbf{II}} & 0 \\ 0 & \mathbf{M}_{32}^{\mathbf{II}} & \mathbf{M}_{33}^{\mathbf{II}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}_2^{\mathbf{I}} \\ \mathbf{u}_2^{\mathbf{II}} \\ \mathbf{u}_3^{\mathbf{I,II}} \\ \mathbf{t}_3^{\mathbf{I,II}} \end{bmatrix}$$

where $\bar{\mathbf{H}}$, $\bar{\mathbf{G}}$ and $\bar{\mathbf{M}}$ are the BEM modified coefficient matrices, **K**, **T** and **M** are the FEM matrices, $\mathbf{u}_2$ and $\mathbf{u}_3$ are displacements at the free boundary and at the interface, respectively, $\mathbf{t}_3$ are tractions at the interface.

3. Numerical example

Both sides of the plate are symmetrically reinforced by four beams of channel sections as shown in Fig.2. The length and height of the plate is L=H=0.5 *m*, the dimensions of each beam are 20×10×2×2 *mm*, and the other dimensions are l=h=0.4 *m*. The material of the plate (Ω^{I}) and beams (Ω^{II}) is PMMA in plane stress and aluminum, respectively. The values of mechanical properties for these materials are: modulus of elasticity E_I=3.3 *GPa* and E_{II}=70 *GPa*, Poisson's ratio ν_I=0.42 and ν_{II}=0.34, density ρ_I=1180 kg/m^3 and ρ_{II}=2700 kg/m^3.

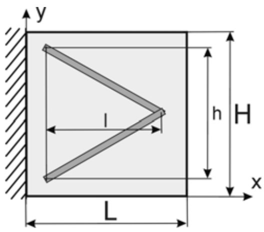

Figure 2. A stiffened plate.

No.	Frequency [*Hz*]		Difference
	BEM/FEM	FEM	[%]
1	398.57	398.86	0.07
2	910.94	897.93	1.45
3	902.06	902.74	0.08
4	1501.0	1500.0	0.07
5	1684.1	1672.6	0.69

Table 1. Results of analysis.

The total number of the boundary and finite elements in the BEM/FEM analysis is 112 (32 elements at the interface) and 64, respectively. The total number of 4-node quadrilateral and beam finite elements in the FEM analysis by the MSC Nastran system is 2604 and 90, respectively.

Table 1 shows the lowest five frequencies computed by the present BEM/FEM formulation and the FEM. A good agreement of the results can be observed, except of 2nd frequency, which corresponds to the longitudinal mode shape, respectively.

4. Conclusions

The coupled DRBEM and FEM is presented in the free vibration analysis of stiffened plates. This approach results in reducing the size of the final system of equations in comparison with the FEM because only the outer boundary and the interface of the body are discretized. Due to simple modification of discretization, the method can be easily used in optimization problems.

5. References

[1] E.L. Albuquerque, P. Sollero and P. Fedelinski (2003). Free vibration analysis of anisotropic material structures using the boundary element method, *Eng. Anal. Boundary Elements*, **27**, 977–985.

[2] D. Nardini and C.A. Brebbia (1983). A new approach to free vibration analysis using boundary elements, *Appl. Math. Modelling*, **7**, 157–162.

[3] R. Górski and P. Fedeliński (2007). Free vibration analysis of non-homogeneous plates by the boundary element method, 17th International Conference on Computer Methods in Mechanics CMM-2007, CD-ROM Proceedings, 4 pages.