222/2011

Raport Badawczy
Research Report

RB/50/2011

Modeling variability of renewable energy sources by a semi-Markov process for power generation prediction

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Warszawa 2011

Abstract

This paper presents an attempt to model the variability of certain environmental factors, such as wind force or water flow rate, by a semi-Markov process with finite state space. The model is based on the following premises. First, the range of possible values of a given factor is divided into a finite number of subintervals. Second, it is assumed that the length of time during which the factor's value remains within one such interval, and the probabilities of transitions to neighboring intervals, depend on the factor's earlier behavior. The model's accuracy is determined by the number of subintervals and the assumed degree of the factor's dependence on its history (the number of previously entered subintervals relevant to predicting the factor's future behavior). According to the presupposed accuracy level the adequately complex state-space and the inter-state transitions diagram of the modeling process are constructed. Subsequently, it is demonstrated how certain parameters of that process, that can be used in power generation forecasting, can be calculated by means of the Laplace transform calculus.

1. Introduction

There exist a considerable number of models for predicting natural phenomena. Due to their nature, such phenomena can only be predicted with greater or smaller degree of uncertainty, hence their modeling is based on various mathematical ways of expressing non-deterministic, uncertain information. Admittedly, there are deterministic models used e.g. in weather forecasting, based mainly on systems of partial differential equations, but their implementation requires a substantial amount of computing power and memory space. In addition, they are only suitable for short term prediction – covering a period of up to one week. Therefore non-deterministic prediction offers a plausible alternative to the deterministic one, as being not much less accurate – as long as certain average values are calculated over medium or long time periods, and significantly less resource consuming. Traditionally, prediction of the former type is carried out with the use of probabilistic or statistical tools (cf. [2], [3], [5], [8], [9]) however methods employing fuzzy logic, theory of possibility, neural networks or wavelet theory are becoming increasingly popular in recent years (cf. [4], [6]). An overview of forecasting methodology is presented in Fig. 1.

This paper is concerned with the issue of medium or long term forecasting of the behavior of renewable energy sources, such as wind or flowing water, based on probabilistic modeling. The short-term forecasting is not addressed herein, as a plethora of well-established methods have been developed for this purpose (e.g. deterministic modeling, time series analysis). A novel stochastic model is presented to describe the variability of a given factor over time. Fluctuations of the factor's value are described by a stochastic process with a finite state space. In order to construct this space, the range of possible values is divided into a finite number of subintervals (e.g. corresponding to energy production levels) denoted $I_1,\dots,I_m$. It is assumed that the length of time during which the factor's value remains within one such interval, and the probabilities of transitions to neighboring intervals, depend on the factor's earlier behavior, i.e. on the sequence of intervals in which the factor's value had stayed before it entered the current interval. The model's accuracy is determined by the total number of intervals (m) and the assumed degree of the factor's dependence on its history (d) defined as the number of previously entered intervals relevant to predicting the factor's future behavior. A state of the process is determined by the current interval and d previously entered ones, thus it incorporates information essential to the future evolution of the factor's value. Further details are given in the next chapter.

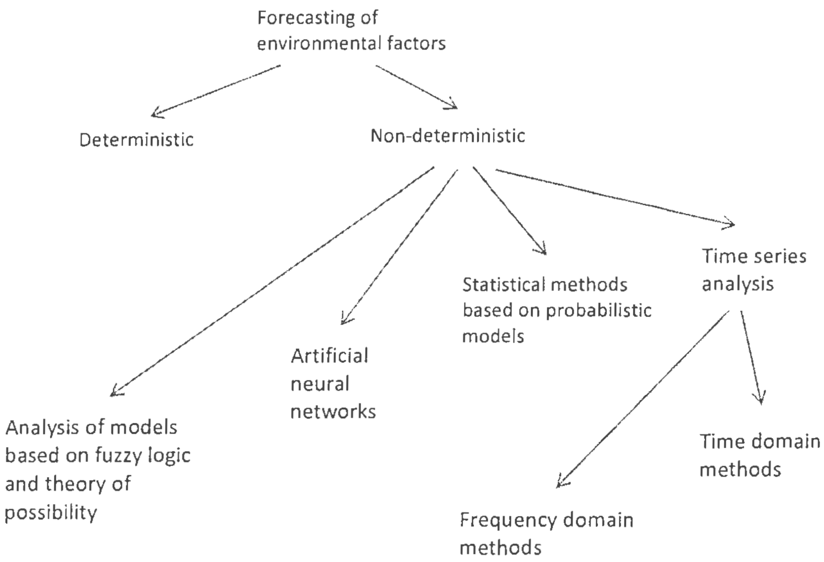

Fig. 1. An overview of forecasting methodology

2. The proposed model in detail

As it is assumed that the time during which the wind speed remains in an interval I_x and the number of the next interval (it can be either x–1 or x+1), depend on the order of some previously entered intervals, the wind speed changes can be modeled by a semi-Markov process with properly constructed state space $Z=\{z_1,\ldots,z_n\}$. To each interval I_x there correspond a number of states in Z, where such a state contains information about the intervals entered by the wind speed before it reached I_x. The number of previously entered intervals taken into account in constructing the state space Z is proportional to the model's degree of accuracy. In consequence, if we neglect the degenerate case "m=2", the cardinality of $\{z_1,\ldots,z_n\}$ exceeds that of $\{I_1,\ldots,I_m\}$, so that $n > m$.

If the first degree of dependence is assumed, the future wind speed values depend on whether the speed recently increased or decreased, i.e. whether the previous interval was I_{x-1} or I_{x+1}, I_x being the current interval. The state space of the modeling process is composed of $n = 2m - 2$ states denoted $z_1,\ldots,z_{2m-2}$. An even-numbered state is entered if the speed increases, and an odd-numbered one – if it decreases. Thus if I_{x-1} and I_x are the previous and current intervals, then the process is in the state z_{2x-2}; if the respective intervals are I_{x+1} and I_x, then the process is in the state z_{2x-1}. Fig. 2 represents the considered state space along with possible inter-state transitions for m=6.

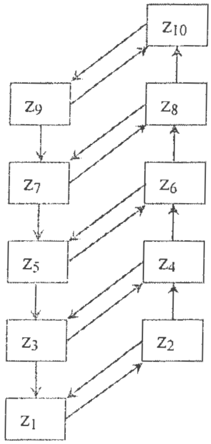

Fig. 2. The state space architecture for the first level of accuracy and m=6

If the second degree of dependence is adopted then future wind speed values depend not only on the recent, but also on the preceding speed change, i.e. the last two speed intervals are taken into account. The state space of the modeling process is now composed of 4m – 6 states which are divided into four groups: $G_1 = \{z_1, z_3, z_6, z_{10},\ldots, z_{4m-14}\}$, $G_2 = \{z_2, z_4, z_7, z_{11},\ldots, z_{4m-10}\}$, $G3 = \{z_5, z_8, z_{12},\ldots, z_{4m-9}, z_{4m-7}\}$, $G_4 = \{z_9, z_{13},\ldots, z_{4m-8}, z_{4m-6}\}$. The states in the first group are entered if the speed decreases twice; in the second – if the speed first increases and then decreases; in the third – if the speed first decreases and then increases; in the fourth – if the speed increases twice. Fig. 3 represents the considered state space along with possible inter-state transitions for m=6.

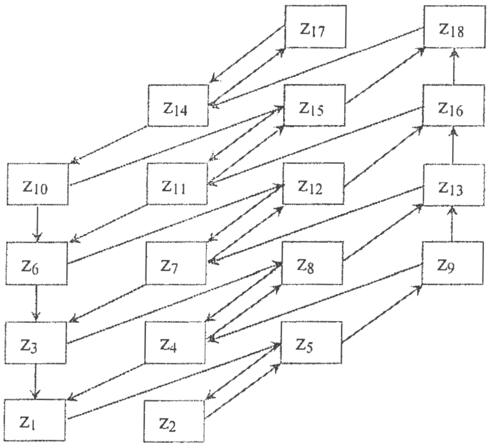

Fig. 3. The state space architecture for the second level of accuracy and m=6

Let $Z = \{Z_t, t\geq 0\}$ be the stochastic process with the state space $\{z_1,\ldots,z_n\}$, modeling the wind speed variability. As Z is assumed to be semi-Markov, the following objects have to be defined:

$X = \{X_k, k\geq 0\}$ – the embedded Markov chain of Z, i.e. X_k is the state of Z at the moment of its k-th state change (X_0 is the initially observed state of Z).

$P = [p_{ij}]_{i,j=1,\dots,n}$ – the transition matrix of X, i.e. $p_{ij} = \Pr(X_k = z_j \mid X_{k-1} = z_i)$. It is assumed that P is the same for every k, i.e. X is homogenous.

T_k – the moment of the k-th state change of Z

S_{ij} – the time spent by Z in the state z_i provided that the next state is z_j

F_{ij} – the distribution function of S_{ij}, i.e.

(1) $$F_{ij}(t) = \Pr(T_k - T_{k-1} \leq t \mid X_k = z_j,\, X_{k-1} = z_i)$$

P and F_{ij} can be obtained from the statistical analysis of wind speed records. For example, in the case of Z with the state space shown in Fig. 1 we have:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{21} & 0 & 0 & p_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{43} & 0 & 0 & p_{46} & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{53} & 0 & 0 & p_{56} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{65} & 0 & 0 & p_{68} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{75} & 0 & 0 & p_{78} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{87} & 0 & 0 & p_{8,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{97} & 0 & 0 & p_{9,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

where $p_{ii}=0$ and $p_{i1} + \dots + p_{in} = 1$ for $i=1,\dots,n$ – an obvious condition for a transition matrix.

Remark: In general case the probability of transition from z_i to z_j depends on the amount of time spent in z_i, thus

$$(2) \qquad \Pr(X_k = z_j \,|\, X_{k-1} = z_i, T_k - T_{k-1} \le t) \neq \Pr(X_k = z_j \,|\, X_{k-1} = z_i, T_k - T_{k-1} \le u)$$

may hold for $u \neq t$. The precise definition of p_{ij} should therefore be as follows

$$(3) \qquad p_{ij} = \Pr(X_k = z_j \,|\, X_{k-1} = z_i, T_k - T_{k-1} < \infty)$$

to underscore the fact that p_{ij} is the probability of transition from z_i to z_j if no information about the sojourn time in z_i is available. However, the event $\{T_k - T_{k-1} < \infty\}$ is irrelevant to p_{ij} as expressed by (3), because $\Pr(T_k - T_{k-1} < \infty) = 1$. It should be noted that this remark pertains to all semi-Markov processes.

3. Equations for parameters characterizing the wind turbine power output

The model constructed in the previous section is useful for determining many parameters that characterize the wind power production process, especially for predicting their future values. For example, it is possible to forecast the total expected energy output during a given time period, the probability that during that period the wind speed will remain within certain limits, or the expected number of times it will cross these limits. It will now be shown in detail how to determine the first parameter. Let π_i be the power generated when Z is in the state z_i, i.e. when the wind speed remains in the respective interval. Let $G_i(u,t)$ be the expected value of the energy produced in the time interval [u,t], provided that at the moment u the process Z enters the state z_i. It can be easily shown that $G_i(0,t)$, $i=1,\ldots,n$ satisfy the following set of equations:

$$(4)\qquad G_i(0,t) = \pi_i t \sum_{j\neq i} p_{ij}[1 - F_{ij}(t)] + \sum_{j\neq i} p_{ij} \int_0^t [\pi_i u + G_j(u,t)] dF_{ij}(u)$$

Indeed, taking into account that

$$(5)\qquad p_{ij}F_{ij}(t) = \frac{\Pr(X_k=z_j, X_{k-1}=z_i)}{\Pr(X_{k-1}=z_i)} \frac{\Pr(T_k-T_{k-1}\leq t, X_k=z_j, X_{k-1}=z_i)}{\Pr(X_k=z_j, X_{k-1}=z_i)} =$$

$$= \Pr(T_k - T_{k-1} \leq t,\ X_k = z_j \mid X_{k-1} = z_i)$$

the first component on the right-hand side of (4) is related to the energy produced in the [0,t] time interval, provided that no state change occurred from 0 to t. In turn, the second component is related to the energy produced in that interval, provided that the first state change occurs at $u \leq t$.

Being a semi-Markov process, Z "forgets" its history at each state change, so that $G_i(s,t) = G_i(0,t-s)$, $i=1,\ldots,n$. In consequence (4) transforms to:

$$(6)\qquad G_i(0,t) = \pi_i \sum_{j\neq i} p_{ij}[t(1 - F_{ij}(t)) + \int_0^t u dF_{ij}(u)] + \sum_{j\neq i} p_{ij} \int_0^t G_j(0, t-u) dF_{ij}(u)$$

Note that

$$(7)\qquad t[1 - F_{ij}(t)] + \int_0^t u dF_{ij}(u) = E[\min(S_{ij}, t)]$$

where E denotes the expected value, hence (7) can be written in the following compact form:

(8) $G_i(0,t) = \pi_i \sum_{j \neq i} p_{ij} H_{ij}(t) + \sum_{j \neq i} p_{ij} \int_0^t G_j(0, t-u) dF_{ij}(u)$

where

(9) $H_{ij}(t) = E\left[\min\left(S_{ij}, t\right)\right]$

As (8) is a system of integral equations, the transform calculus can be used to find its solution. Let

(10) $\Gamma_i(s) = \mathcal{L}\{G_i(0,t)\},\ \Phi_{ij}(s) = \mathcal{L}\{f_{ij}(t)\} = \mathcal{L}^*\{F_{ij}(t)\},\ \Psi_{ij}(s) = \mathcal{L}\{H_{ij}(t)\}$

where $\mathcal{L}$ and $\mathcal{L}^*$ denote Laplace and Laplace-Stieltjes transforms respectively, and f_{ij} is the probability density function of S_{ij}. From the basic properties of the Laplace transform it follows that

(11) $\Psi_{ij}(s) = [1 - \Phi_{ij}(s)]/s^2$

Applying $\mathcal{L}$ to both sides of (8) we obtain:

(12) $\Gamma_i(s) = \pi_i \sum_{j \neq i} p_{ij} \Psi_{ij}(s) + \sum_{j \neq i} p_{ij} \Phi_{ij}(s) \Gamma_j(s)$

Note that the integral in (8) is the convolution of G_j and f_{ij}, which, after applying Laplace transform, becomes the product of Γ_j and Φ_{ij}. As (12) is a system of linear algebraic equations, it can be presented in the following matrix form:

$$(13)\qquad A(s)\begin{bmatrix}\Gamma_1(s)\\ \vdots \\ \Gamma_n(s)\end{bmatrix}=\begin{bmatrix}\pi_1\sum_{j\neq 1}p_{1j}\Psi_{1j}(s)\\ \vdots \\ \pi_n\sum_{j\neq n}p_{nj}\Psi_{nj}(s)\end{bmatrix}$$

where the elements of A(s) are given by:

$$(14)\qquad a_{ij}(s)=\delta_{ij}-p_{ij}\Phi_{ij}(s)$$

4. Determining the sought parameter from the obtained equations

A closed-form solution of (13) should express Γ_i, i=1,...,n in terms of π_i, p_{ij}, Φ_{ij}, and Ψ_{ij}, i, j=1,...,n. However, it is practically impossible to find such a solution in general. For example, in case of the system with the state space shown in Fig. 1 we have the following matrix A(s):

$$\begin{bmatrix}
1 & -\Phi_{12}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-p_{21}\Phi_{21}(s) & 1 & 0 & -p_{24}\Phi_{24}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
-p_{31}\Phi_{31}(s) & 0 & 1 & -p_{34}\Phi_{34}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -p_{43}\Phi_{43}(s) & 1 & 0 & -p_{46}\Phi_{46}(s) & 0 & 0 & 0 & 0\\
0 & 0 & -p_{53}\Phi_{53}(s) & 0 & 1 & -p_{56}\Phi_{56}(s) & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -p_{65}\Phi_{65}(s) & 1 & 0 & -p_{68}\Phi_{68}(s) & 0 & 0\\
0 & 0 & 0 & 0 & -p_{75}\Phi_{75}(s) & 0 & 1 & -p_{78}\Phi_{78}(s) & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -p_{87}\Phi_{87}(s) & 1 & 0 & -p_{8,10}\Phi_{8,10}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & -p_{97}\Phi_{97}(s) & 0 & 1 & -p_{9,10}\Phi_{9,10}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Phi_{10,9}(s) & 1
\end{bmatrix}$$

As A is a sparse matrix (in this case a tri-diagonal one), the equation (13) can be solved by Gauss elimination (GE) with lower than maximal complexity which is $O(n^3)$. However, even applying the Thomas algorithm (a simplified form of GE especially designed for tri-diagonal matrices) to the above given A, results in very complex closed formulas which for greater n are practically impossible to derive.

As follows from the above considerations, (13) has to be solved numerically rather than analytically. For example, it is possible to find $\Gamma_i(s)$ for a number of discrete values of s along a vertical line in the complex space, thus obtaining data which allow to compute $G_i(0,t)$ by numerical integration using the following well-known formula for the reverse Laplace transform:

$$(15) \quad G_i(0,t) = \frac{1}{2\pi i} \lim_{y\to\infty} \int_{x-iy}^{x+iy} e^{st}\Gamma_i(s)ds$$

Here, x is the point where the vertical line crosses the real axis.

Other parameters, e.g. the probability that during a given time period the wind speed will remain within certain limits, or the expected number of times it will cross these limits, can be found in a way similar to that described in the last two chapters.

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