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A MODIFIED GEOMETRIC DISTRIBUTION ARISING IN TRAPPING STUDIES

ZMODYFIKOWANY ROZKŁAD GEOMETRYCZNY ODŁOWÓW

In a series of studies of the length of residence of small mammals in defined study areas, Andrzejewski & Wierzbowska (1961) and Wierzbowska & Petrusewicz (1963) present data of the number of animals that remain in the area for 0, 1, — — complete weeks. They found that the complete data was not well fitted by an exponential distribution, since there were too many observations in the zero class, which they called "ephemeral" animals. However if that class were omitted, the remaining data was well fitted by an exponential distribution with its zero class truncated, which is again an exponential distribution. In the latter paper in particular, this is made the basis of a suggestion that the ephemeral animals do not form a homogeneous group, but are made up partly of "resident" mice who by chance have only stayed in the study area for less than a week, and partly of "migrants" whose nature is not to settle in the area. The number of the former were estimated by extrapolating the successfully fitted exponential distribution.

Trapping took place weekly, and the length of residence of an individual was measured by the interval between its first and last capture. Petrusewicz & Andrzejewski (1962) point out that since not all the animals were caught on every trapping occasion, this must underestimate the true length of residence. In this note I show that this feature of the recording could account for the excess of animals in the zero class.

Let the random variables T and X respectively denote the true period of residence of an individual in the study area, i.e. the interval between the first and last occasions when it was exposed to capture, and the recorded period of residence, i.e. the interval between the first and last occasions it was actually captured. The random interval between the animals entry to the area, and the first trapping occasion may be ignored mathematically. I have preferred to use the discrete geometric distribution as a model for the true period of residence, rather than the exponential.

$$\Pr(T = t) = (1-k)k^t \qquad t = 0,1, \_\_ \tag{1}$$

k is the probability that the animal does not leave the area at some time during a period between trappings. If the continuous exponential distribution is used, the data should strictly be considered as a grouped sample, and the maximum likelihood estimator obtained by the methods discussed by Kulldorf (1961, ch. 2).

According to Petrusewicz & Andrzejewski (1962), the probability that an animal is caught on a given occasion remains fairly constant. Denote it by p, and let $q = 1 - p$ be the chance of evading capture. An animal that remains in the area for t complete weeks, and is therefore exposed to capture on $t + 1$ occasions, has a probability of not being caught at all, and consequently X being undefined, given by

$$\Pr (X \text{ undefined} \mid T = t) = q^{t+1} \tag{2}$$

The probability that it will be caught only once, and consequently recorded as ephemeral is

$$\Pr (X = 0 \mid T = t) = (t + 1) pq^t \tag{3}$$

The probability that it will be caught first on occasion i, and for the last time on occasion $i + x$, counting the occasions it was exposed to capture as $0, 1, \_\_, t$ is $q^i p \cdot pq^{t-i-x} = p^2 q^{t-x}$, and i can take the values $0, 1, \_\_, t - x$. Hence

$$\Pr (X = x \mid T = t) = (t - x + 1) p^2 q^{t-x} \quad x = 1, 2, \_\_ \tag{4}$$

On multiplying (2), (3) and (4) by (1) and summing, the marginal distribution of X is obtained.

$$\Pr (X \text{ undefined}) = (1-k)q \sum_{t=0}^{\infty} (qk)^t = \frac{(1-k)q}{1-qk} \tag{5}$$

$$\Pr (X = 0) = (1-k)p \sum_{t=0}^{\infty} (t+1)(qk)^t = \frac{(1-k)p}{(1-qk)^2} \tag{6}$$

$$\Pr (X = x) = p^2 q^{-x}(1-k) \sum_{t=x}^{\infty} (t-x+1)(qk)^t = \frac{p^2(1-k)k^x}{(1-qk)^2} \quad x = 1, 2, \_\_ \tag{7}$$

If (6) and (7) are divided by one minus the probability (5),

$$\begin{aligned} \Pr (X = 0 \mid X \text{ defined}) &= \frac{1-k}{1-qk} \\ \Pr (X = x \mid X \text{ defined}) &= \frac{(1-q)(1-k)k^x}{1-qk} \quad x = 1, 2, \_\_ \end{aligned} \tag{8}$$

The distribution (8) is geometric except that its first term is modified. It depends on two parameters k and q, with physical interpretations in the present case, and has mean and variance:

$$\mathrm{E}x = \frac{k(1-q)}{(1-k)(1-qk)}, \quad \mathrm{var}\ X = \frac{k(1-q)(1-qk^2)}{(1-k)^2(1-qk)^2}$$

Distributions of this type occur in several branches of population dynamics (Kendall, 1949, p. 238; Moran, 1962, p. 14), although the parameterisation is appropriate to the particular problem. At the end of the period of recording, the animals still in the area will be ascribed a duration of residence which is too short, thus biassing the results, but if the whole study period is long compared with the average residence, this effect will be negligible and I have ignored it.

If a sample of n is taken from this distribution, of which n_x have value x, the likelihood is

$$L = \left(\frac{1-k}{1-qk}\right)^n (1-q)^{n-n_0} \prod_{i=1}^{\infty} k^{n_x x}$$

$$\log L = n\left\{\log(1-k) - \log(1-qk)\right\} + (n-n_0)\log(1-q) + n\bar{x}\log k$$

On setting the derivatives of log L with respect to k and q equal to zero, and solving the resulting equations, the maximum likelihood estimators are found to be

$$\hat{k} = 1 - \frac{\Phi}{\bar{x}}, \quad \hat{q} = \frac{\hat{k} - \Phi}{\hat{k}\,\Theta}$$

where Θ and Φ denote the sample proportions of zero and non-zero observations. The same estimators are obtained by equating the sample mean and proportion of zeros to the population values. The variance-covariance matrix conditional on n fixed, as $n \rightarrow \infty$, is obtained by inverting the matrix of expectations of negative second derivatives of log L, and is

$$\mathrm{var}\ \hat{k} = \frac{1}{n}\ \frac{(1-k)^2(1-qk)}{1-q}$$

$$\mathrm{var}\ \hat{q} = \frac{1}{n}\ \frac{(1-q)(1-qk^2)(1-qk)}{k^2(1-k)}$$

$$\mathrm{cov}\ (\hat{k}, \hat{q}) = \frac{1}{n}\ \frac{(1-k)(1-qk)}{k}$$

Applying these results to the data given by Wierzbowska & Petrusewicz (1963), the estimates of the parameters and their sampling standard deviations are, for the attic population

$$k = .9027 \pm .0069, \qquad q = .7524 \pm \cdot 039$$

and for the house population

$$k = .9105 \pm .0059, \qquad q = .8559 \pm .019$$

The goodness of fit is the same as that for the geometric distribution fitted to the data omitted to the data omitting the zero class, since the method of estimation results in the zero class being fitted exactly.

The above theory does not of course show that the ephermeral animals are not a mixture of two populations. It merely shows that the excess in the zero class could have arisen from the sampling method even if the population had been homogeneous.

SUMMARY

If the number of weeks of residence of an animal in a given study area has a geometric distribution, then the length of residence as recorded by a trapping system which does not catch all the animals on every occasion follows a modified geometric distribution. Estimators of the parameters of this distribution are obtained, and applied to the data given by Wierzbowska & Petrusewicz (1963).

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