A C T A T H E R I O L O G I C A

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Planning of Deer Population Management by Non-linear Programming*

[With 3 Tables]

In order to determine an optimum plan for hunting of deer populations, non-linear programming with a numerical solution on a digital computer was applied. The model given here was developed for a roe-deer population, but it can be also used for other deer species. The model assumes a population at the steady state, with no changes in number from year to year and a stable sex and age group distribution. The aim of the optimization is a maximization of the amount of meat or other profits from the harvested animals, without increase of the population food intake during the winter time. An example of a numerical solution is given here, the practical application of the method is discussed and also the possibilities and prospects of further studies of the problem.

I. INTRODUCTION

Several different management goals for a deer population are possible. In most cases we have to prevent forest damages due to overgrazing, then we can manage the population for a maximum amount of meat yielded from harvested individuals, for the best trophies or for other goals. Deer management should allow obtaining a constant yield from the deer population, therefore population extermination must be excluded and long-term optimization should be applied. Depending on the system of hunting we shall be either nearer to or further from attaining these goals. The aim of this paper is to show how to use information about deer populations in order to determine the optimum system of hunting for obtaining highest yield and avoidance of damage in forests.

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To find an optimum system of hunting the non-linear programming method and computer calculations were applied. The model developed for the programming describes a roe-deer population, but it can be also applied to other deer species, and after some modification to other game animals too.

Ecological studies of the big game mammals are usually difficult and expensive, therefore application of mathematical methods and computers seems especially useful here: a mathematical model allows us to study many different alternatives and to decide which of the empirical data are the most important in order to determine optimum management, and which are of minor importance; it allows us to avoid the expense of unnecessary field investigations. In addition both non-linear and linear programming enable us to use all known data, without rejecting *á priori* some of them.

The only example of a similar study known to the author is the application of linear programming for optimum management of a deer population made by Davis (1970). Davis was concerned with deer management for a period of twenty years and he did not take into account differences due to age among adult individuals. The model given here concerns long term optimization: it was assumed that the population is at the steady state, with no changes in number from year to year and stable sex and age distribution. The application of non-linear programming made it possible to take age differences among bucks and does into account. This seems important, because differences in mortality, reproduction and other parameters among different ages are fairly large. On the other hand, it is rather difficult to determine the age of an individual when hunting it, therefore it was assumed that when harvesting deer we are able to distinguish only three categories of individuals: bucks, does and fawns. Thus, when working out an optimum system of hunting we may take into account full information about the population, with the data for each age separately, but a hunter does not need to distinguish these age groups.

II. THE MODEL

It is assumed that winter and early spring is a critical period for a deer population: all mortality is assumed to occur between the end of the autumn and before the fawns are born the following spring; food shortages can occur only during winters and early springs, and there is a surplus of food in other seasons. If this is so, the hunting season should take place in autumn; earlier hunting interferes with population reproduction and later hunting brings about an unnecessary food intake, at a time of food shortage.

The aim of this investigation is to find out how many bucks, does and fawns should be removed during the hunting season and how many of them left for the winter. Let us therefore define the following activity variables:

X_1 = number of fawns left for the winter,
X_2 = number of fawns removed by hunting,
X_3 = number of bucks left,
X_4 = number of bucks removed,
X_5 = number of does left,
X_6 = number of does removed,

which can assume non-negative values only.

Using the activity variables and data concerning the deer population, it is possible to determine numbers of individuals in every age group within the population at the steady state as follows:

$a_{K,i}$ = number of i-year-old males (*i.e.* those which have survived i winters),
$a_{S,i}$ = number of i-year-old females after the hunting season,
$b_{K,i}$ = number of i-year-old males before the hunting season,
$b_{S,i}$ = number of i-year-old females before the hunting season.

Number of bucks and does which have survived one winter are at summer time given by

$$b_{K,1}=X_1(1-U)(1-M_D),$$
$$b_{S,1}=X_1U(1-M_D),$$

where M_D denotes fawn mortality during winter time, and U is a fraction of females among the fawns which have survived the winter.

After the hunting season the number of one-year-old individuals left for the second winter is

$$a_{K,1}=X_1(1-U)(1-M_D)X_3/(X_3+X_4),$$
$$a_{S,1}=X_1U(1-M_D)X_5/(X_5+X_6).$$

Numbers of two-year-old individuals before the hunting season are

$$b_{K,2}=X_1(1-U)(1-M_D)(1-M_{K,1})X_3/(X_3+X_4),$$
$$b_{S,2}=X_1U(1-M_D)(1-M_{S,1})X_5/(X_5+X_6),$$

where $M_{K,i}$ and $M_{S,i}$ denote mortalities of i-year-old bucks and does. respectively, during their $i+1$ winter of life. Consequently, the numbers of i-year-old bucks and does, before and after the hunting season, are given by

$$b_{K,i}=X_1(1-U)(1-M_D)\left(\frac{X_3}{X_3+X_4}\right)^{i-1}\prod_{j=1}^{j=i-1}(1-M_{K,j}), \tag{1}$$

$$b_{S,i}=X_1U(1-M_D)\left(\frac{X_5}{X_5+X_6}\right)^{i-1}\prod_{j=1}^{j=i-1}(1-M_{S,j}), \tag{2}$$

$$a_{K,i}=b_{K,i}\,X_3/(X_3+X_4), \tag{3}$$

$$a_{S,i}=b_{S,i}\,X_5/(X_5+X_6). \tag{4}$$

The formuale given above are valid only if the sums X_3+X_4 and

$$X_3>0, \tag{5}$$

$$X_5>0. \tag{6}$$

The activity variables can have non-negative values only, therefore from (5) and (6) it follows that the sums given above are different from zero.
X_5+X_6 are not equal zero. As extermination of the deer population is not, in any case, our goal, two conditions can be set, that

Numbers of bucks X_3 and does X_5 left for the winter are sums of individuals of all age groups, after the hunting season, starting from age $i=1$, to the maximum age of the deer, *i.e.*

$$X_3=\Sigma a_{K,i}, \tag{7}$$
$$X_5=\Sigma a_{S,i}. \tag{8}$$

Population food intake during the winter and early spring cannot exceed the amount of the food supply F_T, which the deer can consume without causing damage in the forest, hence

$$F_D X_1 + F_K X_3 + F_S X_5 \leqslant F_T, \tag{9}$$

where F_D, F_K and F_S denote amounts of food intake during the winter and early spring by an individual fawn, buck and doe, respecively, measured in the same units as F_T, *e.g.* in hundredweights of biomass.

To assure proper reproduction of the deer population a sufficient number of bucks should be maintained in the population during the mating season in the summer time. Let S_i denote an average number of does which can be mated by a single i-year-old buck, while Z_i is a fraction of females which require to be mated by males among the i-year-old females. Thus we can assume that all fertile females are mated if the inequality

$$\Sigma b_{S,i} Z_i \geqslant \Sigma b_{K,i} S_i \tag{10}$$

is satisfied.

When every i-year-old doe gives birth to, on the average, R_i fawns, which survive until the autumn, *i.e.* until the beginning of the hunting season, then

$$X_1 + X_2 = \Sigma b_{S,i} R_i. \tag{11}$$

Equations and inequalities from (5) to (11) constitute a mathematical model of the deer population at the steady state. They are conditions which should be satisfied by the activity variables when maximizing the yield from the deer which are harvested during the hunting season. The yield is described by the objective function

$$f = T_D X_2 + \frac{X_4}{X_3 + X_4} \Sigma b_{K,i} T_{K,i} + \frac{X_6}{X_5 + X_6} \Sigma b_{S,i} T_{S,i}, \tag{12}$$

where T_D denotes the value of a harvested fawn, while $T_{K,i}$ and $T_{S,i}$ values of harvested i-year-old buck and doe, respectively.

By substituting formulae from (1) to (4) into the equations and inequalities from (5) to (12) we obtain a set of conditions and the objective function of non-linear programming (Table 1) described by the activity variables and empirical data. The problem of the programming is to find out such a non-negative values of the activity variables, which fulfill all conditions from (5) to (11) and give the maximum value of f, as defined by the formula (12).

Table 1

Non-linear programming model for a deer population: the set of conditions (5')—(11') and the objective function (12'). Explanation of symbols in the body of the paper.

$$X_3 > 0, \tag{5'}$$

$$X_5 > 0, \tag{6'}$$

$$X_3 = X_1 (1-U)(1-M_D) \sum_{i=1} \left(\frac{X_3}{X_3+X_4} \right)^{i} \prod_{j=1}^{j=i-1} (1-M_{K,j}), \tag{7'}$$

$$X_5 = X_1 U (1-M_D) \sum_{i=1} \left(\frac{X_5}{X_5+X_6} \right)^{i} \prod_{j=1}^{j=i-1} (1-M_{S,j}), \tag{8'}$$

$$F_D X_1 + F_K X_3 + F_S X_5 \leqslant F_T, \tag{9'}$$

$$U \sum_{i=1} Z_i \left(\frac{X_5}{X_5+X_6} \right)^{i-1} \prod_{j=1}^{j=i-1} (1-M_{S,j}) \leqslant$$
$$\leqslant (1-U) \sum_{i=1} S_i \left(\frac{X_3+X_4}{X_3} \right)^{i-1} \prod_{j=1}^{j=i-1} (1-M_{K,j}), \tag{10'}$$

$$X_1 + X_2 = X_1 U (1-M_D) \sum_{i=1} R_i \left(\frac{X_5+X_6}{X_5} \right)^{i-1} \prod_{j=1}^{j=i-1} (1-M_{S,j}), \tag{11'}$$

$$f = X_2 T_D + X_1 (1-M_D) \left[(1-U) \frac{X_4}{X_3+X_4} \sum_{i=1} T_{K,i} \left(\frac{X_3}{X_3+X_4} \right)^{i-1} \prod_{j=1}^{j=i-1} (1-M_{K,j}) + \right.$$
$$\left. + U \frac{X_6}{X_5+X_6} \sum_{i=1} T_{S,i} \left(\frac{X_5}{X_5+X_6} \right)^{i-1} \prod_{j=1}^{j=i-1} (1-M_{S,j}) \right] \tag{12'}$$

III. NUMERICAL SOLUTION

The problem of non-linear pr)gramming as shown on the Table 1 has, of course, no analytical solution, but for each set of data a numerical solution has to be found by a digital computer. To obtain this solution a set of data (Table 2) prepared by Mr. B. Bobek from the Department of Animal Genetics and Organic Evolution, Jagiellonian University, was used. These data are partially based on Mr. Bobek's own studies, partially on other sources. They will be probably changed and made more precise during further studies.

The aim of this optimization is a maximization of the amount of meat obtained from the harvested animals, therefore the values of the harvested animals are given by the quite clean weights in kilograms. Heads of bucks are not included with this weight, therefore the $T_{K,i}$ and $T_{S,i}$ values are identical here. No differences in mortality between males and females have so far been found, and for this reason the coefficients of mortality $M_{K,i}$ and $M_{S,i}$ are identical within each age group.

It must be pointed out that the only limit set for the deer population size is given by inequality (9). The maximum value of the objective function f is attained, if the whole food supply F_T is completely used. It implies that the number of individuals, *i.e.* activity variables are linearly related to the value of F_T. If we intend to change the value of F_T, into another value, say F'_T, we can calculate the value of each of the activity variable X', from the equation

$$\frac{x'}{x} = \frac{F'_T}{F_T},$$

where X denotes an arbitrary activity variable calculated using the F_T value. Therefore, when changing the F_T value only, there is no need to repeat the calculations on the computer, as the new values of the activity variables can be calculated by the above given equation.

Using the data from the Table 2 the following solution of the problem described on Table 1 was obtained:

$$X_1=110.9 \qquad X_2=0.3$$
$$X_3=38.8 \qquad X_4=45.2$$
$$X_5=135.7 \qquad X_6=39.2$$

It implies that at the steady state, the maximum yield is attained when every year more than one half of the bucks and about a quarter of does are harvested, and when all fawns are left for the winter. When hunting, according to the activity variables given above, 1394 kilograms of meat should be obtained every year from the harvested deer. This means that the efficiency coefficient E, defined by

$$E=f/F_T$$

equals 1.39.

It is interesting to note that an intuitive explanation can be found for every increase or decrease of the values of the activity variables. For example: it is reasonable to harvest more fawns, because they are not much smaller in weight than the one-year-old adults and as the food intake is limited only during the winter, it is better to remove them before their first winter of life. It is also reasonable to leave more fawns

Table 2

Empirical data for the numerical solution

$M_D=0.15$, $T_D=11$, $U=0.5$

$F_D=3.0$, $F_K=4.6$ $F_S=3.6$ $F_T=1000.0$

i	R_i	$M_{K,i}$	$M_{S,i}$	Z_i	S_i	$T_{K,i}$	$T_{S,i}$
1	0	0.05	0.05	1	2	15	15
2	0.9	0.05	0.05	1	2	17	17
3	0.9	0.05	0.05	1	2	18	18
4	0.9	0.05	0.05	1	2	18	18
5	0.9	0.05	0.05	1	2	19	19
6	0.9	0.05	0.05	1	2	18	18
7	0.9	0.05	0.05	1	2	18	18
8	0.9	0.05	0.05	1	2	18	18
9	0.9	0.10	0.10	1	2	17	17
10	0.9	0.17	0.17	0	2	16	16
11	0	0.20	0.20	0	2	15	15
12	0	0.25	0.25	0	2	15	15
13	0	0.33	0.33	0	2	15	15
14	0	0.50	0.50	0	2	15	15
15	0	1.00	1.00	0	2	15	15

for the winter, because their rather small increase in weight can be balanced by the ability of the older individuals to take part in reproduction. Other similar intuitive reasonings concerning the activity variables are possible, but they cannot supply us with a definite answer: what are the optimum values of these variables? It can be done only by taking into account all empirical data and applying the non-linear programming method.

The optimum solution, *i.e.* the values of the activity variables depends theoretically on all empirical data and on the object of the optimization. Neverthelles, some changes of the data do not influence the activity variables. If we change, for example, the data included in the objective function in such a way that one harvested buck is worth two harvested does or three harvested fawns, so that $T_D=1$, $T_{K,i}=3$ and $T_{S,i}=2$, then we obtain exactly the same solution as that with the values of the animals given by their quite clean weights.

Sex ratio is usually a controversial issue in game management. For this reason calculations for the data from Table 2 were made with different values of the numbers of does, which can be mated by one i-year-old buck, S_i. These calculations (Table 3) show how a change in a single datum influences the activity variables and the coefficient of efficiency: if one buck can mate more does, we may leave a smaller number of bucks for every winter and the food supply formerly taken by them can be used for the larger number of does left, this in turn increases the number of offspring, therefore part of the fawns can be harvested

Table 3

Activity variables and efficiency coefficient E as determined by the different values of S_i

S_i	X_1	X_2	X_3	X_4	X_5	X_6	E
0.5	45.0	4.8	139.9	6.3	61.5	15.4	0.42
1	78.1	0.9	91.0	27.9	96.4	27.5	0.95
2	110.9	0.3	38.8	45.2	135.7	39.2	1.39
3	127.5	1.4	11.1	53.6	157.4	44.9	1.60
4	120.4	22.1	0.1	51.2	177.4	40.0	1.69
8	115.3	29.5	0.0	49.0	181.7	37.3	1.69

every year. Together with the increase of S_i, there is an increase in the efficiency E, which means that with the same population food intake it is possible to obtain a higher yield of harvested deer. When S_i is very high, then all bucks can be harvested every year, as the mating is done by one-year-old bucks, which according to the data (Table 2) are already able to mate.

IV. REMARKS ON THE FIELD APPLICATION OF THE DESCRIBED SYSTEM OF HUNTING

The model given here is simplified: neither random events, nor individual variations among the same age and sex groups were taken into account; it was assumed that mortality, reproduction and food supply are the same every year; it was also assumed that within the three categories which are distinguished by hunter the deer are harvested at random. Nevertheless, it seems that the method described here should be applied for deer management, as it is better to use even a less precise method, than to make a plan of hunting without applying the information on the deer we already possess.

When applying the method described here, we have to take into account changes in population mortality and reproduction from year to year. If we kill X_2 fawns, X_4 bucks and X_6 does every year, without knowing how many of them actually live in the field, we can easily either exterminate the population or allow its unlimited increase. It can be avoided by making a population census every year before the hunting season in order to determine the actual number of fawns N_D, bucks N_K and does N_S. It should be noted that the method given here cannot be applied, even with the most precise data, when the actual number of deer in the field is not known. If we know the actual numbers, then the numbers of fawns, bucks and does which should be harvested are given by the differences N_D-X_1, N_K-X_3 and N_S-X_5, respectively. If it is a good year we may harvest more deer than predicted by the model, if it is a bad one — fewer deer are harvested, but by leaving the same number of animals every year, we avoid both population extermination and unlimited population growth.

As shown above, the only activity variables used are X_1, X_3 and X_5. The other variables X_2, X_4 and X_6 should be used to verify the empirical data: if the sums X_1+X_2, X_3+X_4 and X_5+X_6 are persistently different from the actual numbers of the animals in the field N_D, N_K and N_S, respectively, it means that there are errors in the empirical data. These errors should be detected, because wrong data give activity variables different than the optimum ones, so that the yield obtained can be lower than the maximum possible.

Everything written above refers to a deer population at a steady state, and this steady state is perpetuated by leaving X_1 fawns, X_2 bucks and X_5 does every year. In a deer population into which the method is just introduced the number of individuals in every age and sex group can be much different than this given by the model. In this case neither the way of checking the data by comparing activity variables with the actual number can be used, nor we can expect the amount of yield as predicted by the model. If the same number of fawns, bucks and does is left every year for winter, then theoretically a steady state and stable age distribution will be attained by the population after 15 years, which is the maximum age of the roe deer. Practically, the old individuals usually form only a small part of deer populations, therefore the steady state as described by the model will be attained after a few years. It may take longer if the actual numbers N_D, N_K and N_S are much smaller than the activity variables X_1, X_3 and X_5, respectively, as it takes time for a population to attain the size described by the model.

As suggested above, transition of the deer population from the actual state to the state described by the model can be achived by leaving X_1 fawns, X_3 bucks and X_5 does every year, or the actual numbers N_D, N_K and N_S, if they are smaller than the respective activity variables. If we know the actual number of deer in every age and sex group and we have good data concerning the population, we can predict changes in the population and the amount of yield from the harvested animals. On the other hand, such a transition may not be an optimum one. Optimization of this transition is a problem in dynamic programming and it lies beyond the scope of this paper.

Practically, it is possible to optimize this transition without applying a special program. For example: if number of bucks N_K is much smaller than X_3, but N_D is higher than X_1 and N_S is higher than X_5, then it does not make sense to reduce the numbers of fawns and does to the levels X_1 and X_5, respectively, because in this way the population does not completely use its food supply. It is better to use the surplus food by leaving more fawns and does, which when harvested next year would have higher weights; also among the fawns there are some males which in the next year will be able to mate with surplus females. We may reduce the numbers of deer to the levels X_1, X_3 and X_5 in the following years.

It is important to note here that according to the model harvesting among fawns, bucks and does is random and in particular is independent of the age of bucks and does. If an old individual has a higher probability of being killed by a hunter than a young one, then we can expect a higher yield than that predicted by the model; the reverse is true when a young individual has a higher probability of being killed. This is due to smaller mortality, higher reproduction rates and larger increase in weight of young individuals. As it was assumed that a hunter can distinguish only three categories, the non-random harvesting within these categories has to be considered as an inaccuracy of the empirical data.

V. PROSPECTS OF FURTHER INVESTIGATIONS

When applying the method described here for deer management, the most difficult and expensive task is to collect the empirical data, which are required for finding the optimum values of the activity variables. It is possible to reduce expenses by finding out which of the data are the most important in determining the values of the activity variables.

For example: the activity variables are only little influenced by the values of harvested animals. If we set $T_D=1$, $T_{K,\ i}=3$ and $T_{S,\ i}=2$, instead of the values given in Table 2, we have received no changes in the activity variables, as shown above. The values of harvested animals are given here by their quite clean weights, it is therefore unnecessary to spend time and money in obtaining more precise weights of harvested animals, because they cannot change the activity variables.

Another example is given in Table 3. If we do not know whether one buck can mate with one doe or two does, it is necessary to find this out, as it makes 50 percent increase in the efficiency coefficient E. On the other hand, if we are not sure whether one buck can mate with four or eight does, it is not worth while investigating this because efficiency E is the same in both cases.

It is possible to examine all the data in respect of their influence on the activity variables and the efficiency coefficient. It is a rather extensive subject, which will be further studied.

As has already been pointed out, the model given here is a simplified one. There are the following possibilities of making it more complex and more realistic at the same time:

(1) It was stated above that the population food intake should not exceed a certain value F_T, without taking into account the fact that damage, in forest for example, is a continuously increasing function of the population food intake. It seems that this function increases slowly at the beginning and exhibits rapid increase at higher values of the food intake. Therefore, the amount of food intake should not be decided *á priori*; we should estimate the damage in units, the same as those used for the yield harvested from the population, then such optimum values of F_T and activity variables by which our profits from the yield minus losses from damage reach their maximum value.

(2) It was assumed that when the males to females ratio in summer exceeds a certain value, all fertile females are mated. But it seems that the fraction of the mated females is an increasing function of the males to females ratio. This function increases linearly at the low values of the ratio, and then at high values its increase becomes slower. It is possible that some fertile females are not mated, even if the males are superabundant, if so, a further increase in the number of males brings about losses without any effect on reproduction. Thus, it does not make sense to assume a certain sex ratio, but we should rather use the function mentioned above for determining an optimum sex ratio in respect of our goals.

(3) It is generally well known that reproduction and mortality within animal populations, especially among mammals, are determined to a high

degree by the population social structure and other intrapopulation processes. These structures and processes were not included into the model, as so far there are no good quantitative data describing them within deer populations. Nevertheless, it seems important to include the above mentioned intrapopulation phenomena in the model, and this may prove possible after making further field studies on the roe deer populations.

(4) It also seems important to introduce genetic variabilities within the population into the model. It is especially important when our goal is to obtain the best trophies. Unfortunately there are as no good quantitative data concerning heredity and selection of deer trophies, but this should be considered as a subject for future studies.

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PLANOWANIE GOSPODARKI ŁOWIECKIEJ W POPULACJACH SAREN METODĄ PROGRAMOWANIA NIELINIOWEGO

Streszczenie

Opisano tu zastosowanie metody programowania nieliniowego, z rozwiązaniem numerycznym na maszynie cyfrowej, dla ustalenia optymalnego planu corocznych odstrzałów w populacjach saren i innych pokrewnych gatunków. Optymalizacja tego planu ma na celu uzyskanie największej ilości tuszki lub innych pożytków z osobników odstrzelonych, przy ograniczonej ilości pokarmu pobranego przez

populację w okresie zimy. Dotyczy ona populacji w stanie równowagi, przy stałym rozkładzie grup płci i wieku. Zakłada się, że w czasie odstrzałów odróżniane są tylko trzy kategorie osobników: koźlęta, kozły i siuty. Opisana metoda ma za zadanie ustalić ile osobników, w każdej z tych kategorii należy pozostawić na zimę, ile zaś odstrzelić; zdefiniowano zatem następujące zmienne decyzyjne określające liczby: X_1 — koźląt pozostawionych, X_2 — koźląt odstrzelonych, X_3 — kozłów pozostawionych, X_4 — kozłów odstrzelonych, X_5 — siut pozostawionych, X_6 — siut odstrzelonych.

Dane empiryczne, w oparciu o które przeprowadzono optymalizację, zebrano w tabeli 2 i oznaczono następującymi symbolami: M_D — śmiertelność wśród koźląt podczas pierwszej zimy ich życia, $M_{K,\,i}$ i $M_{S,\,i}$ — śmiertelność wśród i-letnich kozłów i siut, odpowiednio, podczas ich $i+1$ zimy życia, U — frakcja samic wśród osobników, które przeżyły jedną zimę, F_D, F_K, F_S — ilości pokarmu pobranego w okresie zimy przez jedno koźlę, kozła i siutę, odpowiednio, F_T — ilość pokarmu, które zużyć może w czasie zimy cała populacja, bez powodowania szkód w łowisku, R_i — średnia ilość osobników urodzonych z jednej siuty w wieku i, które dożywają do jesieni, Z_i — frakcja samic biorących udział w rui, wśród wszystkich i-letnich samic, S_i — liczba samic, które może zapłodnić jeden samiec w wieku i. T_D — wartość tuszki (podana w kg) koźlęcia, $T_{K,\,i}$ i $T_{S,\,i}$ — wartości tuszki, odpowiednio kozła i siuty, w wieku i.

Tabela 1 zawiera równania i nierówności (5′)—(11′), określające warunki, które winny być spełnione przez zmienne decyzyjne przy maksymalizacji ilości uzyskanych pożytków, opisanych funkcją celu f (12′). Optymalne rozwiązanie dla danych z tabeli 2 dane jest następującymi wartościami zmiennych decyzyjnych: $X_1=110.9$, $X_2=0.3$, $X_3=38.8$, $X_4=45.2$, $X_5=135.7$, $X_6=39.2$. Przy zastosowaniu innych danych empirycznych, na przykład przy zmianie wartości S_i, która określa ile siut może zapłodnić jeden kozioł, uzyskujemy inne wartości zmiennych decyzyjnych i wydajności $E=f/F_T$ (Tabela 3).

Przy wprowadzaniu opisanej tu metody ustalania wielkości odstrzałów do praktyki łowieckiej, trzeba wziąć pod uwagę duże uproszczenia modelu, na którym metoda ta jest oparta. Aby zapobiec eksterminacji populacji lub jej nadmiernemu wzrostowi należy corocznie pozostawiać na zimę w łowisku X_1 koźląt, X_3 kozłów i X_5 siut. Pozostałe zmienne decyzyjne winny być używane dla sprawdzenia dokładności zastosowanych danych, ponieważ gdy populacja jest w stanie równowagi sumy X_1+X_2, X_3+X_4 i X_5+X_6 powinny być zbliżone do liczby koźląt, kozłów i siut, odpowiednio, przed okresem polowań. Z praktycznym zastosowaniem opisanej tu metody łączy się sprawa przejścia populacji od dowolnego stanu, zastanego w momencie wprowadzania tej metody, do stanu równowagi oraz sprawa losowego odstrzału w obrębie wyróżnionych kategorii.

Opisana tu metoda może być bardzo przydatna dla ustalenia, które z danych empirycznych mają największe znaczenie dla ustalenia optymalnego planu odstrzałów (porównaj Tabelę 3, która opisuje wpływ stosunku płci na wydajność E i wartości zmiennych decyzyjnych), a co za tym idzie, które powinny być najbardziej szczegółowo badane.

Uprecyzyjnienie modelu i dalsze jego badanie powinno iść w kierunku zastąpienia wartości F_T i S_i odpowiednimi funkcjami ciągłymi, uwzględnienia regulacji wewnątrzpopulacyjnej oraz włączenia do modelu zjawisk genetycznych.