EVOLUTIONARY COMPUTING IN MULTI-OBJECTIVE OPTIMIZATION OF LAMINATES

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1. Formulation of the problem

The paper is devoted to the multi-objective optimization of fibre-reinforced, multi-layered laminates. Interply hybrid laminates are considered [1]. External plies of the laminates are made of a stronger and more expensive material while internal plies are made of a weaker but less expensive material.

The aim of the paper is to find the optimal set of ply angles and the number of plies made of particular materials in order to satisfy contradictory criteria. In order to solve a multi-objective optimization (MOO) task, the multi-objective evolutionary algorithm is employed. In the present paper the Pareto attitude to the multi-objective optimization is used [2]. A MOO problem can be expressed as searching for the vector of non-dominated (efficient) solutions **x**, which minimizes the vector of *k* objective functions. The vector **x** is required to satisfy the *m* inequality and *p* equality constrains.

A fibre-reinforced, symmetric hybrid laminates are considered. It is also assumed that laminates are symmetrical - as a result there is no coupling between shell and bending states. The ply orientations (fibre ply angles) and the number of external plies of the laminate are the design variables. Two objective functionals are taken into account:

1. The minimization of the structure cost. It is assumed that the thicknesses of plies h_i, the number of plies *N* and area of the plate A_i are fixed. The dimensionless cost *C* is calculated as follows:

$$C = [n_e c_e + (N - n_e) c_i]\, h_i A_i \tag{1}$$

where: n_e - the number of external plies; c_e, c_i - the unit costs of the external and internal ply materials, respectively $[1/m^3]$.

2. The maximization of the fundamental eigenfrequency:

$$arg \max\{\omega_1(\mathbf{x}); \mathbf{x} \in \mathbf{D}\}. \tag{2}$$

2. Multi-Objective Evolutionary Algorithm

Traditional, typically gradient optimization methods are fast and precise, but usually lead to local optima. To increase the possibility of reaching the global optimum the global optimization methods [4] are employed. Evolutionary Algorithms (EAs) are also very useful if the information about the objective function gradient is hard or impossible to obtain. The only necessary information for the EA to work is the objective (fitness) function value. As EAs work on a population of possible solutions of the problem, the searching is multidirectional. Each possible solution is called a chromosome and it consist of genes. In the real-value coding each gene typically represents one design variable.

To solve presented multi-objective optimization problem the Non-dominated Sorting Genetic Algorithm (NSGA-II) [3] has been used. In order to calculate the objective functions values the boundary-value problem for laminates must be solved. The Finite Element Method (FEM) commercial software has been used to solve the boundary-value problem for laminates.

3. Numerical example

The aim is to find the optimal number of external plies and the optimum values of ply angles to i) minimize the cost of the structure; ii) maximize the 1^{st} eigenfrequenciy. A symmetric rectangular hybrid laminate plate 0.5x0.2m stacked up of 18 plies of the same thickness *h*=0.0002m is considered. The plate is divided into 200 4-node plane finite elements. The material properties and unit costs are: i) for external material M_e: E_1 =181 GPa, E_2 =10.3 GPa, ν_{12} =0.28, G_{12} =7.17 GPa, ρ =1600 kg/m^3, c_e =6.0 1/m^3; ii) for internal material M_i: E_1 =38.6 GPa, E_2 =8.27 GPa, ν_{12} =0.26, G_{12} =4.14 GPa, ρ =1800 kg/m^3, c_e =1.0 1/m^3.

The parameters of NSGA-II are: the population size $p_s = 50$; the number of genes $n_g = 10$; the mutation probability $p_m = 0.1$; the crossover probability $p_c = 0.8$; the number of generations $gen = 100$. Each ply angle could vary in the range of $\langle -90°; 90° \rangle$ every 5°, 15°, 45° or continuously. The results in the form of Pareto solutions are presented in Figure 1.

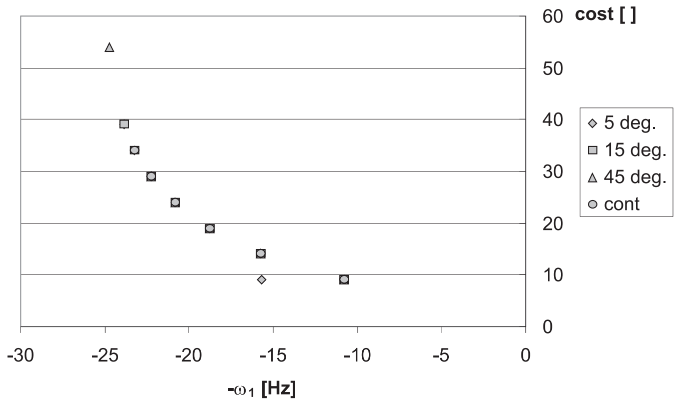

Figure 1. Optimization results for 5^o, 15^o, 45^o and continuous variants.

4. Final conclusions

The optimization of hybrid laminates has been performed. To satisfy contradictory criteria the multi-objective optimization method in the form of multi-evolutionary algorithm has been used. Positive optimization results have been obtained for all considered variants (discrete and continuous).

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6. References

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