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An Empirical Fuzzy Model for Maximizing Profit in Electricity Spot Market

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Abstract—**This paper discusses the use of context based fuzzy optimization approach to maximize profit in the context of joint quantity-price bidding in today's deregulated electricity market. The proposed method is innovative because it utilizes sets of empirical (market) data that reflect functional dependence between the input and output variables. The paper describes the structure of the fuzzy system and its components: the context variables, the input membership function and the output membership function, as well as the use of aggregation operators in aggregating the context variables. The paper also presents a numerical example to demonstrate the application of the proposed method and compare the result with analytical solution.**

Index Terms—**Aggregating operators, Deregulated electricity market, Fuzzy optimization, Strategic bidding**

I. INTRODUCTION

THE introduction of competition in the electricity industry is aimed to improve efficiency in production, transmission and consumption of electrical energy. Restructuring is also intended to attract players and investments in the markets as well as to ensure competitive electricity price. Electricity industry reform has been the theme throughout electricity markets around the globe since 1990s.

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The Australian National Electricity Market (NEM) has undergone restructuring progressively since 1998. The market consists of inter-connected electricity networks on the Eastern side of Australia, including Australian Capital Territory, New South Wales, South Australia, Victoria, Queensland and Tasmania. Generating companies (gencos) in each power network region produce electrical energy traded into the spot "pool" whereby the spot price for electricity is calculated by the central dispatch operator. Gencos compete by providing dispatch offers while market customers submit dispatch bids, detailing the price and demand quantity to be supplied. The central operator determines which gencos are required to satisfy demand at what time, and at what production level. The objective of this scheduling process is to offer the supply quantity while minimizing the cost in meeting the demand based on the offer and bid prices. The clearing price to match supply and demand (or the spot price) is calculated using the bid stacking method [1]: the central operator stacks the cumulative supply offers in increasing price order against decreasing price stack of demand-bids.

Previous research [2], [3] suggested that an optimal strategy to bidding in a perfectly competitive market is bidding at marginal costs. In real life operation, such a strategy is not necessarily profitable for the suppliers. The electricity market is not perfectly competitive, but closer to an oligopoly system. Because of the rather large market shares, generators can exercise market power. A supplier can withdraw some of its capacity from the market but gain more profit since its offer bid can outweigh its loss of market share. The success of a generator in exercising its market power depends on an accurate assessment of market conditions such as load forecasts, network constraints and the bidding behavior of rival generators. In some market mechanisms, a generator must internalize its dispatch scheduling and unit commitment in a bid formulation. Optimal bidding strategies are fundamental to the survival of generators in a competitive electricity market. There have been a number of studies of optimal bidding strategies in competitive electricity markets: using residual demand analyses [4]-[10], using estimation of market parameters by probability distribution functions [11]-[15], Dynamic Programming [16]-[18], Genetic Algorithm [19]-[26] and other heuristic approaches [27]-[31]. This paper introduces a systematic bidding strategy for gencos in formulating its daily bid using a fuzzy methodology. The proposed strategy is innovative because it is able to handle uncertainty in market parameters, such as load demands and expected trading prices, using membership functions and aggregating operators.

II. Context based fuzzy system

A. *Context-based Fuzzy System*

Up to date the literature presents use with two common approaches to develop models, namely the mathematical model and the empirical models. Each method has its own advantages and disadvantages, and thus is suitable for different type of problems and environments. For modeling the electricity spot market, we propose the empirical approach to develop the market model because the market is a complex system featuring non linear characteristics. Mathematical models cannot capture the underpinning factors without drawing many limiting assumptions. Often simplified model, such as using linearised constant for loss factor at nodal points, is not desirable because the accuracy of the system suffers tremendously, not to mention the impact of such model on pricing issue and other aspects in the market operation.

A context based fuzzy approach has been introduced in [32]. Unlike conventional fuzzy systems which are based on fuzzy rules, the context based fuzzy system extracts implicit rules from the market data and uses the empirical data to define the functional dependence between the input and the output. Each data set serves as a rule that carry certain weight depending on the relevant context. In effect, we do not need to compute large data sets because only selected data sets defined by the specified context are relevant for processing. The input to the contextual fuzzy system consists of context variables denoted by X_k and input variable U. The context variables serve as the means to filter the data sets according to pre-defined context while the input variable serves as the independent variable to determine the output. Fig. 1 illustrates the proposed context based fuzzy approach to determine bidding strategy of gencos in the deregulated electricity market.

In short, the proposed context based fuzzy system has been designed to extract meaningful relationship between the input and the output variables using the empirical data set. The retrieval process involves selection of data sets based on their degree of relevance to a given context. This context may be defined by more than one context variable.

B. Context Membership Function

Context variables allow some leeway to describe uncertainty surrounding a particular variable by means of context membership function. For example, in specifying the temperature of the day for the purpose of predicting the load demand and formulating supply bid curve, "temperature is about 35 degree Celcius" is more precise than "high", "medium" or "low" temperatures. In some cases, the uncertainty can be statistically estimated using normal (Gaussian) probability density functions that can be transformed directly into Gaussian-type membership functions. The transformation can be done easily because both functions employ the same parameters: mean value (center), and standard deviation (sigma). A Gaussian type membership function can be expressed as a normalized normal probability density function: a Gaussian type membership function always has a maximum value of one while a normal probability density function always has a total area of one. So, in the above example, the temperature variation expressed using the word "about" can be quantified into Gaussian type membership functions with peak (mean) value of 35 degree Celcius.

In short, context variables assign values between zero and one to each element of empirical data based on its adherence to a given context. These weights are set by membership functions of the context variables. Data that adhere more closely to the given context will have more weights than those that are less relevant. Fig. 2 illustrates the extraction of relevant data from the empirical data set based on a context membership function *Around-5*. The empirical data consists of 100 data points that are uniformly distributed between 0 and 10. The context variable *Around-5* is defined by a Gaussian type membership function with a center of 5 and a sigma of 0.5. The weighted data, which is filtered by the context membership function *Around-5*, is shown in Fig. 2(c).

In some cases, the object to be modeled could be time-varying. Therefore, a time-based context variable, in which more recent data has more influence than older data, can be added to make the contextual fuzzy system adaptive to a time-varying system. Addition of new data to the model is very straightforward since it only requires re-filtering data sets by the context membership functions.

A. Aggregating Context Variables

Several context membership functions can be aggregated using fuzzy connectives. A range of fuzzy connectives with different degree of compensation has been documented in [33]. The aggregation operators include all triangular norms (AND operators) and triangular conorms (OR operators). The selection of which aggregation operation to use depends on the context. A set of context variables can be combined into a single context by an aggregating operator. For example, combining context variables X_1, X_2,..X_n defined by AND method can be expressed as follows:

$$W = X_1 \cap X_2 \cap \ldots \cap X_k$$

The AND method can be implemented by using triangular norms such as logical product, Hamacher product, algebraic product, Einstein product, bounded product or drastic product. Each of these operators has different degree of compensatory. It was found in [33] that the logical product has the greatest degree of compensatory while the drastic product produces least degree of compensatory. The choice of fuzzy operator is usually determined on a per-case basis. For example, if the algebraic product operator is used in place of the AND method, then the weight assigned by the aggregate context w to the i^{th} data set is given as:

$$w_i = \prod_{k=1}^{M} \mu_{Xk}(x_{k,i}) \quad \text{for } i = 1, 2, \ldots N \qquad (1)$$

where μx_k is the membership function of the context variable x_k; $x_{k,i}$ is the i^{th} data for the context variable x_k; M is the total number of context variables; and N is the total number of data sets. The result of the aggregation is normalized in order to have a normal fuzzy set.

It is noted that the function of context variables is to define fuzzy relations between the inputs and outputs by weighting the data sets according to their adherences to a pre-defined context. In some cases, however, the context can be too stringent; as a result, there are not enough activated data sets to define the input-output relationship clearly. In order to activate more data sets, the context membership functions should be relaxed. Intuitively, the context membership functions that have weaker correlations with the output are relaxed more than those that have stronger correlations. This relaxation will allow for more data sets to be activated at a given context so that the system becomes more robust.

A coefficient of correlation which measures a linear association between two variables can be used as an indicator of the strength of the correlation between the context and the output variables. It is noted that the function may not be necessarily linear, but it must be linear in the context variables. The standard correlation coefficient between the context variable x_k and the output variable y is given as:

$$r_k = \frac{\sum_{i=1}^{N}(x_{k,i}-\bar{x}_k)(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{N}(x_{k,i}-\bar{x}_k)^2}\sqrt{\sum_{i-1}^{N}(y_i-\bar{y})^2}} \quad k=1, 2,.. M \qquad (2)$$

where $x_{k,i}$ is the i^{th} element of the context variable x_k; $\bar{x}_k$ is the mean of the context variable x_k; y_i is the i^{th} element of the output variable; $\bar{y}$ is the mean of the output variable; N is the total number of data sets; and M is the number of context variables. It is suggested in [34] that, in order to have a stable correlation coefficient (that is, a small standard error), the sample size must be greater than 30.

For the context variable x_k, a coefficient of relaxation denoted by ρ_k is defined as:

$$\rho_k = \left(\frac{r_k}{\max\{r_1, r_2,..r_M\}}\right)^{\beta} \text{ for } \beta \geq 1 \qquad (3)$$

where β is the relaxation factor that modifies the relaxation coefficient at different strengths. It is noted that the correlation coefficients are normalized so that the context membership function which has strongest correlation is not relaxed (that is, $\rho_k = 1$).

Given the relaxation coefficients, the context membership functions that are relaxed using a fuzzy hedge *More or Less* and is defined as:

$$\mu_{\tilde{X}k} = (\mu_{Xk})^{\rho_k} \qquad (4)$$

where $\mu_{\tilde{X}k}$ is the relaxed membership function of the context variable x_k; μx_k is the membership function of the context variable x_k; and ρ_k is the relaxation coefficient for the context variable x_k.

In some cases, there are functions that have a nonlinear relationship between the context variables and the output. Since a correlation coefficient defines a linear association, it cannot be used directly to measure data sets that have a non-linear relationship. Therefore, the calculation of the coefficient must be done locally where the non-linear function is locally approximated by a linear function. It is done by calculating the correlation coefficient only on data sets with weights above a given threshold value.

The local correlation coefficient $\hat{r}_k$ for the context variable x_k is defined as:

$$r_k = \frac{\sum_{i=1}^{N} v_i (x_{k,i} - \bar{x}_k)(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N} v_i (x_{k,i} - \bar{x}_k)^2} \sqrt{\sum_{i=1}^{N} v_i (y_i - \bar{y})^2}} \quad k=1,2,..M \quad (5)$$

where

$$v_i = \begin{cases} 1 : w_i \geq \alpha \\ 0 : w_i < \alpha \end{cases}$$

and w_i is the weight assigned to each data set by the aggregate context membership function, and α is the minimum support level of the membership function in order to activate the data sets.

This is based on the assumption that, in the sub-space defined by the context membership functions, the function is relatively smooth and can be approximated by a linear function.

B. Input Membership Function

The input Membership Functions (MF) can be of any type as long as they are symmetric and have a single maximum point (for example, Gaussian or triangular). In general, however, Gaussian-type membership functions are preferred since they are more natural in handling empirical data.

The width of the membership function depends mainly on the smoothness of the function being approximated and the density of data at a region of interest. For example, a wide membership function is not suitable for approximating a function with high nonlinearity as it tends to over-generalize the function. On the other hand, a narrow membership function may fail or over-fit the function in a region with a low density. In general, for a region with a high density of data, the width of the input membership function could be made relatively narrow. In addition, in a region with low density, the width must be sufficiently wide to have an adequate coverage of the empirical data.

A method to determine the width of the input MF is to use a standard sigma-count, which measures the coverage of a membership function. Hirota and Pedrycz [35] used the standard sigma count to serve as a plausible measure of granularity that effectively summarized the number of elements embraced (at least partially) by a given fuzzy set. For normal fuzzy sets, the standard sigma count over a continuous variable x is defined as:

$$\sigma(A) = \int_X \mu_A(x)dx \qquad (6)$$

where A is the fuzzy set defined over a universe of discourse X. Similarly, the sigma count of the input membership function over a set of input data (discrete) is defined as:

$$\sigma(U) = \sum_{i=1}^{N} \mu_U(u_i) \qquad (7)$$

where U is the input fuzzy number, u_i is the i^{th} input data, and N is the number of rows.

In the proposed fuzzy model, however, the empirical data is filtered by the context membership function. Therefore, the coverage of the input membership function must be measured only on the filtered data, not on all data. Accordingly, the standard sigma count is modified and given as:

$$\tilde{\sigma}(U) = \sum_{i=1}^{N} \min\{\mu_U(u_i), w_i\} \qquad (8)$$

where $\widetilde{\sigma}$ is the modified sigma count, and w_i is the weighting factor as defined by the context variable W for the i^{th} data.

It is noted that the minimum operator (also known as a logical product operator) is used here since the modified sigma count measures the absolute coverage of the empirical data by both the input and context membership functions.

In order to have a balanced support on both wings, the modified sigma count is calculated at three parts: $\widetilde{\sigma}(U)$, $\widetilde{\sigma}(U_{left})$ and $\widetilde{\sigma}(U_{right})$. It is noted that U is the input fuzzy number; U_{left} and U_{right} are the left and right wings of the input fuzzy number (as shown in Fig. 3). The minimum support on each wing is set to a certain value (for example, 10% of the total sigma count). This guarantees that the coverage of the input membership function on the empirical data is not one-sided but supported on both sides. Additionally, in order to handle input near boundaries where one-sided coverage is inevitable, an additional rule is added to reduce the threshold values on the wing close to the boundaries so that the width will not be unnecessarily wide.

In order to handle data with a non-uniform probability density function, an adaptive mechanism with a set of rules is introduced to adjust the width of the input membership function. Initially, the width is set to the minimum and then gradually expanded until the membership function has a sufficient coverage of the empirical data. The process of adjusting the width of the input membership function is summarized in the following steps:

1. *Initialization:* the width is initialized with a pre-defined minimum value.
2. *Evaluation:* the coverage of the input membership function is evaluated on three parts: total, left and right, using the modified sigma count given in (8). Then they are checked against a set of threshold values: minimum total sigma count, minimum sigma count on the left wing, and minimum sigma count on the right wing.
3. *Expansion:* If any one of the three conditions is not satisfied and the current width is still less than the

maximum value, then the width is expanded by a given percentage (for example, 10%) and Step 2 is repeated.

To test the robustness of the proposed adaptive input membership function, a fuzzy model is built to approximate a nonlinear function based on unevenly-spaced empirical data. The nonlinear function is given as:

$$y = 4 \sin(10\ln(u)) + 10 : u \text{ in } [1,10] \qquad (9)$$

where the empirical data for u is a row vector of fifty logarithmically-equally-spaced points between decades 10^0 and 10^1.

Fig. 4 shows the performance of fuzzy systems for three different kinds of input membership functions: narrow, wide and adaptive. Each was evaluated based on the measured absolute errors (as shown in Fig. 4b), indicating how well they approximated the nonlinear function. The wide membership function performed well only in the region with low frequency of changes, but over-generalized the function in the region with high frequency of changes. On the contrary, the narrow membership function performed well in the region with high density of empirical data but over-fitted the function in the region with low density of empirical data. The adaptive membership function overcame these problems by properly adjusting its width based on the level of density of the empirical data in the region of interest. As indicated by smaller absolute errors, the proposed adaptive membership function outperformed both narrow and wide membership functions.

C. Output Membership Function

The output of the contextual fuzzy system is a membership function constructed from the empirical data y weighted by the context **w** and the input $\mu_U(\mathbf{u})$. In other words, the output membership function can be viewed as a projection of the input membership function to a stochastic function represented by a set of input-output data filtered by the context variable. This approach is similar to translating a fuzzy number through a fuzzy relation using *the compositional rule of inference* [36].

If the output membership function is chosen to be of Gaussian-type, then the parameters can be estimated using the weighted normal fitting. Consequently, the center is defined as:

$$Y_{center} = \frac{\sum_{i=1}^{N} y_i . z_i}{\sum_{i=1}^{N} z_i} \qquad (10)$$

and the sigma is given as:

$$Y_{sigma} = \sqrt{\frac{\sum_{i=1}^{N} y_i^2 . z_i}{\sum_{i=1}^{N} z_i}} \qquad (11)$$

where

$z_i = t - norm\{\mu_U(u_i), w_i\}$

and w_i is the weight for the i^{th} element assigned by the context; and u_i is the i^{th} element of the input data, and μ_U is the input membership function. The determination of *t-norm* to be employed in the operation is determined on a per-case basis.

In those cases in which the resulting outputs are not symmetric, non-symmetric membership functions such as triangular or two-sided Gaussian should be used in place of Gaussian-type membership functions. However, this may involve more complex computations.

III. Numerical Example

In this section, a numerical example is presented to show the implementation of the proposed technique to solve an optimization problem. The example also shows the effects of the relaxation of context membership functions and the choice of fuzzy connectives on the fitness of the model. This section is composed of two main parts: the definition of the mathematical model, and the development and optimization of the contextual fuzzy model.

A. Mathematical Model

The mathematical model of the nonlinear system is defined as:

$$y = -u^2 + 10u + u(x_1 + 0.5x_2) : \; u, x_1, x_2, x_3 \in [0,10] \quad (12)$$

The system has three context variables x_1, x_2 and x_3 where x_3 is an irrelevant context variable. It is important to know that x_1 is more dominant than x_2 because the unit coefficient of x_1 is twice that of x_2 while both have the same range. It is also noted that the function is linear in the context variables. This means that, for a fixed input, the output is simply a linear combination of the context variables.

If x_1 and x_2 are assumed to be known, then the optimum value of u for maximizing the output y can be derived from (12) and it is given as:

$$u^{opt} = \frac{10 + x_1 + 0.5x_2}{2} \qquad (13)$$

For example, if $x_1 = 4$ and $x_2 = 2$, then the optimum value of $u = 7.5$ which gives a maximum output ($y =$ 56.25).

This model is actually a simplified version of a profit maximizing problem for generators participating in competitive electricity markets. The input variable u corresponds to the output level of a generator; the context variables x_1, x_2 and x_3 are the regional load demands; and the output y is the profit earned by the generator.

A regional load demand usually has a daily profile that can be estimated accurately using a normal probability density function based on its historical data. Individual generators offer their capacities in the form of supply functions. At any given trading period, the intersection between the aggregate supply function and the actual demand will determine the price of electricity for this trading period. An individual generator can maximize its profit by strategically adjusting its supply function based on the load forecast and the expected behaviors of its competitors [37]-[39].

A detailed description of the spot pricing mechanism in competitive electricity markets is described in

[40].

B. Contextual Fuzzy Model

Based on this mathematical model, sets of empirical data were generated for developing an equivalent empirical model. For each context variable (that is, x_1, x_2 and x_3) and the input variable u, there were 1000 data points that were uniformly distributed between 0 and 10. Given these sets of data, the output data y was calculated using (12). Consequently, there were 1000 sets of data (x_1, x_2, x_3 and y).

Each context membership function was defined to be of Gaussian-type with initial sigma 0.2. For example, if the context was defined by a linguistic value *Around-4*, then the membership function would have center of four and sigma of 0.2. If the context was given as:

"X_1 is *Around-4* AND X_2 is *Around-2* AND X_3 is *Around-7*"

then, the input-output relationship was revealed by the weighted input-output data points according to this context. It is noted that the weight assigned to each data point is proportional to the size of the circle in the scatter plots which are given in Fig. 5 and Fig. 6.

Fig. 5 shows the scatter plots of y versus u when the context membership functions were aggregated by using six different triangular norms: logical product, Hamacher product, algebraic product, Einstein product, bounded product and drastic product. The input-output relationship were not clearly defined in these plots, especially scatter plots (e) and (f), because there were not enough activated data points. It is shown that the approximated function is an interpolation of these weighted data points. The estimated output is a projection of the input, which is defined by a Gaussian-type membership function with a fixed sigma of 0.5, through the weighted data points. The center of the input membership function was paired with the center of the output membership function, then these data pairs were interpolated and plotted (indicated by a solid line) on the scatter plot along with the actual function (indicated by the dotted line).

For the given context, the corresponding actual function, which is based on the mathematical model (x_1 = 4 and x_2 = 2), is given as:

$$y = -u^2 + 15u \qquad (14)$$

The fitness of each pattern is indicated by the measured Root-Mean-Square (RMS) error between the actual and estimated outputs. The measured RMS errors and the revealed patterns in the scatter plots suggested that the first four t-norms were preferred to the last two. However, we could not determine which one of the four t-norms was the best choice since several runs gave inconclusive results among the four t-norms.

Relaxation of context membership functions will allow for more data points to be activated. The degree of relaxation depends on the calculated correlation coefficients between the context variable and the output.

The context membership functions were relaxed by using the relaxation coefficients tabulated in Table 1. For the context membership function X_k, which is of Gaussian-type, the relaxed sigma $\tilde{\sigma}_k$ is simply given as:

$$\tilde{\sigma}_k = \frac{\sigma_k}{\sqrt{\rho_k}} \qquad (15)$$

where σ_k is the initial sigma of the context membership function X_k, and ρ_k is the relaxation coefficient of the context membership function X_k. Therefore, the sigma of the context membership function X_1 was fixed while those of X_2 and X_3 were relaxed to 0.3561 and 3.6515 respectively.

The relaxed membership functions allowed for more data points to be activated; as a result, the patterns, which showed the input-output relationship, became more distinctive than the previous ones (for comparison, see Fig. 5 and Fig. 6). The calculated RMS errors were significantly reduced, especially for the case of bounded product. However, the drastic product failed to activate any data point in both cases due to its rigid degree of compensation; therefore, it is not recommended to be used for aggregating the context membership functions.

TABLE 1 CALCULATED COEFFICIENTS FOR THE CONTEXT VARIABLES (x_1, x_2 and x_3).

	Context Variables		
	x_1	x_2	x_3

Standard Correlation Coefficient (r_k)	0.451	0.2533	0.0246
Normalized Correlation Coefficient	1	0.5617	0.0545
Relaxation Coefficient (ρ_k) when β=2	1	0.3155	0.0030

In order to statistically conclude the best t-norm with its relaxation coefficient for this particular model, one hundred trials were conducted and the results are tabulated in Table 2.

The results show that logical and Hamacher product operators with the relaxation coefficient (β=2) gave the least RMS errors. Therefore, either one of these should be used as the aggregation operator for context membership functions.

TABLE 2
THE STATISTICAL RMS ERRORS (avg/std) FOR A SET OF TRIANGULAR-NORMS

Context Membership Function	Triangular Norms				
	logical	hamacher	algebraic	einstein	bounded
Initial	(3.44/0.86)	(3.38/0.82)	(3.96/0.86)	(4.14/0.88)	(12.71/6.75)
Relaxed $\beta = 1$	(1.83/0.39)	(1.82/0.37)	(2.01/0.42)	(2.12/0.45)	(3.62/2.18)
Relaxed $\beta = 2$	(1.68/0.37)	(1.67/0.38)	(1.71/0.40)	(1.74/0.41)	(2.01/0.55)
Relaxed $\beta = 3$	(2.09/0.69)	(2.03/0.64)	(1.87/0.51)	(1.84/0.46)	(1.81/0.35)
Relaxed $\beta = 4$	(3.04/1.13)	(2.89/1.04)	(2.50/0.88)	(2.38/0.81)	(2.05/0.60)

The output membership function is constructed from the output data filtered by the context membership function and the input membership function. The filtering process is illustrated in Fig. 7.

Fig. 7(a) shows the output data filtered by the aggregate context membership function, and Fig. 7(b) shows the output data filtered by the input membership function *Around-7*. It is noted that activated data has weight greater than zero, and the degree of significance depends on the assigned weight. The most significant data has a weight of one. The output data filtered by the context membership function and the output data filtered by the input membership function were combined by using the algebraic product operator. Then, the resulting output was a Gaussian-type membership function which was fitted to the filtered output data as shown in Fig. 7 (c). The output membership function had center of 55.67 and sigma of 5.03.

Theoretically, if x_1 = 4, x_2 = 2, and u = 7, then the output will be 56. In comparison, based on the developed fuzzy model, the input-output relationship can be described linguistically as follows: if X_1 is *Around-5*, X_2 is *Around-2* and the input variable is *Around-7*, then the expected output should be *Around-56.*

Optimization of the developed fuzzy model can be done using any standard optimization technique, such as Gradient-Descent or Newton methods. Fig. 8 illustrates the process of maximizing y for the given context. The input was initialized randomly (for example, *Around-4*), then it was iteratively optimized. The width of the input membership function varied corresponding to the density of data points. The optimum value was achieved when u was *Around-7.25* represented by the Gaussian membership function (center = 7.25 and sigma = 0.34). This result was close to the analytical solution (u = 7.5) which was calculated using the mathematical model for the case when $x_1 = 4$ and $x_2 = 2$.

IV. Conclusion

An empirical fuzzy model based on a context-based fuzzy filtering technique has been presented in this paper. Unlike conventional fuzzy systems that are based on fuzzy rules, the proposed contextual fuzzy system relies on sets of empirical data to describe the functional dependence between the inputs and the output. A simplified profit maximization problem for generator's bidding in competitive electricity market has been presented as a numerical optimization problem example. The application of the proposed technique to this problem indicates that it can optimize the supply function of a generator in which the contextual fuzzy model is developed using historical market data. That is, by utilizing historical load demand data and trading period as the context variables, generators can determine the optimal dispatch schedule that results in maximum profit. This proposed model has been used as the basic component in developing bidding strategies for generators participating in competitive electricity markets.

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